Strengthening Discu

ssions

Teachers can foster strategy sharing by attending to the cognitive demands that students experience while talking, listening, and making mistakes.

By Allison B. Hintz

Strategy sharing is a certain type of discussion that centers on students' ideas and occurs when children present different approaches to problems and provide information about how they solved the problem (Wood, Williams, and McNeal 2004). A teacher may orchestrate a strategy-sharing discussion to achieve one or more of the following goals (NCTM 2000):

- Elicit students' mathematical ideas
- Represent students' diverse strategies
- Make connections between mathematical ideas
- Compare similarities and differences across strategies
- Develop students' repertoire of strategies and flexible thinking

During my years of teaching and research in elementary school classrooms, I have focused on understanding and supporting students' experiences during strategy-sharing discussions. Building on the work of other researchers, I wanted to understand how to cultivate a classroom atmosphere that encourages problem solving and inquiry (e.g., Yackel and Cobb 1996) and that fosters mathematical discourse where students' problem-solving strategies, not the answers, are the foundation of the discussion (e.g., Lampert 1990). I also wanted to understand how strategy sharing could be mathematically productive and socially supportive for students.

In this article, I draw on a larger study in which I spent a year observing and interviewing first-grade and fourth-grade mathematics students who regularly enacted strategy-sharing discussions (Hintz 2010). Data were collected at three points across an academic year (fall, winter, and spring) for a week each trimester. Data included videotaped lessons and individual interviews with focal students from each class. During the interviews, these students narrated their experiences, describing general aspects (e.g., what typically happens during strategy-sharing lessons in their classroom) as well as specific occurrences (e.g., what the student was thinking about as he or she listened to a classmate share a particular strategy). Several analytic passes were conducted for lessons and interviews, to identify and describe the mathematical and social demands that students experience during strategy sharing. I learned that particular components of strategy sharing place noteworthy demands on students, specifically, discussing mathematical mistakes, sharing strategies, and listening to classmates' solutions.

Discussing math mistakes

During strategy-sharing lessons, students are expected to engage in discussion about their own (and their classmates) mathematical mistakes. Discussing incorrect answers plays an essential role in mathematical discussions. As Staples and Colonis (2007, p. 259) describe,

> If only "correct" ideas regularly receive attention, the mathematics that gets explored is limited, and the students whose original ideas were incorrect may hold on to incorrect mathematics.

Furthermore, investigating errors provides a unique opportunity to examine mathematical principles, and children can collectively work toward a sound solution. However, sharing incorrect answers publicly, especially in the discipline of mathematics—where attention to precision is important—can be difficult (Staples 2008). The following vignette, which portrays the typical nature of the discussions documented in my study, offers an opportunity to drop in on an episode that entailed a mathematical mistake.

First-grade students in Ms. K's class were sitting on the carpet for a warm-up session. Ms. K had chosen to pose the problems 5 + 5, then 4 + 5 to highlight the idea of using what you know about one problem to solve a new problem. As the first sharer approached the easel, Ms. K reminded her, "When you explain how you thought about 5 + 5, please turn to face your classmates and talk loudly so everyone can hear your thinking. When you tell us about your strategy, tell us how you got your answer and why you chose to solve it in that way. I might stop you to ask questions to make sure we understand your ideas." Then, turning to the class, Ms. K said, "When your classmate shares her thinking, your job is to face her and listen carefully to her ideas. I want you to think about each step she took to solve in order to understand how she thought about this problem."

After a few students shared their solutions for 5 + 5, Ms. K chose to highlight the idea of knowing doubles facts and moved on to posing 4 + 5. The first sharer, Macy,

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said her answer was 7, explaining, "Instead of starting with 5 + 5, I started by using what I know about the doubles fact 4 + 4, which equals eight, and then I took one away."

"Interesting," replied Ms. K. Continuing, she asked, "Macy, can you say more about why you took one away?"

"The problem I want to solve is 4 + 5, but I did 4 + 4, and then since 4 + 5 is one away from 4 + 4, I had to take one away," Macy described.

"I see," said Ms. K. Turning to the class, she continued, "Smart mathematicians are people who begin by using what they know to solve new problems, and that is what Macy did. She used a doubles fact she knows, 4 + 4, to think about a new problem, 4 + 5. Then, since she knew she had changed the numbers in the problem, she knew she had to change her answer. Macy, I think I can see just what you did. Can you tell us again how you changed the numbers?"

"I made the 5 a 4, and so I had to [pausing] wait, um, I think I did something wrong," Macy looked puzzled.

Ms. K inserted, "If you want to change your answer, that is OK. In our classroom, you say, 'I'd like to revise my thinking,' and you tell us how your ideas are changing. Revising your thinking is something mathematicians do."

"Yes, I got confused," said Macy. "It is not 7; it is 9. I was supposed to add on one more, not take it away."

"I see," said Ms. K. Turning to the class, she continued, "Who can repeat what Macy just said?"

The discussion continued by investigating Macy's thinking about adding one versus taking one away and using her ideas to examine important mathematical ideas of addition. Ms. K concluded the discussion by thanking Macy for her contribution and persistence and then highlighting the mathematical learning that happened as a result of Macy's sharing.

What is important to consider in the vignette is the way Ms. K framed Macy's mistake as a "desirable contribution" (Staples 2008). Framing mistakes as such entails the following:

- Looking for the logic within the error (e.g., "Since [Macy] knew she had changed the numbers in the problem, she knew she had to change her answer.")
- Positioning the contribution as competent (e.g., "Smart mathematicians are people who begin by using what they know to solve new problems.")
- Teaching students what to say when they make a mistake (e.g., "In our classroom, you say, 'I'd like to revise my thinking.'")

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Consider also the way that Ms. K is cultivating a classroom culture where mistakes are valued as a productive part of mathematical learning and where students know they will be supported in making sense of, and persevering with, problems. Even within the same classroom, students experience mistakes differently, which is no surprise, because students are unique individuals with their own history of learning experiences. Drawing from children's narrations across the varied group of students that I studied, I learned that making mistakes feels acceptable for some

children but challenges others. First grader Salvador said, "If we mess up, we can just try it again. You say, 'Oh, no; it is not that number' and do it again, and it's OK," and Macy added, "You just say, 'I want to revise my thinking.'"

Similarly, Jamal (a fourth grader) explained, "I can fix it quickly. I'm good at working through mistakes."

For these students, who know what to do and say when an error occurs, mistakes seemed to have little effect. For other students, such as fourth graders Norah and Kole, mistakes were "embarrassing," especially if they had to work through an error publicly, on the spot. Norah described, "I don't like to make mistakes, because it's embarrassing when you thought you got it right but you got it wrong and you have to keep working out loud."

Kole worried about embarrassing himself by "messing up" publicly.

Sharing strategies

When a student is the *sharer*, or speaker, during strategy sharing, she may use words, gestures, written numbers, and/or mathematical representations (such as a number line or array) to communicate her thinking. Sharers are expected to make their thinking public as they explain their strategy aloud and offer their ideas for discussion. They must respond to questions (most commonly from the teacher) about their thinking and the steps in their strategy.

Students' descriptions of their sharing experiences revealed insights into why students may (or may not) share, as well as how participation through sharing can change over time. Children choose to share (or not) for a number of reasons. For some children, sharing their ideas is something they like to do. Jamal said, "I like math a lot, and it makes me want to explain to other people what I'm thinking."

Similarly, Salvador explained, "My classmates really want to learn about my thinking."

From these two students, we can hear that some children like to share. However, a student who likes to share may chose not to share. For example, Salvador does not share if he feels that his idea is no different from the ideas already being discussed, "I was going to say 4 + 4 = 8, but I couldn't say it again because Macy already said it."

For other children, affective dimensions of sharing (such as nervousness) might make it hard for them to share their ideas publicly. Norah explained, "Sharing my strategy, unless it is really easy, like a fact, makes me nervous because there are people listening."

Similarly, first grader Andy got "stage fright" when he shared his thinking. Although he found the math to be "easy-peasy," sharing his ideas

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in front of his classmates was "one of the last things" Andy chose to do. Students like Norah and Andy help us know that some students may avoid sharing to mitigate the risks of making a mistake in front of others.

Students also described ways that their sharing changed over time. For example, Macy, who was new at her school at the beginning of the academic year, at first did not share often. She explained, "I was new, and I wasn't used to this. Once I got used to this, I wasn't nervous. I just listened to everyone else and thought it was OK, and then I started sharing." As Macy learned what was normative about sharing in her classroom, her sharing increased.

These student reflections point to important implications for teaching practices that support children in the role of sharer. In particular, as Macy's comments highlight, supporting students in learning *how* and *what* to share is crucial. To support a student in knowing *how* to share, consider phrases such as the ones Ms. K used (e.g., "When you explain . . . , please turn to face your classmates, and talk loudly so everyone can hear your thinking"). To support students in knowing *what* to share, a teacher might remind children what to include in their explanation (e.g., "When you tell us how you solved, tell us how you got your answer and why you chose to solve in that way").

Listening to peer solutions

Only one student speaks at a time during strategy sharing, so at any given moment, most students are taking on the role of listener. In mathematical discussions, listening is important. As Cazden (2001, p. 89) wrote,

> Students have to listen to and learn from each other as well as the teacher. That's the only way for them to learn during the time spent solving problems in a group.

Wood (1999) also placed high priority on a student's role as a listener and believed that listening enables students to follow the mathematical thinking and reasoning being discussed.

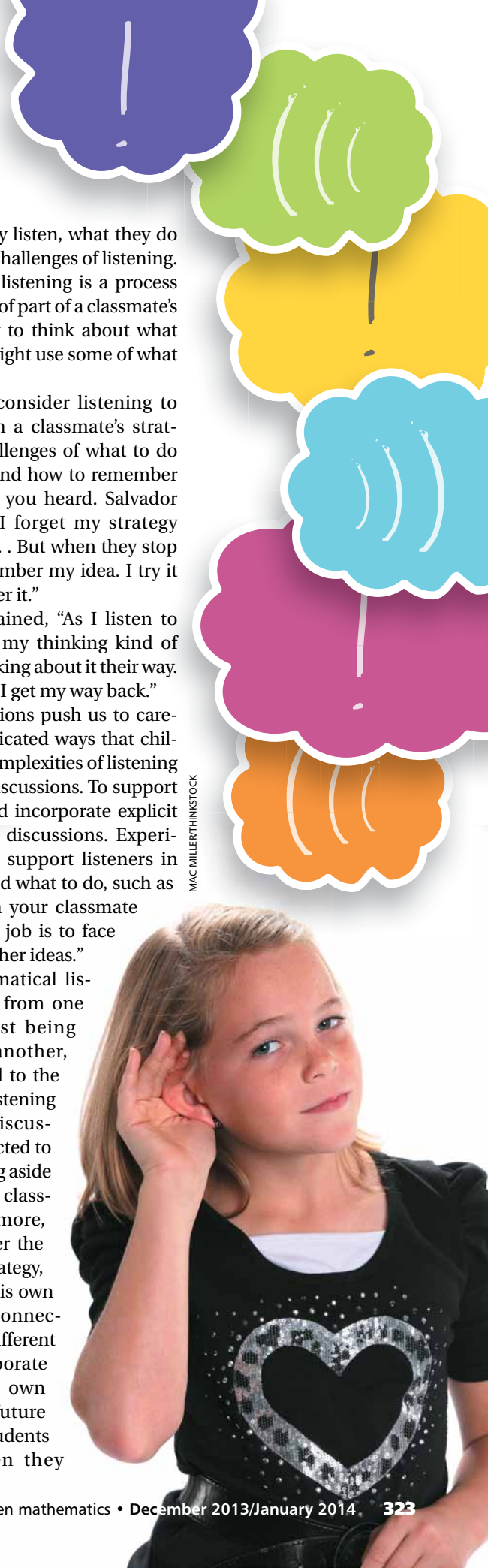
Listening is a difficult dimension of classroom discussion to study because little observable behavior occurs. Therefore, carefully considering students' descriptions of their listening experiences is essential. Students described differing reasons why they listen, what they do with what they hear, and challenges of listening. For a student like Jamal, listening is a process that entails consideration of part of a classmate's idea. He explained, "I try to think about what they're explaining . . . . I might use some of what they said."

Salvador and Norah consider listening to be about thinking within a classmate's strategy; with that comes challenges of what to do with your own thinking and how to remember the steps of the strategy you heard. Salvador described, "Sometimes I forget my strategy because I have to hear . . . . But when they stop sharing their idea, I remember my idea. I try it their way if I can remember it."

Likewise, Norah explained, "As I listen to other peoples' thinking, my thinking kind of goes away, and I start thinking about it their way. Then, if this isn't working, I get my way back."

These students' reflections push us to carefully consider the sophisticated ways that children must navigate the complexities of listening during strategy-sharing discussions. To support listeners, a teacher should incorporate explicit talk about listening into discussions. Experiment with prompts that support listeners in knowing where to look and what to do, such as when Ms. K said, "When your classmate shares her thinking, your job is to face her and listen carefully to her ideas."

Also, because mathematical listening is about learning from one another's ideas, not just being good listeners of one another, support must also extend to the specific expectations of listening during mathematical discussions: The listener is expected to briefly set his own thinking aside so he can understand a classmate's strategy. Furthermore, he is expected to consider the logic within the shared strategy, compare the strategy to his own strategy while making connections between the two different solutions, and/or incorporate the heard idea into his own thinking when solving future problems. As the focal students helped us realize when they



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described toggling between their own ideas and their classmates' ideas, young mathematicians practice a particular type of listening that requires unique skills. Certain questions and prompts can foster these mathematical listening skills. For example, when Ms. K asked, "Who can repeat what Macy just said?" she communicated to students that they are expected to listen to their classmate's strategy and be able to describe it. Possible follow-up questions Ms. K could use include these:

- "How is your solution different from Macy's?"
- "What is the same about the way you approached the problem and the way Macy approached the problem?"
- "Why do both of these solutions work for the same problem?"
- "I am going to give you a new problem, and I want you to use Macy's strategy when you solve it."

Making implicit discussion facets explicit

Productive mathematical discussions, such as strategy sharing, help students develop understanding and fluency with mathematical concepts and procedures (Chapin, O'Connor, and Anderson 2009). Strategy-sharing discussions are a common and important part of learning in mathematics classrooms, and thinking about how participating in discussions can place demands on students is essential. Certain demands, such as making and articulating mistakes, sharing ideas, and listening, can be noteworthy for children. By using the ideas offered in this article, teachers can make the too-often implicit facets of discussion explicit as they teach children how to participate in and learn from strategy sharing.

Common Core Connection

SMP 3

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