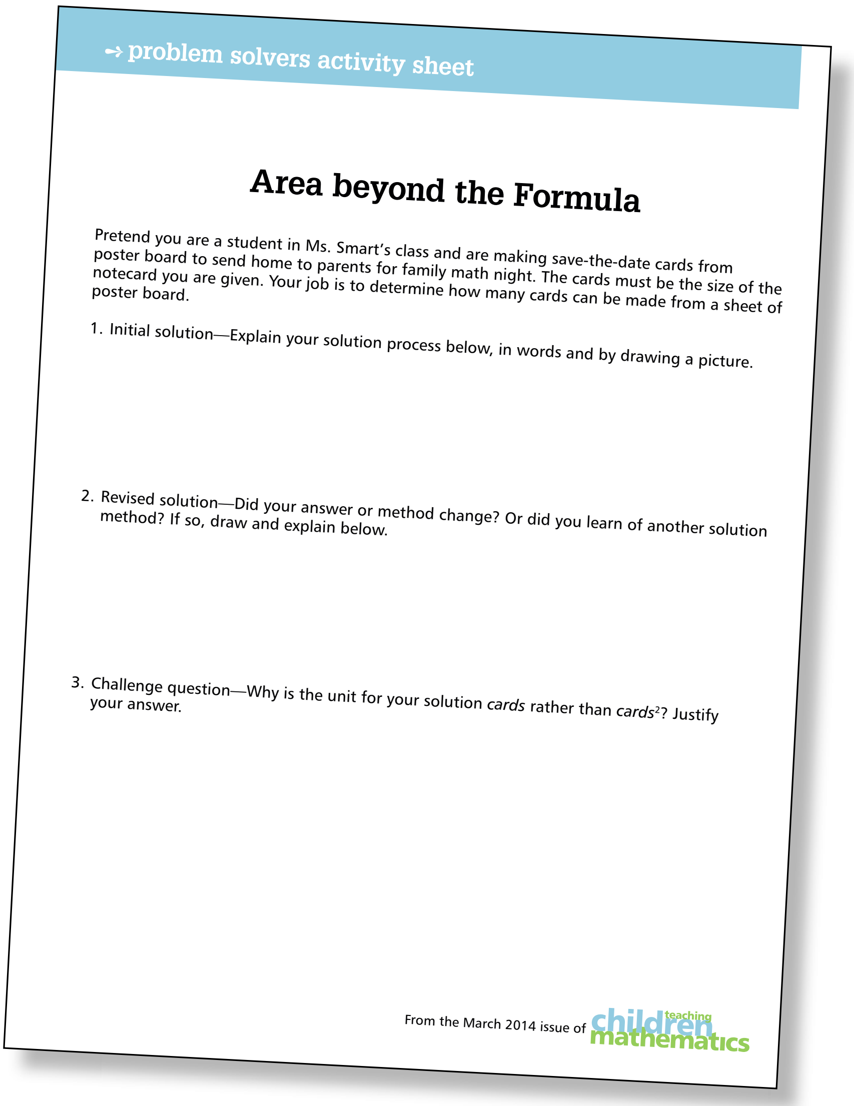
Chrystal Dean

Area beyond the formula

The **March 2014 problem** scenario has students investigate area in a conceptual manner that goes beyond tiling and formulas. To access the full-size activity sheet, go to **www.nctm.org/tcm**, Back Issues.


→ problem solvers activity sheet

Area beyond the Formula

Pretend you are a student in Ms. Smart's class and are making save-the-date cards from poster board to send home to parents for family math night. The cards must be the size of the notecard you are given. Your job is to determine how many cards can be made from a sheet of poster board.

1. Initial solution—Explain your solution process below, in words and by drawing a picture.

2. Revised solution—Did your answer or method change? Or did you learn of another solution method? If so, draw and explain below.

3. Challenge question—Why is the unit for your solution *cards* rather than *cards*^{2}? Justify your answer.

From the March 2014 issue of teaching children mathematics

Walter Stark wanted his fifth-grade class to explore the Area beyond the Formula problem so that he could assess and support his students' depth of understanding of the mathematical ideas underlying the concept of area.

Introducing the problem

The whole class began with a conversation to review the names and characteristics of different polygons, which was followed by a discussion of the perimeter and how to calculate it. Some students suggested listing "shortcuts" for determining perimeter. One such shortcut was to multiply the length of one side by four to calculate the perimeter for a square. Students argued that this approach made sense because squares have four congruent sides.

Next, Stark launched the Area beyond the Formula activity by first asking students, "What is area?"

Students discussed the question, focusing on space filling and connections to operations with side lengths:

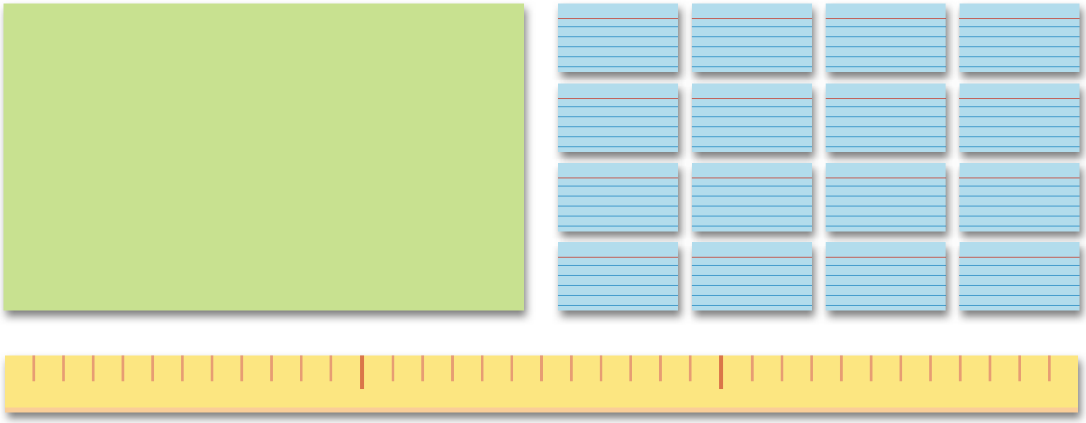
Matthew: Area is the amount of space inside the polygon.

Stark: What do you mean?

Sophia: It is the number of things that fill up the inside. Like when you tile a floor.

Max: Yeah, you multiply two sides to get the area.

Following this exchange, Stark distributed a copy of the activity sheet to each student. The class read the problem aloud, and students individually underlined pertinent information. Working in groups of two or three students, each group received an index card, a poster board, and a yardstick. Students were asked to use these tools to determine how many invitation cards the size of the index card they could make from the poster board without changing the size of the card (i.e., no overlapping, cutting, taping, or folding the cards).



Solution strategies

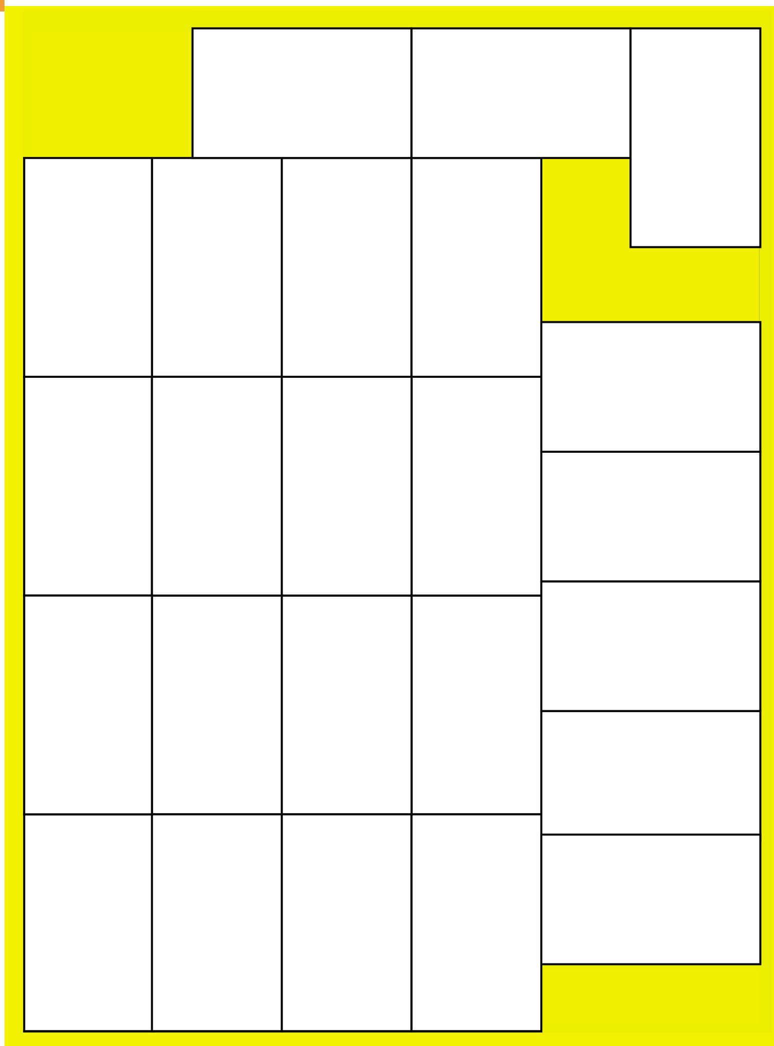
Students in Stark's class used three different strategies when solving the problem. One solution was to trace the index card on the poster board until "it was full" and then count the number of spaces representing a whole card. Students using this strategy did not change their method after seeing others. They were convinced that this was an effective method because it seemed to best fit the context for the problem (how they would actually cut cards from the poster board for invitations). Noticing one student lining the cards along the edge, Stark asked if there was a more efficient way to proceed rather than tracing all the cards. In response, a second student suggested tracing along each edge and then "adding up [i.e., counting] the number of cards that fill the middle of the paper." Other students used a similar method, but rather than counting cards individually, they described "using area" to determine the number of cards. They meant that they had multiplied the number of cards—that they had traced—along one edge by the number of cards they had traced along an adjacent edge to get the total number of cards, while maintaining the orientation of the card. However, the most common strategy that students used was to find the area of the poster board and divide by the area of the index card.

During the whole-class discussion, each group presented its solution strategies. Stark prompted further discussion of the strategies. Dominick's group noticed that if they held the poster board like a portrait instead of a landscape while maintaining the original orientation of the index card, they were able to fit a different number of cards. This comment led to more discussions about multiplying the number of cards across one side of the poster board by the number of cards down an adjacent side of the poster board. The children concluded that drawing the cards was not necessary when using this strategy.

The class then turned to the method of dividing the area of the poster board by the area of the index card, a calculation that yields a noninteger quotient. Students were puzzled by the remainder. Several students suggested ignoring the remainder, claiming the whole number would be the answer to how many index cards you could get out of the poster. Ryan pointed out that the number of cards found with this method was greater than any answers obtained by the other methods. Darwin noticed that attempting to tile the poster with index cards leaves extra poster board at the edges that is not big enough to fit full cards. Without considering the physical dimensions of the index cards, Darwin was comparing the number of cards generated by division with the answers generated by physically placing the card on the poster board. He could see a "border" of unused space on some of the sides. He claimed that the "spaces could not be put together to make other cards because they would have to be taped" to accomplish the creation of more cards. Thus, students correctly concluded that the larger number when dividing the two areas was the result of (incorrectly) assuming that the extra space around the border could be combined to create cards.

FIGURE 1

A student noticed that attempting to tile the poster with index cards leaves a border at the edges that is not big enough to fit full cards. The teacher had to push students toward a multiplicative approach.



Variations on the classroom setup

Stark changed the implementation of the activity in two ways. First, students were allowed to trace the index card on the poster board. This seemed to support students in thinking deeply about the context of the problem. However, because of this option, students were inclined to use a tiling method—tracing the card and counting the tracings—rather than iterating the index card along the length and width of the poster to find the number of cards that fit on each dimension. Although tiling can support visualizing area as an array, it is not as sophisticated as iterating. In addition, students often are inconsistent with the orientation of the card and are simply trying to maximize the number of cards they can fit onto the poster board (see **fig. 1**). Stark had to push students to move to a multiplicative approach.

A second variation was that Stark allowed the use of a yardstick or ruler. The original classroom setup suggested that students not know the dimensions of the index card or poster board and not be allowed to use a ruler. However, students' use of measuring devices yielded a noninteger answer that forced them to focus intently on the task. The brief confusion over the decimal portion of the quotient supported a rich discussion about how an answer could *seem* correct via calculation yet be implausible given the context of the problem.

If you use these variations, follow up by challenging students who traced the cards to find a more efficient method for coming to a solution. This might encourage students to think beyond their first additive approach to the development of a multiplicative strategy. In addition, although the conversation that resulted from dividing the area of the poster board by the area of the index card definitely had value, teachers using this variation of the task would want to encourage students to determine how they would solve the problem if rulers or yardsticks were unavailable. Challenging the children, as Stark did, to explain the different quantities of cards they had found would be important.

In Stark's class, the issue of orientation arose only with poster board. The orientation of the index card could also be brought up during whole-class discussion. This could support conversations about multiplicative reasoning as well as the concept of unit. For example, if the index card was iterated on one side of the poster board in portrait orientation and iterated on an adjacent side in landscape orientation, the unit of measure is no longer the "index card," but instead is a square.

The third question on the activity sheet deals with units of measure. Given that Stark's students were focused on the specific context

of the problem, they would be unlikely to find it problematic that the unit of measure should be cards rather than cards². Thus, to encourage a rich debate about unit, it might be useful to reword the final question as follows:

> When finding the area of a piece of paper that is 7 inches by 9 inches, the answer would be 63 inches². You are working with the concept of area for this problem. Why is the unit for your solution "cards" rather than "cards²"? Justify your answer.

REFERENCE

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

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