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this emerging
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to attend to
children's
actions, make
interpretations,
and respond
with robust
instruction.



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Professional **NOTICING:** Developing Responsive Mathematics Teaching

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Thoughtful implementation of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) presents an opportunity for increased emphasis on the development of mathematical understanding among students. Granted, ascertaining the mathematical understanding of an individual student is highly complex work and often exceedingly difficult. Although textbooks may provide practitioners with considerable overarching instructional guidance, to complete the picture, mathematics teachers must often focus on individual children. In some instances, we might consider verbal explanations or work samples to gain insight into one's thinking; however, quite often these avenues do not provide a complete or accurate portrayal of a student's understanding. Indeed, a student may unwittingly offer an explanation that differs from her actual strategy (CCSSI 2010). Similarly, work samples may feature deceptive or insufficient details to truly gauge the student's thinking. In these instances, a systematic approach to appraise the mathematical moment is required to fully appreciate the student's true understanding and then respond with effective instructional tactics.



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The process of professional noticing is predicated on solidly linking the practices of attending, interpreting, and deciding.

Following a framework

Professional Noticing of Children's Mathematical Thinking (Jacobs, Lamb, and Philipp 2010) provides a structure for teachers to better understand and act on their students' mathematical conceptions and practices. Building on the noticing pedagogies of van Es and Sherin (2008), Jacobs, Lamb, and Philipp characterize responsive teaching (professional noticing) as a progression through three interrelated phases:

1. Attending
2. Interpreting
3. Deciding

Attending involves noting aspects of a mathematical moment as a way to gather meaningful evidence. This might include a child's body language, how he manipulates a tool, or the presence of excessive background noise. The list of possible items to which a teacher might attend may be exhaustive; however, the goal is to attend closely to actions most significant to the mathematical learning at hand—for example, how a child uses her fingers, changes inflection during counting, or forms a numeral when writing.

Interpreting involves coordinating the observed actions (attending) with what is known about mathematical development in a particular area. Jacobs, Lamb, and Philipp (2010, pp. 172–73) wrote,

On the basis of a single problem, we do not expect a teacher to construct a complete picture of the child's understandings, [but rather] are interested in the extent to which the teacher's reasoning is consistent with both the details of the specific child's strategies and the research on children's mathematical development.

The key to meaningful interpretation is making a strong connection to the evidence gathered while attending.

Deciding refers to conceiving (and executing) an effective tactic drawn from the interpretation of a child's mathematical thinking.

We are not arguing that there is a single best response, but we are interested in the extent to which teachers use what they have learned about the children's understandings from the specific situation and whether their

reasoning is consistent with the research on children's mathematical development. (Jacobs, Lamb, and Philipp 2010, p. 173)

Here, Jacobs, Lamb, and Philipp are interested in the extent to which decisions connect interpretations (which are direct results of the attending process). As these authors note, decisions may come in many forms. Some may be more diagnostic in nature. For example, a teacher may *attend* to a child who works on the task $34 + 9$, pauses for some time, and then sequentially raises nine fingers to solve the task. From this, the teacher *interprets* that the child is enacting a counting-on strategy (Steffe 1992; Wright, Martland, and Stafford 2006). On the basis of this interpretation, the teacher *decides* to ask the child where she began her count so that he may gather more information about whether the child was, indeed, counting on or perhaps had to begin counting at one (during the pause) and simply continued the count using her fingers.

Alternatively, the level of evidence gathered while attending may be sufficient for an instructional decision. In the example above, perhaps the teacher observes the child consistently counting on when dealing with arithmetic tasks. Thus, the teacher might decide to pose similar tasks or slightly more sophisticated arithmetic tasks (e.g., $34 + 14$) and emphasize models or tools to support the development of non-count-by-ones strategies, sometimes referred to as “composite” strategies (Thomas and Tabor 2012; Wright, Martland, and Stafford 2006). For example, the teacher might introduce an empty number line and invite the child to use this tool to model $34 + 14$ by making an initial jump of 10. In either event, the process of professional noticing is predicated on solidly linking the practices of attending, interpreting, and deciding.

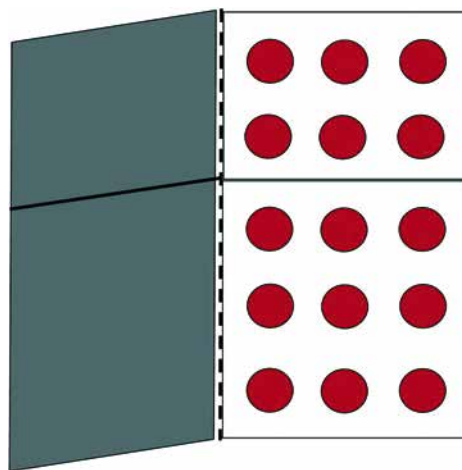
Putting it into practice

In a second-grade classroom, a teacher working to develop foundations for multiplication has designed an instructional experience aimed at supporting CCSSM Operations and Algebraic Thinking Standard 2.OA.4 (CCSSI 2010). This standard specifies that children will—

use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns [and] write

FIGURE 1

One student determines the dot configuration on the 3×5 array tool, then closes both “doors” to conceal it, places the tool on the table, and describes the dot pattern to his or her partner.



an equation to express the total as a sum of equal addends. (p. 19)

In the classroom, children are working in pairs to determine how many dots are on screened array tools of differing configurations (see fig. 1). One student is tasked with examining the array to determine the configuration of dots, being careful to conceal the configuration from his or her partner. This student then closes both “doors” to conceal the dot configuration, places the tool on the table, and describes the dot pattern to the partner (i.e., “There are five rows of three dots”). The partner is tasked with determining the numerosity of the array pattern and writing an equation featuring repeated addends (i.e., $3 + 3 + 3 + 3 + 3 = 15$). If the partner needs a hint, the first student can open the smaller “door” to reveal a portion of the array. This task presentation offers considerable flexibility in that, ideally, the array remains concealed for much of the task, prompting students to enact solution strategies that are more abstract; however, the possibility to reveal portions of the array (as well as the entire array, if necessary) allows students to engage with the materials in varying ways—and for teachers to professionally notice these varying strategies.

It is important to understand that, for some children, accurate interpretations will require looking beyond the current grade level.

As the teacher moves about the classroom observing each pair of students, he notices Julie staring blankly at a 3×5 array tool and that the smaller “door” is open on the tool, leaving two rows exposed. He then moves between Julie and her partner. Consider the following exchange:

Teacher: How many rows are there? [*He motions across the two exposed rows, each containing three dots.*]

Julie: Two rows of three [*confidently*].

Teacher: Absolutely. So, what if I told you there were three more rows hidden on this card? [*He motions down the card—over the top of the concealed portion of the array tool.*] How many dots would there be on the card altogether? [*He closes the top panel on the array tool, concealing the original two rows.*]

Julie: Six [*touching the card in the approximate location of the third row of dots and continuing to touch the card in a linear pattern*], seven, eight, nine, ten. Can I look at the top part again?

Teacher: Sure [*opening the top panel to reveal two rows of dots*].

Julie: [*Touching each of the six exposed dots*] One, two, three, four, five, six.

Teacher: [*Closing the top panel of the array tool*]

Julie: Seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen [*touching the card in a linear pattern in the approximate location of the rows of dots*].

Julie: Fifteen [*confidently*].

Teacher: How did you know that?

Julie: Because there were three rows hidden, and another two rows [*pausing*], so I counted by threes [*pausing*]. Yeah, I counted by threes, and there were fifteen altogether.

Before reading on, stop and take a few moments to attend to the exchange above. What do you find noteworthy in Julie's words or actions?

Attending

Recall that the aim of attending is to identify aspects of the exchange that are mathematically salient. Perhaps you identified some of the following features in Julie's exchange with her teacher:

- Julie confidently identified the exposed top portion of the array tool as “two rows of three” dots.
- Julie then touched the array tool in the approximate location of each individual dot and counted these dots by ones.
- At one point, Julie attempted to “assign” four dots to a row but stopped herself and asked to see the top portion of the array tool again.
- Julie ultimately negotiated the posed task successfully, arriving at a response of fifteen.
- Julie explained her strategy as “counting by threes.”

Although additional, salient mathematical features may be in this exchange, the evidence outlined above provides sufficient information to continue the process of professional noticing.

Interpreting

Having attended to several salient mathematical aspects of the exchange, we must now coordinate these aspects with what we know about developing mathematical knowledge.

Before reading on, consider Julie's words and/or activities to which you attended, and try to formulate an interpretation of her mathematical understanding. What do you think she understands about multiplication and arrays? What types of strategies does Julie use when attempting multiplicative tasks?

Certainly, any number of progressions and frameworks related to multiplication might inform this process (Griffin 2004; Mulligan and Mitchelmore 1997; Wright, Ellemor-Collins, and Tabor 2012). However, in this instance, the research-based progressions of CCSSM afford a practical and sufficiently complex backdrop for us to consider Julie's mathematical thinking (Confrey et al. 2012). Here, Julie enacted an additive strategy to determine the numerosity of the array pattern; however, her strategy was

apparently unitary (count-by-ones) in nature rather than composite (group based). That is, Julie focused on single units, and did not capitalize on the inherent groups within the array. Although Julie is in the second grade, we may have to look to prior grade levels along the CCSSM progression to locate her mathematical strategies and thinking. It is important to understand that, for some children, accurate interpretations will require looking beyond the current grade level.

Returning to the evidence gathered during the attending phase, we note that Julie appeared to touch the array tool in the supposed location of each concealed dot to arrive at her response of fifteen. In addition to being a unitary (count-by-ones) strategy, the manner in which she touched the cover in the approximate location of the concealed dots (referred to as *a motor representation*) is strongly associated with mental images (Steffe 1992; Thomas and Tabor 2012); thus, Julie's thinking may be found in the Kindergarten Operations and Algebraic Thinking (OA) portion of CCSSM. Specifically, Standard K.OA.1 seems applicable:

Represent addition and subtraction with objects, fingers, mental images [emphasis added], drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (CCSSI 2010, p. 11)

Interestingly, Julie purports to have “counted by threes.” However, close attention to her strategy reveals that this was not the case. This illustrates the need for mathematics teachers to base interpretations on not only verbal explanations but also the varied nonverbal cues that often manifest during students' mathematical work.

Deciding

Having attended and interpreted, we address the next phase of the professional noticing framework, which deals with thoughtful decision making, either diagnostic or instructional. Diagnostically, the teacher may elect to pose similar tasks involving arrays to confirm the child's unitary (count-by-ones) strategies. Let us suppose, though, that the teacher observed the child repeatedly count by ones when dealing with arrays. What, then, might constitute a thoughtful instructional decision?

FIGURE 2

Revealing some of or the entire array allows students to engage with this flexible task in various ways while teachers professionally notice these strategies.



Before reading on, consider the interpretation of Julie's mathematical thinking, and try to envision a specific mathematical task that would advance her thinking. What could you do to help Julie develop arithmetic strategies that are more sophisticated? What tasks might help move Julie's understanding of quantity from unitary (count by ones) to composite (grouping)?

Of course, there are likely many instructional decisions that would help Julie advance her thinking in this area. One decision might be to focus on strengthening Julie's verbal aspect of number (Thomas, Tabor, and Wright 2010/2011) and engage her in some brief skip-counting practice (by threes in this instance). Given the teacher's apparent aim to develop quantitative foundations for multiplication (i.e., composite understanding of quantity), however, we might opt for an instructional decision involving actual quantities with a particular emphasis on composite units. Here, one may follow progressions embedded within CCSSM to help plan instructional decisions. There is likely more work to be done in the Kindergarten Operations and Algebraic Thinking (OA) Standards. Specifically, tasks associated with Standard K.OA.3, which emphasizes partitions to ten, would likely provide useful instructional experiences and help Julie move away from a unitary conception of quantity. Once Julie develops facility with the kindergarten standard, then her teacher might follow portions of the OA trajectory into first grade. Here, emphasis on 1.OA.1 and 1.OA.2 would be applicable, as these standards further develop group-based strategies in the context of addition and subtraction as well as provide a basis to reconnect, ultimately, with the original, partially screened array task (CCSSM 2.OA.4) (see fig. 2).

Having decided on a productive pathway to develop composite quantity and support Julie's foundations for multiplication, her teacher

The CCSSM progression and tasks in Standard 1.OA.6 move from unitary to composite conceptions of quantity.

TARGET

Work with equal groups of objects to gain foundations for multiplication.

2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

MATHEMATICAL INSTRUCTION AND DEVELOPMENT

Represent and solve problems involving addition and subtraction.

1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawing, and equations with a symbol for the unknown number to represent the problem).

Possible task

Gather three novelty insects (or pictures of individual insects) and have the student place them in a styrofoam cup—this is the “colony”—and place a lid on the cup. Ask the student how many **beetle wings** are in the colony. Ask the student how many **beetle legs** are in the colony. Play similar games with other animals (penguins, horses, etc.) that are placed in hidden spaces (igloos, stables, etc.).

Represent and solve problems involving addition and subtraction.

1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).

Possible task

1. Present the student with a drawing of a large picnic table that seats 10 students per side (20 total). Use counters to “fill in” some of the seats, and tell the student that those seats are taken. Then cover the drawing with an opaque cloth (e.g., a napkin)—this is the “tent.” Ask the student how many more children can sit at the picnic table under the tent. If needed, the teacher may, **very briefly**, lift the tent (cloth) so the child can get another look.

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).

Possible task

1. Construct a “tower” of linking cubes (≤ 10). Let the student count how many cubes are in the “tower.” Then, without letting the student see, split the tower into 2 sections and keep one of the sections hidden behind your back. Give the other section to the student and ask him or her to determine how many cubes are hidden behind your back. Have the student keep a record of the different combinations.

JULIE

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

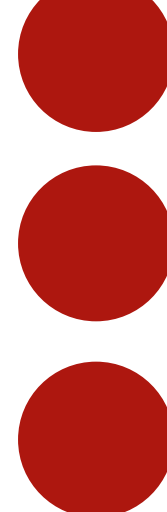
may now consider specific instruction within each of these standards. Certainly, one could choose from many good tasks as supplemental instructional experiences (see **fig. 3**). First, we see instruction (K.OA.3) aimed at helping Julie develop an understanding of relatively small composite structures (e.g., 10 comprises 3 and 7; 6 and 4, etc.). Certainly, Julie might possibly engage in unitary (counting) strategies to negotiate these types of tasks; however, with experience, she will soon be able to construct and leverage relatively small composites for arithmetic thinking. Moving along the identified pathway, we next see instruction that extends Julie's work with composite structures up to 20. Finally, Julie is presented with tasks featuring materials organized into natural, equal groups (e.g., bug legs, etc.). Here, instruction is aimed at helping Julie use her constructed knowledge of composites to make quantitative determinations involving equal groups. Note that each of these tasks involve some type of screening or conceal-

ment of materials, and these presentations are designed to capitalize on Julie's capacity to imagine hidden objects and materials.

When implementing a particular task, the teacher may begin the process of professional noticing again—attending to Julie's mathematical practices, forming an interpretation, and making or revising diagnostic and instructional decisions.

Professional noticing over time

We have presented professional noticing as a responsive teaching practice that occurs swiftly in conjunction with a child's mathematical activity. However, a more delayed approach to interpreting and deciding might be beneficial at times. When working with children who do not present many nonverbal cues or who struggle to explain their thinking, teachers might find it advantageous to spend several lessons *attending* to the activities of these children. Perhaps the teacher keeps a small notebook handy to



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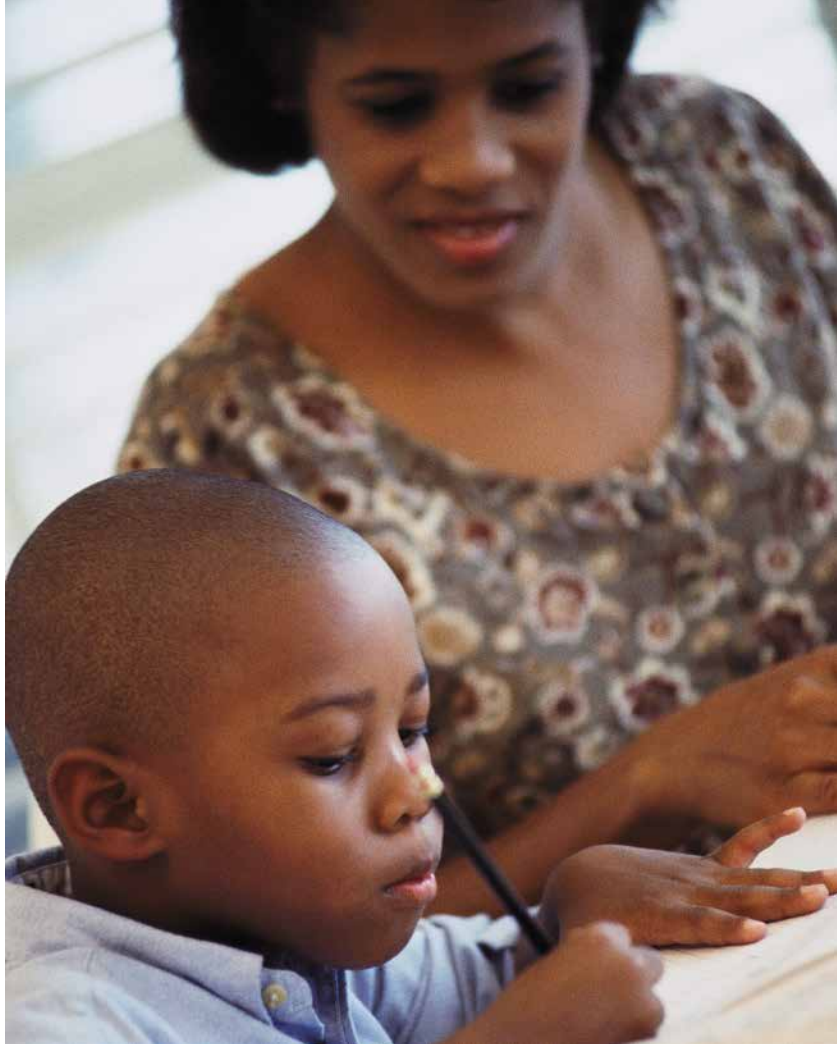
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In addition to verbal explanations, students often manifest various nonverbal cues as they work. Mathematics teachers must attend to both.

record any salient cues or explanations with respect to particular tasks. Digital video cameras (often found on mobile devices) are excellent tools for capturing photographs and video of key mathematical moments and work. For children whose true mathematical understanding remains something of a mystery, prolonged attending can greatly inform subsequent interpreting and deciding phases.

Teaching responsively

Indeed, we find that professional noticing establishes a powerful platform using mathematical progressions, such as CCSSM, to teach responsively in a variety of contexts. Deliberately connecting instructional decisions to interpretations based on attending evidence increases the likelihood of better understanding our students and giving them thoughtful, individualized, and effective mathematical experiences.

REFERENCES

- Thomas, Jonathan N., and Pamela D. Tabor. 2012. "Developing Quantitative Mental Imagery." *Teaching Children Mathematics* 19 (October): 174–83.
- Thomas, Jonathan N., Pamela D. Tabor, and Robert J. Wright. 2010/2011. "First Graders'

- Number Knowledge." *Teaching Children Mathematics* 17 (December/January): 299–308.
- Common Core State Standards Initiative (CCSSI). 2010. *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf
- Confrey, Jere, Kenny H. Nguyen, and Alan P. Maloney. 2011. *Hexagon Map Learning Trajectories for the K–8 Common Core Mathematics Standards*. <http://www.turnonccmath.com/>
- Griffin, Sharon. 2004. "Building Number Sense with Number Worlds: A Mathematics Program for Young Children." *Early Childhood Research Quarterly* 19 (1): 173–80. doi:<http://dx.doi.org/10.1016/j.ecresq.2004.01.012>
- Jacobs, Victoria R., Lisa L. C. Lamb, and Randolph A. Philipp. 2010. "Professional Noticing of Children's Mathematical Thinking." *Journal for Research in Mathematics Education* 41 (2): 169–202.
- Mulligan, Joanne T., and Michael C. Mitchelmore. 1997. "Young Children's Intuitive Models of Multiplication and Division." *Journal for Research in Mathematics Education* 28 (3): 309–31. doi:<http://dx.doi.org/10.2307/749783>
- Olive, John. 2001. "Children's Number Sequences: An Explanation of Steffe's Constructs and an Extrapolation to Rational Numbers of Arithmetic." *The Mathematics Educator* 11 (1): 4–9.
- Steffe, Leslie P. 1992. "Learning Stages in the Construction of the Number Sequence." In *Pathways to Number: Children's Developing Numerical Abilities*, edited by Jacqueline Bideaud, Claire Meljac, and Jean-Paul Fischer, pp. 83–88. Hillsdale, NJ: Lawrence Erlbaum Associates.
- van Es, Elizabeth A., and Miriam Gamoran Sherin. 2008. "Mathematics Teachers' 'Learning to Notice' in the Context of a Video Club." *Teaching and Teacher Education* 24 (1): 244–76. doi:<http://dx.doi.org/10.1016/j.tate.2006.11.005>
- Wright, Robert J., James Martland, and Ann K. Stafford. 2006. *Early Numeracy: Assessment for Teaching and Intervention*. 2nd ed. London: Paul Chapman Publications/Sage.
- Wright, Robert J., David Ellemor-Collins, and Pamela D. Tabor. 2011. *Developing Number Knowledge: Assessment, Teaching and*

Intervention with 7–11 Year Olds. London, UK: Paul Chapman Publications/Sage.



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