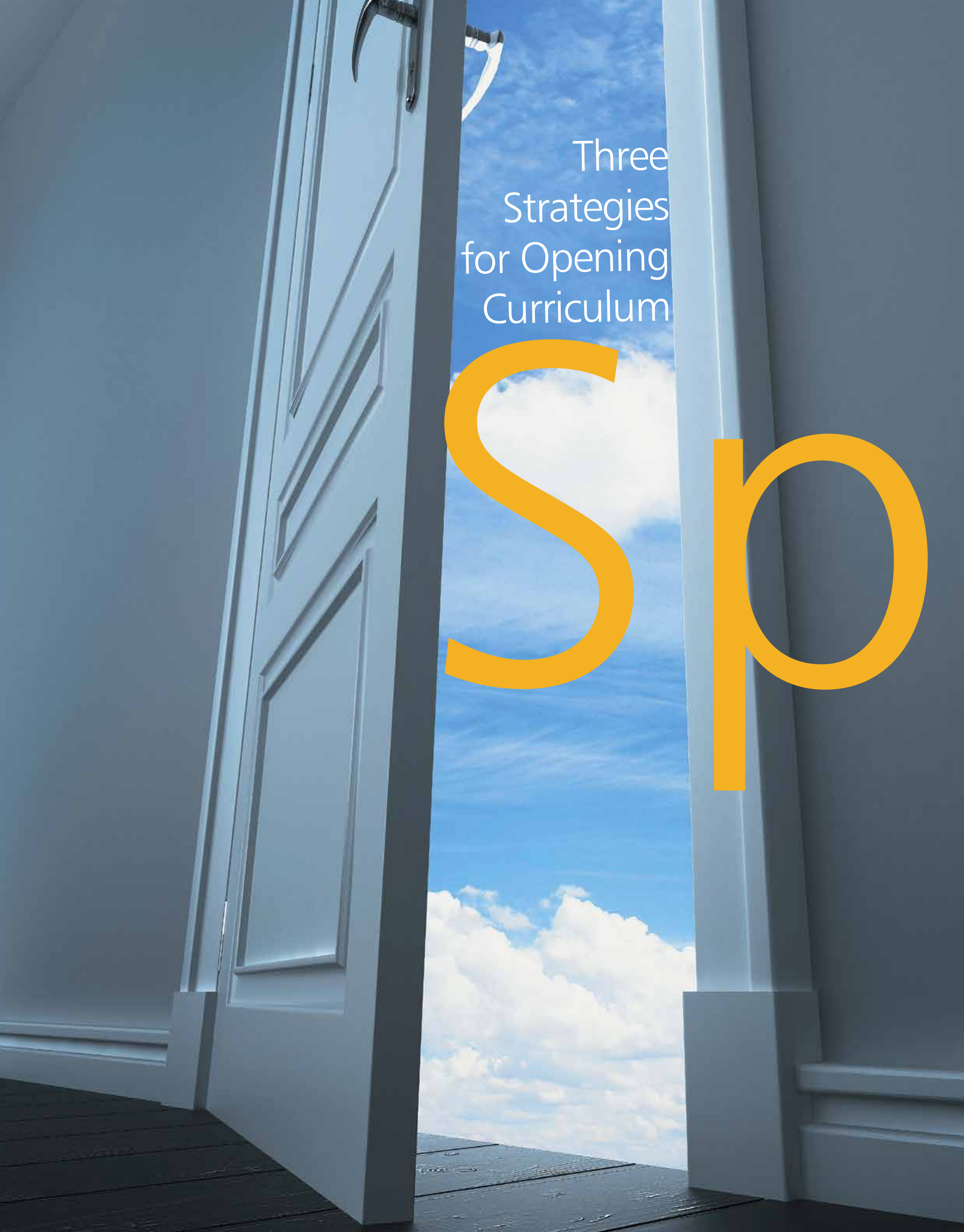
Three Strategies for Opening Curriculum Sp



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By Corey Drake, Tonia J. Land, Tonya Gau Bartell, Julia M. Aguirre, Mary Q. Foote, Amy Roth McDuffie, and Erin E. Turner

Make these small adjustments to your syllabus and watch spaces open to connect to children's multiple mathematical knowledge bases.

Imagine the following scenario: A third-grade teacher opens the teaching guide for her district-mandated textbook to a multidigit subtraction lesson. The lesson shows the standard algorithm for subtraction, using the problem 377 – 187 = ? This example is followed by ten practice problems and, at the end of the lesson, two word problems. Reflecting on her students' participation in a mental math routine earlier in the week, the teacher knows that her students can think about these problems in at least three other ways (see **table 1**). She also knows that, by the end of the lesson, most of her students will assume (correctly) that the word problems involve subtraction situations and will automatically apply the standard algorithm to solve those problems.

How can this teacher present a more meaningful, engaging lesson that builds on what she knows about her students' understanding of subtraction and their prior experiences with subtraction, both in and out of school, while still adhering to her district's mandate to use the textbook?

Competing expectations

Many teachers confront this tension between using published curriculum materials and teaching in ways that are responsive to children. Teachers are often expected to use a particular mathematics curriculum series, but they still want to be able to build on and connect to children's multiple mathematical knowledge bases (MMKB). Children's MMKB include children's mathematical thinking and children's home- and community-based mathematical funds of knowledge (Carpenter et al. 1999; Gonzalez, Moll, and Amanti 2005; Turner et al. 2012). Children's experiences using mathematics as part of home or community activities, as well as family practices that involve mathematics, are all part of children's home- and community-based mathematical funds of knowledge. Children's mathematical thinking includes the multiple strategies that students use to solve problems, if given the opportunity, as well as common confusions or misconceptions that children might have. In several of its standards, the Common Core State Standards for School Mathematics (CCSSM) calls for students to use multiple solution strategies (CCSSI 2010). Students make sense of problems and develop multiple solution strategies by connecting problems to their own experiences both in and out of school and by using and building on all of their MMKB.

TABLE 1

Reflecting on her third graders' participation in a previous mental math exercise, the teacher knows that her students can think about problems in at least three ways other than what their math book presents.

Solutions for 377 – 187 = ?

Solution A	Solution B	Solution C
300 – 100 = 200	187 + 3 = 190	"I know 377 take away 200 is 177. 177 is ten less than 187, so 377–187 must be 190."
70 – 80 = –10	190 + 10 = 200	
7 – 7 = 0	200 + 177 = 377	
200 – 10 + 0 = 190	177 + 10 + 3 = 190	

Small adjustments, big dividends

Eliciting and building on children's MMKB while using mandated curriculum materials is a significant challenge, because curriculum materials often focus on single strategies for solving problems and single meanings for problem contexts. In this article, we present three strategies for making small changes to curriculum materials that can open spaces for eliciting, building on, and connecting to children's MMKB. Each strategy requires small adjustments that are both mathematically impactful and reasonable for teachers to implement.

We illustrate several of these strategies using a lesson from Grade 4 *Everyday Mathematics* (UCSMP 2007, pp. 406–11). We chose this lesson because *Everyday Mathematics* promotes *Standards*-based content and practices and is used widely in schools and classrooms across the country. Our point is that *all* curriculum materials, even high-quality materials, require teachers to make changes to open spaces for children's MMKB. In part, this is because any set of published curriculum materials must be written for a generic classroom, so teachers have an important role in adapting and using the materials to meet the strengths, needs, and experiences of specific children.

The stated objective of the *Everyday Mathematics* lesson is to "guide the exploration of strategies to solve equal-grouping division number stories" (UCSMP 2007, p. 406). At the beginning of the lesson, in the Math Message, students read the following problem:

> A box holds 6 chocolate candies. How many boxes are needed to hold 134 chocolates?

The teacher is then directed to ask several students to share their solutions. Four possible strategies are provided in the teaching materials to give teachers an idea of what to expect from their students:

1. Direct modeling
2. Drawing a picture

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Teachers are expected to build on and connect children's multiple mathematical knowledge bases (MMKB).

3. Break[ing] 134 into smaller "friendly numbers"
4. Thinking of the problem as a multiplication problem with a missing factor

The next three pages in the teaching guide direct teachers to explain the "multiples strategy" as "one way" to solve these types of division problems. The multiples strategy involves using decade multiples to find the answer to a division problem. For example, to solve the problem in the Math Message using the multiples strategy, students would figure out how much ten sixes, twenty sixes, and thirty sixes equal and then use that information to find the solution to the problem.

After the explanation of the multiples strategy, students receive several problems for their math journals. The teaching guide states that teachers should "encourage students to use a variety of strategies to solve the problems … " (p. 410), but the format of the journal pages supports the use of the multiples strategy. The pages have little room for students to explore other solution strategies, including those that were shared after the Math Message. Below, we suggest three strategies for opening spaces for children's MMKB in curriculum materials, and we provide examples of those strategies, using the *Everyday Mathematics* lesson. We focus on lesson changes that stay consistent with the stated lesson objective while opening spaces for children's MMKB.

Strategy 1. Rearrange lesson components

Most curriculum lessons have several components, including opening routines or messages, a variety of student tasks, differentiation suggestions, and homework. In reviewing many textbooks, we found that more open spaces tend to be located on the periphery of lessons, including opening messages, textbook margins, teacher notes, and differentiation ideas. Often, teachers can open spaces for children's MMKB by moving these components around or omitting some of them and focusing on others. In general, the goal is to find those components that focus on (1) having students make connections between the task and their prior knowledge and experiences, (2) providing support for students to develop their own strategies, and (3) encouraging students to share and explain their strategies. After these components have been identified, strategies for rearranging them to open curriculum spaces and support children in making meaning of the mathematics could include the following two options.

Frontload problem solving

In many textbooks, the tasks that demand complex problem solving are located in the textbook margins or at the end of the lesson as an

application or as a problem-solving section. Or they might appear in the beginning of the lesson as a warm-up activity, as in the Math Message component of the *Everyday Mathematics* lesson described above. Or they might be located in teaching notes related to homework, enrichment activities, or advanced learner sections that are not part of the main lesson. Focusing the majority of the lesson on these problem-solving tasks and making these the "main" lesson will open spaces for children's MMKB, while still maintaining the mathematical goal of the lesson. The key to frontloading problem-solving tasks is to engage children in those tasks *before* introducing a preferred solution strategy.

Cut components

Consider omitting sections of a lesson that *tell*, *direct*, or *show* children how to make sense of and solve problems. Use this extra time to make connections to children's MMKB, exploring ways in which children's previous experiences and home- and community-based mathematical funds of knowledge can be leveraged to support new solution strategies. For example, consider using the extra time to listen to what children already know about the lesson topic and when and how they have encountered that topic outside school.

In the *Everyday Mathematics* lesson, we would rearrange the curriculum by omitting the teacher's explanation of the multiples strategy but retaining the Math Message, strategy sharing, and some kind of independent practice (although with adaptations, described below as part of strategy 2). We would begin the Math Message by asking students about their out-of-school experiences with items that come in packages, such as boxes of chocolate. Students could discuss how many chocolates fit in boxes of different sizes and shapes, the arrangement of chocolates in rows and layers, other items that come in boxes, and so on. We might ask students how they would know or figure out how many chocolates were in ten boxes or twenty boxes and how they might decide to share those chocolates with one or more people. Students could discuss their experiences of sharing items fairly, including strategies they have used to make sure each person receives the same amount. We might also ask students how they could organize chocolates into boxes (i.e., different dimensions of rows and columns of twelve or twenty-four chocolates) and how they could predict, estimate, or figure out how many boxes they would need for a given number of chocolates. By re-arranging the curriculum in this way, the space opened in the Math Message is maintained throughout the entire lesson and children are able to develop and explore a variety of strategies, as stated in the lesson objective, using their prior experiences with the content.

Teachers have an important role in adapting and using materials to meet the strengths, needs, and experiences of specific children.

Strategy 2. Adapt tasks to open spaces for children's MMKB

Adapting textbook tasks in ways that open spaces for children's MMKB—particularly children's mathematical thinking—is often possible with specific strategies, such as those that follow.

Adjust numbers or offer choices

Adjust the numbers in the problem or provide multiple number choices. By adjusting the number choice, you can open access to struggling learners and fast finishers while maintaining the cognitive demand and mathematical goals of the lesson. All students are able to work on the same mathematics but in ways that connect to their own prior knowledge. This often can be done by providing multiple number choices for a single problem and allowing students to work on the numbers that are "just right" for them.

Encourage multiple representations and strategies

Encourage multiple representations and solution strategies; having more than one tool in their toolbox is important for students. By asking them to solve a problem in two different ways or to use multiple representations, you can increase their capacity to solve problems, their practice justifying their solutions, and their ability to compare and contrast solutions to deepen mathematical understanding—all important mathematical practices highlighted in CCSSM.

In the *Everyday Mathematics* lesson, the first journal page presents three story problems with significant scaffolding for using the multiples strategy to solve them. The first problem reads as follows:

> José's class baked 64 cookies for the school bake sale. Students put 4 cookies in each bag. How many bags of 4 cookies did they bake? (p. 409)

Instead of having students work on all three problems, we might provide students with only this one problem along with number choices tailored to student needs. We could present the problem without the scaffolded activity sheet and encourage students to use at least two strategies or representations to solve the problem. This problem could be solved either after or instead of the Math Message:

> José's class baked _____ cookies for the school bake sale. Students put _____ cookies in each bag. How many bags did they make?
> A. (24, 4)
> B. (64, 4)
> C. (180, 6)
> D. (276, 6)
> E. (191, 5)

In this example, we used only one of the problems on the workbook page, but we included the number choices provided in all three problems. We also offered two additional number choices. Generating appropriate number choices for a classroom can prove difficult when you do not know a particular group of students. However, for the purposes of this example, we assumed that some students might find 64 ÷ 4 difficult.

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Therefore, we included an easier number choice. We included the third number choice to provide a transition to the fourth number choice so that the leap between the choices is not as big. If number choices of higher difficulty are needed for a particular class, you could supply them. In summary, we are saying that number choices can be used to differentiate a curricular task and fit the needs of any classroom. (See Land et al. [forthcoming] for more examples of number choice as a differentiation strategy.)

Strategy 3. Make authentic connections

Look critically at the real-world contexts presented in the textbook. Do these contexts actually help your students make sense of the mathematics? Are they meaningful to the students in your classroom, and do the mathematical practices in the problem or task actually connect to an activity in which children might engage in that real-world context? The word problem given in the student workbook of the *Everyday Mathematics* lesson and adapted above reflects experiences students might have in or out of school, but these may not be experiences that your students have had. Are there other experiences or situations that would be more authentic? One way to answer this question is to ask students to find multiplication and division situations at home or in the community, write about them or take a picture, and share them with the class. Possible situations include the number of children playing in a soccer club on a Saturday morning or the numbers associated with planning a large event, such as a wedding, quinceañera, or family reunion. These situations can be used as contexts for future word

problems. A related adaptation, as suggested above, would be to begin the lesson by having students share examples of items that come in packages in which each package has the same number of items. Then, consider situations when it might be useful to know how many packages are needed or how many packages can be filled with a given number of items.

Curriculum spaces

In our work, we have found that curriculum materials have "spaces" for students to make sense of mathematics, but those spaces are often *closed* when the materials require (or strongly suggest) that students use a particular strategy or when the materials present solution strategies *before* connecting to children's MMKB. We have developed a set of strategies for teachers to use to *open* curriculum spaces through relatively small changes to elementary mathematics curriculum materials. We think of these *curriculum spaces*, or *third spaces* (Moje et al. 2004), as places for teachers and students to create bridges for using children's MMKB to support school mathematics learning.

Enacting these strategies is likely to be easier or more difficult depending on the particular materials. However, these strategies can be applied to any lesson by focusing on tasks that provide openings for children's MMKB and by engaging students in rich discussions. Many curriculum series include opportunities for "higher-order thinking" or "problem solving" at the end of lessons. Choosing one or two of these problems and opening the lesson by allowing children to develop and share strategies for solving those problems provides an opportunity for teachers and children to build on children's MMKB in ways that closely align with CCSSM. Similarly, almost all lessons can be opened by eliciting children's out-of-school stories and experiences with the mathematical content and task, again opening space for children to make sense of the mathematics through their own MMKB (See also Butterworth and Lo Cicero 2001). Curriculum spaces—or potential curriculum spaces—exist in all curriculum materials; and with small changes, can be powerfully leveraged to support the learning of all children.

Common Core Connections

SMP 1	3.NBT.2
SMP 3	4.NBT.6

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Corey Drake is an associate professor and Director of Teacher Preparation at Michigan State University. Her research interests include supporting preservice teachers in productively using curriculum materials and other resources. **Tonia J. Land**, an assistant professor in the Drake University Teaching and Learning Department, is currently interested in examining ways to support preservice teachers in their use of curriculum materials and how teachers use number choice in their problem posing. **Tonya Gau Bartell** is an assistant professor in the Department of Teacher Education at Michigan State University. Her research focuses on the tools and experiences that can support math teachers' development of equitable pedagogical practices with explicit attention toward social justice, culture, race, and power. **Julia M. Aguirre**, an associate professor in the Education Program at the University of Washington–Tacoma, is interested in working with K–grade 12 teachers to connect children's mathematical thinking and children's funds of knowledge to teach equity-based math lessons to K–grade 12 students. **Mary Q. Foote** is an associate professor in the Department of Elementary and Early Childhood Education at Queens College–CUNY. For fourteen years, she was a classroom teacher at PS84M in New York City, a Title I school serving a population of children of whom 55 percent were Latino and 35 percent of whom were African American. **Amy Roth McDuffie**, a professor in the Washington State University–Tri-Cities Department of Teaching and Learning, focuses on teachers' learning in and from practice with attention to equity and teachers' use of curriculum resources. **Erin E. Turner** is an associate professor in the Department of Teaching, Learning, and Sociocultural Studies at the University of Arizona as well as a previous bilingual elementary school teacher in Phoenix. She currently focuses on issues of equity and social justice in mathematics education.