

Which is the better deal?



Which cell phone plan should I purchase?
Should I buy or lease a new car?
Which store has the better price on this particular item?

We are constantly presented with choices that require mathematical comparisons to determine which offer would provide us with the best deal. In this problem scenario, students are presented with two options for receiving an allowance. They must determine which option is the most beneficial.

Problem scenario

You are going to receive an allowance for the next twelve weeks. You must choose how you would like to be paid. These are your choices:

- A.** You get \$2 for the first week, but you get \$0.25 more every week after that.
- B.** You get \$0.01 for the first week, but the amount doubles every week after that.

Which choice would give you more money?

See the **activity sheet** on **page 330** for additional questions.

Classroom setup

Before introducing the problem to your students, you will need to gather some materials:

- A large piece of paper for each pair or group of students to record their solutions
- Pens or markers
- A digital camera or a smartphone or tablet with a camera
- Calculators (optional)

Present the problem scenario to the class. Have your students turn to an “elbow partner” to discuss the definition of an allowance and the problem they will solve today. To ensure understanding of the task, ask one student to share his or her understanding of today’s problem with the rest of the class. Before students begin to formally solve the problem, ask the class to make a prediction and take a quick vote as to which choice would provide more money.

Organize students in pairs or triads to solve the problem. Distribute the activity sheet or the paper for students to record their answers. Calculating the total amounts mentally or by hand may be appropriate for most students, but if you think that the computations may be too difficult for others, provide them with calculators.

As students are working, walk around the classroom and observe the strategies that students are using to solve the problem. You may want to take pictures to help gather evidence of student thinking during the solution process. Try not to tell students how to do the math, but use questions to provoke their thinking:

- What is the rule for how much money you will receive?
- How much money do you get in the first week? How much in the second week?
- How could you keep track of how much money you receive each week?
- Is that amount in cents or dollars?

- How much money would you have altogether after twelve weeks?

When students have solved the problem, select solutions to share with the whole class. Begin by focusing on different solutions for choice A. Consider comparing solutions with different answers or frameworks. For example, one solution may have erroneously started at \$2.25 on week 1 instead of \$2.00. You might also consider comparing and contrasting solutions that were organized differently.

Continue with discussions concerning solutions for choice B. In some cases, students may have incorrectly calculated the increase by starting with \$0.01, \$0.02, \$0.04, \$0.06 and adding 2 each time. You could contrast this solution with one that correctly doubled each time. To decide which would be the better way to get paid, look at solutions and frameworks that compare the two choices. Ideally, a solution that shows a week-by-week comparison should be used for the final discussion.

As you display solutions, you could ask questions of the class to prompt discussion:

- Do you have any questions about this solution?
- Which answer is correct? Why?
- Which solution is easier to read? Why?
- What does *double* mean?
- Is this how much allowance I have altogether?
- So, which choice is better?
- Did the answer surprise you? Why or why not?
- How can doubling a penny end up being more than choice A?
- How many weeks did it take to double the original amount in choice A?
- What else do you notice?

As the class shares ideas, you can begin to create a list on the board or on chart paper as a record of student thinking.

Extensions and modifications

You might offer choice C to students who finish early; it involves flipping a coin each week to determine the amount of allowance. If the coin flip result is heads, they get \$6; and if the result is tails, they get no allowance.

Where's the math?

This scenario affords students a context to compare two different growing patterns; one is additive, and the other is multiplicative. Many students will be surprised by the solution to this problem. Despite having a much higher starting point of \$2, adding a constant increase of \$0.25 each week for twelve weeks results in less money than starting with a mere penny and doubling the total amount each week. Through discussion, students may notice that in this scenario, it takes eight weeks to double the original amount in choice A; whereas the total amount doubles every week in choice B. They may also notice that the total amount remains below a dollar in choice B until week eight, but the total amount quickly overtakes choice A after this amount of time.

This scenario also promotes the use of a table as an organizing framework for recording and comparing weekly results for the two choices.

If students do not use calculators, this problem presents them with the opportunity to use a variety of strategies to add monetary amounts.

A further extension would be to determine if or when choice A or C would provide more money than choice B.

Share your students' work

Try this problem in your classroom. We are interested in how your students responded to the problem, which problem-solving strategies they used, and how they explained or justified their reasoning. Send your thoughts and reflections—including information about how you posed the problem, samples of students' work, and photographs showing your problem solvers in action—by **April 1, 2015**, to Problem Solvers department editor **Ed Enns**, Waterloo Region District School Board, Learning Services, 51 Ardelt Avenue, Kitchener, ON N2C 2R5; or email him at ed_enns@wrdsb.on.ca. Selected submissions will be published in a subsequent issue of *TCM* and acknowledged by name, grade level, and school name unless you indicate otherwise.

Edited by Ed Enns, ed_enns@wrdsb.on.ca, who works as an elementary learning services consultant with the Waterloo Region District School Board in Kitchener, Ontario, Canada. Submit problems to be considered for future columns. *TCM* will not acknowledge receipt of problems; however, those we select for publication we will credit to the author. Find detailed submission guidelines for all departments at www.nctm.org/tcmdepartments.

Name _____

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- A. You get \$2 for the first week, but you get \$0.25 more every week after that.
- B. You get \$0.01 for the first week, but the amount doubles every week after that.
 - 1. How much would you get paid if you picked choice A?
 - 2. How much would you get paid if you picked choice B?
 - 3. Which choice would give you more money?