

# Comparing fractions

## Problem set examples

These are examples of the problem sets we used to help introduce students to various ways of comparing fractions. We did not give them the category titles; we simply gave them the problem sets. They worked on the problem sets in groups of two or three students. As students progressed through the sets, they often noticed the common feature. Even if they did not notice it with their partners, we brought it to everyone’s attention in subsequent large-group sessions.

### 1. Both fractions are one unit less than a half.

- a. Mark and Vivian wanted to race from the fence to the edge of the parking lot. Mark’s shoe laces came loose when he was  $\frac{3}{8}$  of the way to the edge of the parking lot. Vivian’s came loose when she was  $\frac{4}{10}$  of the way to the edge of the parking lot. Who was closer to the edge of the parking lot?
- b. Stephanie and Brian were both cutting long strips of paper. Brian got a cramp in his hand when he was  $\frac{4}{10}$  of the way to the end of his strip. Stephanie dropped her scissors when she was  $\frac{5}{12}$  of the way to the end of her strip. Who cut more of their paper strip?
- c. Ariel and Jessica were helping paint long lines on the cafeteria wall as part of a mural. They both started in the same corner and wanted to paint all the way to the opposite corner. Ariel ran out of paint when she was  $\frac{1}{4}$  of the way to the other corner. Jessica ran out when she was  $\frac{2}{6}$  of the way to the other corner. Who was closer to the other corner?
- d. Josiah’s family was planning a day at the lake. His mom and dad had to drive separate cars to get all the family members to the lake. His mom drove  $\frac{4}{12}$  of the way to the lake and then got caught at a train crossing. His dad had to stop and get gas  $\frac{2}{6}$  of the way to the lake. Who got closer to the lake before making the first stop?
- e. Maria was untangling lights to go on her Christmas tree. She had two strings of lights that were exactly the same length. She started to unwind the first string but found a huge knot when she was  $\frac{5}{12}$  of the way along the string. She gave up and tried the second string. She started to unwind the second string but found a huge knot when she was  $\frac{3}{8}$  of the way along that string. On which string did she make the most progress?
- f. A carpenter is about to lay carpet in a house. He has two large rolls of carpet that are the same length. He begins to unroll the first roll in the den and stops when he has unrolled  $\frac{4}{10}$  of the carpet. He begins to unroll the second roll in the master bedroom and stops when he has unrolled  $\frac{1}{4}$  of the carpet. In which room did he unroll the most carpet?

### 2. Both fractions are one unit more than a half.

- a. Mark and Vivian wanted to race from the fence to the edge of the parking lot. Mark’s shoe laces came loose when he was  $\frac{5}{8}$  of the way to the edge of the parking lot. Vivian’s came loose when she was  $\frac{6}{10}$  of the way to the edge of the parking lot. Who was closer to the edge of the parking lot?
- b. Stephanie and Brian were both cutting long strips of paper. Brian got a cramp in his hand when he was  $\frac{6}{10}$  of the way to the end of his strip. Stephanie dropped her scissors when she was  $\frac{7}{12}$  of the way to the end of her strip. Who cut more of their paper strip?
- c. Ariel and Jessica were helping paint long lines on the cafeteria wall as part of a mural. They both started in the same corner and wanted to paint all the way to the opposite corner. Ariel ran out of paint when she was  $\frac{3}{4}$  of the way to the other corner. Jessica ran out when she was  $\frac{4}{6}$  of the way to the other corner. Who was closer to the other corner?
- d. Josiah’s family was planning a day at the lake. His mom and dad had to drive separate cars to get all the family members to the lake. His mom drove  $\frac{7}{12}$  of the way to the lake and then got caught at a train crossing. His dad had to stop and get gas  $\frac{4}{6}$  of the way to the lake. Who got closer to the lake before making the first stop?
- e. Maria was untangling lights to go on her Christmas tree. She had two strings of lights that were exactly the same length. She started to unwind the first string but found a huge knot when she was  $\frac{7}{12}$  of the way along the string. She gave up and tried the second string. She started to unwind the second string but found a huge knot when she was  $\frac{5}{8}$  of the way along that string. On which string did she make the most progress?
- f. A carpenter is about to lay carpet in a house. He has two large rolls of carpet that are the same length. He begins to unroll the first roll in the den and stops when he has unrolled  $\frac{6}{10}$  of the carpet. He begins to unroll the second roll in the master bedroom and stops when he has unrolled  $\frac{3}{4}$  of the carpet. In which room did he unroll the most carpet?

## 3. Both fractions are one unit less than one.

- a. Mark and Vivian wanted to race from the fence to the edge of the parking lot. Mark's shoe laces came loose when he was  $\frac{7}{8}$  of the way to the edge of the parking lot. Vivian's came loose when she was  $\frac{9}{10}$  of the way to the edge of the parking lot. Who was closer to the edge of the parking lot?
- b. Stephanie and Brian were both cutting long strips of paper. Brian got a cramp in his hand when he was  $\frac{9}{10}$  of the way to the end of his strip. Stephanie dropped her scissors when she was  $\frac{11}{12}$  of the way to the end of her strip. Who cut more of their paper strip?
- c. Ariel and Jessica were helping paint long lines on the cafeteria wall as part of a mural. They both started in the same corner and wanted to paint all the way to the opposite corner. Ariel ran out of paint when she was  $\frac{3}{4}$  of the way to the other corner. Jessica ran out when she was  $\frac{5}{6}$  of the way to the other corner. Who was closer to the other corner?
- d. Josiah's family was planning a day at the lake. His mom and dad had to drive separate cars to get all the family members to the lake. His mom drove  $\frac{11}{12}$  of the way to the lake and then got caught at a train crossing. His dad had to stop and get gas  $\frac{5}{6}$  of the way to the lake. Who got closer to the lake before making the first stop?
- e. Maria was untangling lights to go on her Christmas tree. She had two strings of lights that were exactly the same length. She started to unwind the first string but found a huge knot when she was  $\frac{11}{12}$  of the way along the string. She gave up and tried the second string. She started to unwind the second string but found a huge knot when she was  $\frac{7}{8}$  of the way along that string. On which string did she make the most progress?
- f. A carpenter is about to lay carpet in a house. He has two large rolls of carpet that are the same length. He begins to unroll the first roll in the den and stops when he has unrolled  $\frac{11}{10}$  of the carpet. He begins to unroll the second roll in the master bedroom and stops when he has unrolled  $\frac{3}{4}$  of the carpet. In which room did he unroll the most carpet?

## 4. Both fractions can be used in the context of money.

- a. Mark and Vivian wanted to race from the fence to the edge of the parking lot. Mark's shoe laces came loose when he was  $\frac{4}{5}$  of the way to the edge of the parking lot. Vivian's came loose when she was  $\frac{6}{10}$  of the way to the edge of the parking lot. Who was closer to the edge of the parking lot?
- b. Stephanie and Brian were both cutting long strips of paper. Brian got a cramp in his hand when he was  $\frac{6}{10}$  of the way to the end of his strip. Stephanie dropped her scissors when she was  $\frac{3}{4}$  of the way to the end of her strip. Who cut more of their paper strip?
- c. Ariel and Jessica were helping paint long lines on the cafeteria wall as part of a mural. They both started in the same corner and wanted to paint all the way to the opposite corner. Ariel ran out of paint when she was  $\frac{3}{4}$  of the way to the other corner. Jessica ran out when she was  $\frac{4}{5}$  of the way to the other corner. Who was closer to the other corner?
- d. Josiah's family was planning a day at the lake. His mom and dad had to drive separate cars to get all the family members to the lake. His mom drove  $\frac{7}{10}$  of the way to the lake and then got caught at a train crossing. His dad had to stop and get gas  $\frac{15}{20}$  of the way to the lake. Who got closer to the lake before making the first stop?
- e. Maria was untangling lights to go on her Christmas tree. She had two strings of lights that were exactly the same length. She started to unwind the first string but found a huge knot when she was  $\frac{1}{2}$  of the way along the string. She gave up and tried the second string. She started to unwind the second string but found a huge knot when she was  $\frac{3}{5}$  of the way along that string. On which string did she make the most progress?
- f. A carpenter is about to lay carpet in a house. He has two large rolls of carpet that are the same length. He begins to unroll the first roll in the den and stops when he has unrolled  $\frac{6}{10}$  of the carpet. He begins to unroll the second roll in the master bedroom and stops when he has unrolled  $\frac{3}{4}$  of the carpet. In which room did he unroll the most carpet?

## 5. More of a bigger unit or less of a smaller unit

By the time we started with the following fractions, most students were competently comparing fractions using all of the previous strategies. Even though these examples could be solved by using the previous strategies, we thought it was important for students to realize the idea that more of a bigger unit is always more than less of a smaller unit. Although using the part-whole subconstruct in this case (to reason about the relative size of the parts) may be easy, this line of reasoning does support the idea of magnitude.

- a. Mark and Vivian wanted to race from the fence to the edge of the parking lot. Mark's shoe laces came loose when he was  $\frac{5}{8}$  of the way to the edge of the parking lot. Vivian's came loose when she was  $\frac{3}{10}$  of the way to the edge of the parking lot. Who was closer to the edge of the parking lot?
- b. Stephanie and Brian were both cutting long strips of paper. Brian got a cramp in his hand when he was  $\frac{6}{10}$  of the way to the end of his strip. Stephanie dropped her scissors when she was  $\frac{5}{12}$  of the way to the end of her strip. Who cut more of their paper strip?
- c. Ariel and Jessica were helping paint long lines on the cafeteria wall as part of a mural. They both started in the same corner and wanted to paint all the way to the opposite corner. Ariel ran out of paint when she was  $\frac{3}{4}$  of the way to the other corner. Jessica ran out when she was  $\frac{2}{6}$  of the way to the other corner. Who was closer to the other corner?
- d. Josiah's family was planning a day at the lake. His mom and dad had to drive separate cars to get all the family members to the lake. His mom drove  $\frac{6}{12}$  of the way to the lake and then got caught at a train crossing. His dad had to stop and get gas  $\frac{5}{9}$  of the way to the lake. Who got closer to the lake before making the first stop?
- e. Maria was untangling lights to go on her Christmas tree. She had two strings of lights that were exactly the same length. She started to unwind the first string but found a huge knot when she was  $\frac{4}{10}$  of the way along the string. She gave up and tried the second string. She started to unwind the second string but found a huge knot when she was  $\frac{5}{8}$  of the way along that string. On which string did she make the most progress?
- f. A carpenter is about to lay carpet in a house. He has two large rolls of carpet that are the same length. He begins to unroll the first roll in the den and stops when he has unrolled  $\frac{2}{10}$  of the carpet. He begins to unroll the second roll in the master bedroom and stops when he has unrolled  $\frac{3}{4}$  of the carpet. In which room did he unroll the most carpet?

## 6. One fraction can be expressed in terms of another.

- a. Mark and Vivian wanted to race from the fence to the edge of the parking lot. Mark's shoe laces came loose when he was  $\frac{5}{8}$  of the way to the edge of the parking lot. Vivian's came loose when she was  $\frac{3}{4}$  of the way to the edge of the parking lot. Who was closer to the edge of the parking lot?
- b. Stephanie and Brian were both cutting long strips of paper. Brian got a cramp in his hand when he was  $\frac{6}{10}$  of the way to the end of his strip. Stephanie dropped her scissors when she was  $\frac{3}{5}$  of the way to the end of her strip. Who cut more of their paper strip?
- c. Ariel and Jessica were helping paint long lines on the cafeteria wall as part of a mural. They both started in the same corner and wanted to paint all the way to the opposite corner. Ariel ran out of paint when she was  $\frac{3}{4}$  of the way to the other corner. Jessica ran out when she was  $\frac{7}{8}$  of the way to the other corner. Who was closer to the other corner?
- d. Josiah's family was planning a day at the lake. His mom and dad had to drive separate cars to get all the family members to the lake. His mom drove  $\frac{3}{10}$  of the way to the lake and then got caught at a train crossing. His dad had to stop and get gas  $\frac{2}{5}$  of the way to the lake. Who got closer to the lake before making the first stop?
- e. Maria was untangling lights to go on her Christmas tree. She had two strings of lights that were exactly the same length. She started to unwind the first string but found a huge knot when she was  $\frac{9}{10}$  of the way along the string. She gave up and tried the second string. She started to unwind the second string but found a huge knot when she was  $\frac{4}{5}$  of the way along that string. On which string did she make the most progress?
- f. A carpenter is about to lay carpet in a house. He has two large rolls of carpet that are the same length. He begins to unroll the first roll in the den and stops when he has unrolled  $\frac{8}{12}$  of the carpet. He begins to unroll the second roll in the master bedroom and stops when he has unrolled  $\frac{2}{3}$  of the carpet. In which room did he unroll the most carpet?