H. Bahadir Yanik and Yasin Memis

Making insulation decisions through mathematical modeling

Much discussion is taking place lately about conserving energy to help create a more sustainable world. Buildings, houses, and other structures lose energy through inadequately sealed windows, doors, and seams—including cracks. The more insulated a house or structure, the more energy is conserved. Heated and cooled air stays within the walls, requiring less energy to produce them.

Engaging students in studies about conservation and sustainability can support students' understanding of making environmentally conscious decisions to conserve Earth. This article aims to contribute to these efforts and direct students' attention to how they can use mathematics to make environmental decisions.

Insulation

In general, heat transfers from warm areas to cool areas. For instance, during winter days in Wyoming, heat flows from the warm indoors to the cold outdoors. On the other hand, on summer days in Arizona, heat flows from the warm outdoors into the cool, air-conditioned indoors. Throughout this heat transfer, vast amount of heat can be lost if a building does not have proper insulation. For example, a house can lose approximately 70 percent of heat through uninsulated windows, roofs, floors, and walls (Mau 1998). Accordingly, this loss of heat increases the power bills; insulation is needed to reduce heating and cooling demands. Insulating is—

> the act of preventing the flow or passage of heat, electric, or sound energy through a material, medium or system. It is the act of protecting a material, medium or system with a material in order to prevent energy escaping from it. (Ogedengbe, Fatomilola, and Bello 2013, p. 326)

Reducing energy bills is possible by insulating to reduce the amount of energy consumed for heating and cooling.

The activity

The purpose of this activity is to find out and compare costs of three separate insulation

packages and determine which would be most economical for the consumer. A modeling task (see **fig. 1**) was implemented with fifth- and sixth-grade students in a public classroom in Turkey. Condensed qualitative results of a study that documented students' thinking are provided to showcase the effectives and student challenges when investigating the modeling task.

Lesson introduction

The lesson started with a brief discussion about how energy is important in today's world and what people can do to save energy in daily life. Following this discussion, students were shown a picture of a building, including how much heat is lost from windows, roof, and external walls (see **fig. 1**). The teacher read the modeling task and asked students to work on it in groups of three or four; they were supplied with calculators. Students who knew how to use Microsoft Excel® were also allowed to use the computers in the classroom.

Task exploration

Initially, some students had difficulties understanding what was meant by "recovering the amount of money Mrs. Candan spent on each type of insulation." The teacher conducted a whole-class discussion to clarify the meaning of that statement. Several questions guided the classroom discussion and helped students understand that people can save money by saving energy:

- Why does Mrs. Candan want to have building insulation?
- What do you think about the effect of insulation on heating and cooling?
- How would insulation effect the power bill?
- What does it mean to save money through insulation?

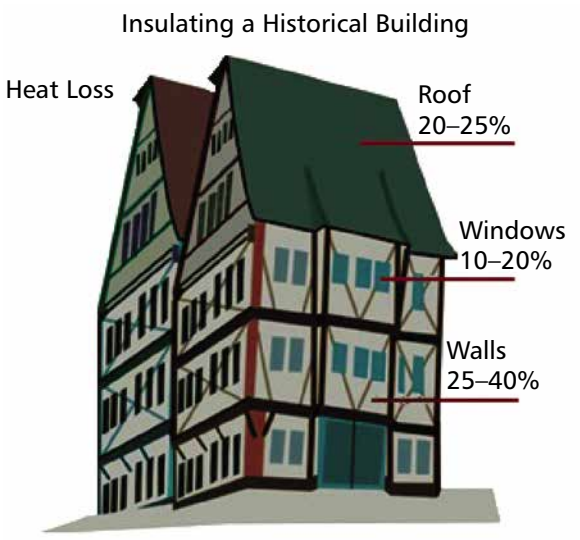
After students developed this initial understanding of the importance and rationale of insulation, they investigated three questions (see **fig. 1**).

Task 1

Initially, all the students tried to determine how much money Mrs. Candan would save monthly for each type of insulation. Some students had

FIGURE 1

Fifth and sixth graders developed an initial understanding of the importance and rationale of insulation before attempting the questions in the modeling task below. Students compared costs of three separate insulation packages and determined which would be most economical for the consumer.



Mrs. Candan bought a historical building. In the first year, she did not spend any money on the insulation of the old building because she thought it was unnecessary. However, after the first year, she realized she had spent a lot of money for the electricity to heat and cool the building. After she examined the electricity bills for one year, she realized that she spent an average of $1500 dollars per month. She talked to several contractors and received some recommendations of how to insulate the building. She decided to focus on three options (given below). Your job is to help Mrs. Candan decide which option for insulation is the best choice. Write a letter to Mrs. Candan addressing the following questions:

1. How many months would it take to recover the amount of money Mrs. Candan spent for each insulation option?
2. How much money would Mrs. Candan save in 5 years with each option?
3. Explain which option would be the best choice for Mrs. Candan.

	Option 1	Option 2	Option 3
Insulation type	**Windows only**	**Windows and roof**	**Windows, roof, and external walls**
Insulation cost	$3,600	$10,800	$21,600
Savings on electricity bill	20%	40%	60%

TABLE 1

When some students had difficulties finding percentages of a number for the first task, the teacher encouraged them to convert percentages to fractions and use them instead.

Student strategies for task 1

Insulation type	Windows only	Windows and roof	Windows, roof, and external walls
Insulation cost	$3,600	$10,800	$21,600
Savings for each month	20% of $1,500 ($300)	40% of $1,500 ($600)	60% of $1,500 ($900)
Group 1	300 + 300 + 300 + 300 + 300 + 300 + 300 + 300 + 300 + 300 + 300 + 300 = 3,600 (12 months)	600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 + 600 = 10,800 (18 months)	900 + 900 = 21,600 (24 months)
Group 2	300; 600; 900; 1,200; 1,500; 1,800; 2,100; 2,400; 2,700; 3,000; 3,300; 3,600 (12 months)	600; 1,200; 1,800; 2,400; 3,000; 3,600; 4,200; 4,800; 5,400; 6,000; 6,600; 7,200; 7,800; 8,400; 9,000; 9,600; 10,200; 10,800 (18 months)	900; 1,800; 2,700; 3,300; 3,600; 4,500; 5,400; 6,300; 7,200; 7,800; 8,100; 9,000; 9,900; 10,800; 11,700; 12,600; 13,500; 14,400; 15,300; 16,200; 17,100; 18,000; 18,900; 19,800; 20,700; 21,600 (24 months)
Group 3	3,600/300 = 12 months	10,800/600 = 18 months	21,600/900 = 24 months

TABLE 2

Several student strategies emerged from task 2.

Student strategies for task 2

Savings for each month	20% of $1500 ($300)	40% of $1500 ($600)	60% of $1500 ($900)
Group 1	300 × 60 = 18,000	600 × 60 = 36,000	900 × 60 = 54,000
Group 2	300 × 60 = 18,000 18,000 – 3,600 = 14,400	600 × 60 = 36,000 36,000 – 10,800 =25,200	900 × 60 = 54,000 54,000 – 21,600 = 32,400
Group 3	60 × 1,500 = $90,000 (five-year cost for uninsulated building) 60 × 900 = $54,000 (five-year power cost for insulated building—Offer 2) 54,000 + 10,800 = $64,800 (total amount of money spent in five years) 90,000 – 64,800 = $25,200 (total savings in five years—Offer 2)		

difficulties finding percentages of a number. The teacher encouraged those students to convert percentages to fractions and use them instead of percentages (e.g., 20% = 20/100; 40% = 40/100; 60% = 60/100). Several student strategies emerged for finding the number of months needed to recover the money for insulation investment (see **table 1**).

Some students added the amounts repeatedly (see **table 1**, **group 1**) or cumulatively (see

table 1, **group 2**) until they calculated insulation cost and determined the number of months needed to recover it. Others directly divided insulation costs by monthly savings (see **table 1**, **group 3**). The following excerpt shows how group 1 students described their strategies (***T:*** Teacher; ***S:*** Student):

T: How did you find out the number of months needed to recover the amount of money Mrs. Candan spent on option 1?

S: We first found out 20 percent of 1,500, which is 300 (dollars). This is what Mrs. Candan would save each month if she just takes the first option. So, if we add 300 twelve times repeatedly, we get 3,600. So, it needs 12 months to recover that money.

Task 2

For the second task, several student strategies emerged. For example, after finding out monthly savings for each type of insulation, some students multiplied each amount of savings by the total number of months (60 months) for five years to find out the amount of money Mrs. Candan would save (see **table 2**, **group 1**). Some students, on the other hand, thought that they would have to take away insulation costs from that amount to find net savings (see **table 2**, **group 2**). Other students compared both five-year uninsulated and insulated costs of power bills to calculate the total amount of savings (see **table 2**, **group 3**).

After students had shared their strategies, group 1 students seemed to be confused about whether they should take away insulation costs from their five-year savings as other students had. After a whole-class discussion, members of group 1 also came to understand that for the net profit in five years, it was necessary to take away the insulation cost from the five-year total savings.

Task 3

For the final task, students were asked to compare and contrast all three options to come to a conclusion about which one of them would be a good choice for Mrs. Candan. For this task, students found it helpful to use Microsoft Excel (see **fig. 2**). In the first row, students entered the total cost for both the first month's power expense and the insulation cost. Then, for the following rows, students entered cumulative addition of monthly power bills.

By using Excel, students were able to compare each option with another. For instance, at the end of the first, second, and third year, students recognized that the different options become more appropriate to choose. The following excerpt shows how Ali compared three options.

T: What do you think about these three options?

A: If she accepts the first option, at the end of the first year, Mrs. Candan would pay the same as the amount of money for the power bill for the uninsulated building.

T: So, it is not a good option then?

A: It would still be a good option, since starting the following month, she will pay 300 dollars less for each month.

T: How about the other options?

A: For the first year, it seems the best option is option 1. But, at the end of second year, Mrs. Candan will pay the same total amount of money with option 1.

T: So?

A: After the second year, she will begin to pay less amount of money if she chooses option 2.

T: How about option 3?

FIGURE 2

Members of group 4 prepared their data set in Microsoft Excel, which allowed them to easily compare the options.

A	B	C	D	E	F	G	H	I	
Months	Uninsulated	Offer 1	Offer 2	Offer 3					
1	1,500	4,800	11,700	22,200					
2	3,000	6,000	12,600	22,800					
3	4,500	7,200	13,500	23,400					
•	•	•	•	•					
•	•	•	•	•					
12	18,000	18,000	21,600	28,800					
13	19,500	19,200	22,500	29,400	First offer seems to be better after the first year.				
•	•	•	•	•					
•	•	•	•	•					
24	36,000	32,400	32,400	36,000					
25	37,500	33,600	33,300	36,600	Second offer seems to be better after the second year.				
•	•	•	•	•					
•	•	•	•	•					
36	54,000	46,800	43,200	43,200					
37	55,500	48,000	44,100	43,800	Third offer seems to be better after the third year.				
•	•	•	•	•					
•	•	•	•	•					
60	90,000	75,600	64,800	57,600					

A: Mrs. Candan needs to wait at least three years to begin paying less amount of money for the power bill among other options.

Whole-class discussion

Through the end of the lesson, the teacher encouraged students to share their thoughts and methods and discuss which one of the three options would be a good choice for Mrs. Candan and whether it would be worth making the insulation investment. The teacher showed the data set prepared in Excel, including the amount of money Mrs. Candan would pay for each type of insulation for five years. The teacher then asked students to come up with a mathematical model to describe how much money Mrs. Candan would spend on power bill and insulation costs together and check whether those models would produce the numbers given in the data set (see **fig. 3**).

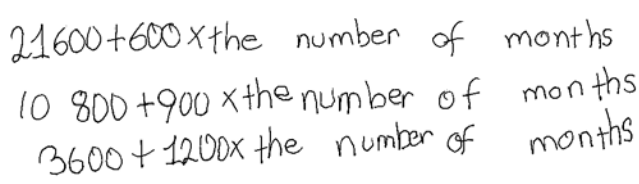
After students had prepared those models, the teacher asked them to write a letter to Mrs. Candan to explain which insulation package would be most economical for her.

Raising awareness

This modeling task presented students with an opportunity to use mathematics to make decisions regarding the insulation of a building with emphasis on reducing energy costs over the long run. Specifically, through this activity, students mathematically explored when insulation investments are financially profitable. Engaging students in such activities can raise their awareness about energy and help them develop mathematical arguments to support their recommendations. People in various fields (e.g., economics, health, and engineering) often encounter similar problematic situations (like the one mentioned in this article) in which they need to make cost-effectiveness analyses by comparing and contrasting several options, evaluating the strengths and weaknesses of alternatives to determine if new tools or machines are good investments over time. In the Insulation activity, students calculated the cost-effectiveness of building insulation by looking at the initial costs of three insulation packages and the yearly savings for each of them. At the end of the activity, students not only made a decision about which insulation type would be an appropriate choice for Mrs. Candan but also recognized that using less energy by insulating has environmental benefits, such as conserving fossil fuels and reducing air pollution.

FIGURE 3

These are several mathematical models that students developed.



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H. Bahadir Yanik, **hbyanik@anadolu.edu.tr**, is an associate professor in the mathematics education program in the Department of Elementary Education at Anadolu University in Eskisehir, Turkey. He is interested in modeling and STEM education. **Yasin Memis**, **ysnmemis@anadolu.edu.tr**, teaches grades 5–8 mathematics at Iki Eylul Middle School and is a graduate student currently working on his Ph.D. studies in the Department of Elementary Education at Anadolu University. He is interested in children's mathematical thinking. Edited by **Terri L. Kurz**, **terri.kurz@asu.edu**, who teaches mathematics and mathematics methodology at Arizona State University at the Polytechnic campus in Mesa; and by **Jorge Garcia**, **Jorge.garcia@csuci.edu**, who teaches at California State University Channel Islands.