teaching children mathematics

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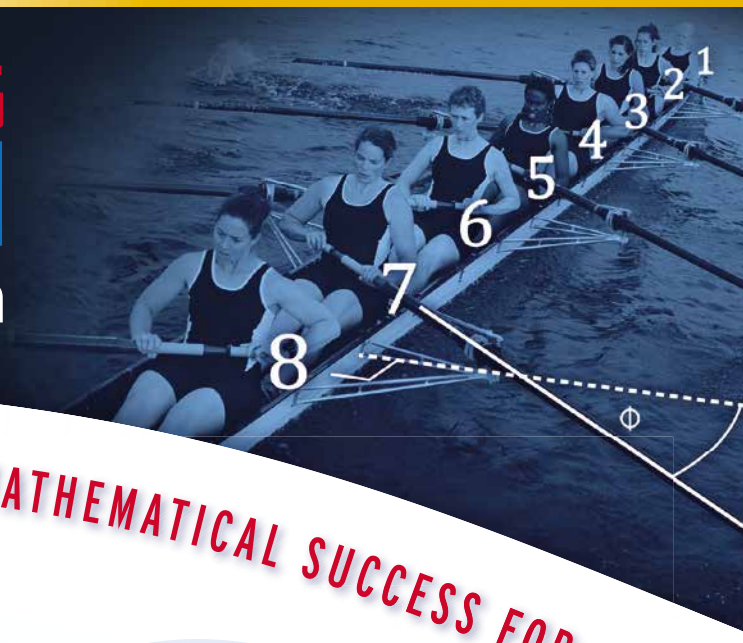

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Classroom-Based Formative Assessments—Guiding Teaching and Learning*

FRANCIS (SKIP) FENNELL, BARBARA ANN SWARTZ, BETH McCORD KOBETT, AND JONATHAN A. WRAY

Principles to Actions: Ensuring Mathematical Success for All (NCTM 2014) recognizes the need to find a way to leverage assessment opportunities to improve teaching and learning at the classroom and school level. And although we know a lot about the importance and potential impact of formative assessment done right and well (NMAP 2008; Black and Wiliam 2010), a disconnect continues to exist among planning, teaching, and assessment—and thus, between teaching and learning—in too many classrooms. Assessment must be linked to the planning and instruction of a lesson—every day—ensuring that lesson activities inform teaching and learning for all students. *Principles to Actions*'s eighth Mathematics Teaching Practice directs teachers to "*elicit* and use evidence of student thinking" (NCTM 2014, p. 53, emphasis added), but what are some ways to elicit this evidence?

We have defined a small set of classroom-based formative assessment (CBFA) techniques that we think of as a "five-color palette" for teachers to use on a regular basis, sometimes mixing the "colors" to consider how best to assess teaching and learning, and using student responses to monitor and adjust instructional decision making during a lesson as well as to aid in planning. Each of the following techniques has roots within the formative assessment literature and should be considered daily as teachers plan and implement instruction:

- Observations
- Interviews
- *Show Me* activities
- Hinge questions
- Exit tasks

ELNAVEGANTE/THINKSTOCK

One way to organize this palette of assessments is by considering observations, interviews, and *Show Me* activities as assessments that monitor and guide classroom instruction that day; whereas responses to hinge-point questions (Wiliam 2011) and exit tasks help assess daily progress and more directly influence planning for the next day's lesson. We will further discuss two of these techniques, the *Show Me* method and the hinge-questions method.

Show Me

The *Show Me* technique is most likely an activity that teachers use to augment an observation. Like the observation, the *Show Me* technique monitors the lesson but also provides the teacher with a glimpse of student performance of particular concepts and skills. Consider the language here: *Show Me* implies that students can demonstrate what they are learning. Our experience has been that the *Show Me* technique is used, typically, within number-related lessons. However, such demonstrations might occur within any content domain. Consider the following *Show Me* examples:

- 1.NBT.A.1—Use base-ten blocks to represent or show me 57 in three different ways.
- 3.NF.A.3b—Using your pattern blocks, show me two different ways to represent 1/3.
- 3.OA.B.6—Use a drawing to show me 32 ÷ 4.

Hinge question

Our work with teachers using the five-technique palette of classroom-based formative assessments has shown the hinge question to be the most challenging technique to plan for and use regularly. The hinge question (Wiliam 2011) provides a check for understanding and proficiency at a "hinge-point" in a lesson. In essence, responses to this question help define not only the success of the lesson but also next steps in planning. However, regular use of the hinge question implies that as teachers plan, they consider and even frame that "deal breaker" of a question that not only helps provide an indication of how the class is doing but also begins to frame the planning needs for the next day's lesson. Daily use of a hinge question also implies a thorough understanding of the intent of the lesson and its mathematical and pedagogical focus. Consider the following examples of a hinge question, noting that the multiple-choice formatted example below includes distractors, which offer clues to possible misconceptions:

- **Primary level (1.G.A.2):** What are ways in which squares and rectangles are similar and different?
- **Primary level (2.NBT.B.9):** Is 45 + 59 > or < 100? How do you know?
- **Intermediate level (4.NF.A.2):** What is the order of 4/5; 4/6; and 4/8 from least to greatest?

  A. 4/5, 4/6, 4/8
  B. 4/6, 4/5, 4/8
  C. 4/8, 4/5, 4/6
  D. 4/8, 4/6, 4/5

- **Intermediate level (5.MD.A.1):** 500 mL of water was needed for the recipe. If that amount were tripled, how many liters (L) would be needed? How do you know?

> CONSIDER OBSERVATIONS, INTERVIEWS, AND *SHOW ME* ACTIVITIES AS ASSESSMENTS THAT MONITOR AND GUIDE CLASSROOM INSTRUCTION THAT DAY; WHEREAS RESPONSES TO HINGE-POINT QUESTIONS AND EXIT TASKS HELP ASSESS DAILY PROGRESS AND DIRECTLY INFLUENCE PLANNING FOR THE NEXT DAY'S LESSON.

Classroom-based formative assessments—frequency and importance

Principles to Actions (NCTM 2014) advocates "ongoing and continual" formative assessment (p. 94), and we believe that such techniques as observations, classroom-based interviews, *Show Me*, hinge questions, and exit tasks are accessible for teachers. However, what is most important is the intentional use of these techniques. This five-option palette of assessment opportunities must become an integral component of planning and teaching every day.

Ed. note: At **http://www.mathspecialists.org**, find recent presentations on the classroom-based formative assessments (CBFAs) noted in this article. For a complete description of the CBFA techniques presented here, see the chapter titled "Classroom-Based Formative Assessments—Guiding Teaching and Learning" in *Annual Perspectives in Mathematics Education 2015: Assessment to Enhance Teaching and Learning* (NCTM 2015).

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Did students "get" it? Teachers can find out now!

BY ROBYN SILBEY, PD AND CAMPUS CONSULTANT

How do your teachers assess students' understanding at the end of a lesson? How do students solidify their learning and assess their own progress?

Lesson closers provide fabulous opportunities to engage students and hold them accountable for learning throughout the lesson. Moreover, lesson closers give teachers immediate feedback about their students' understanding so that programming is as accurate as it can be. Encourage your teachers to design lesson closers by asking questions that relate to the lesson's goal and assess conceptual understanding rather than having students perform computation. For example, "How can I determine whether my students reached the lesson's learning goal?"

- "How can I help students digest and reflect on their new learning?" Questions should help students identify a concept's "big ideas."
- "How will the data that I collect from this lesson closer help me plan tomorrow's instruction?" Questions will give teachers information about what students know and what they have yet to learn, so that teachers can tailor instruction specifically to students' needs.

Example

Goal: 4.NF.6—Find whole-number quotients (and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models).

Lesson closer: Partner talks or a group talk, a demonstration, or whiteboard responses. Questions might include these:

- How are division and place value connected?
- What are some ways you can check to see if you divided correctly? Why do your checking methods make sense?
- You divide 402 by 3 and by 6. Without actually dividing, predict which quotient will be greatest. Explain your thinking.
- What pictures would you draw to show 121 ÷ 5?

The value of lesson closers is the immediate feedback provided to students and teachers alike. Summarizing knowledge by articulating it verbally reinforces pride in learning as it celebrates accomplishment. Starting tomorrow, help teachers make lesson closers part of their daily teaching practice.

Questions? Comments? Contact **robyn@robynsilbey.com**.

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In our sister journal *

Student Explorations in Mathematics

Do your students struggle with division? Many students' misunderstandings about division relate to its definition. The "Demystifying Division" exploration in the March *SEM* takes the concept of division from the basics of whole numbers through polynomials, using a variation on the standard division algorithm and manipulatives and linking division and factoring with integers to algebra—a useful progression for students in grades 5–10.