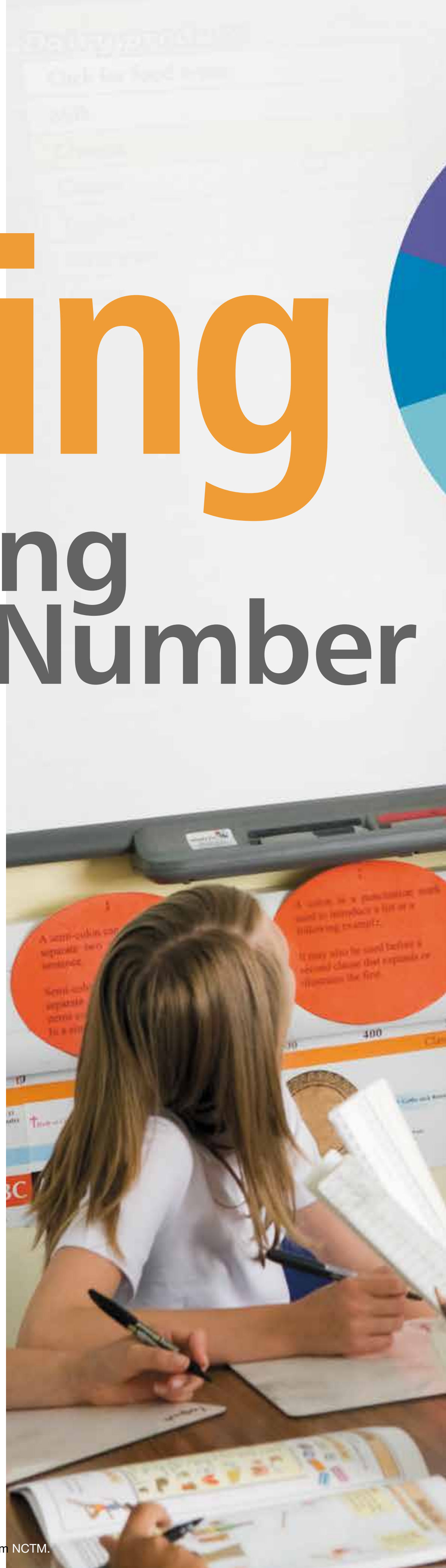
Counting on Using a Number

By Paul Betts

Learn how educators can help children who persistently use the counting-all strategy shift to using counting on for adding quantities.

Teachers often say to me, "I don't know what to do for my students who struggle to learn math. No matter what I do, they don't seem to get it."

I can relate to what these teachers are saying. This article considers how to help children who persistently use the counting-all strategy for adding quantities, despite reform-based interventions intended to shift them to using counting on. A number board game with a special rule for moving a token seems to help these children take up counting on as a strategy within various numerical contexts.



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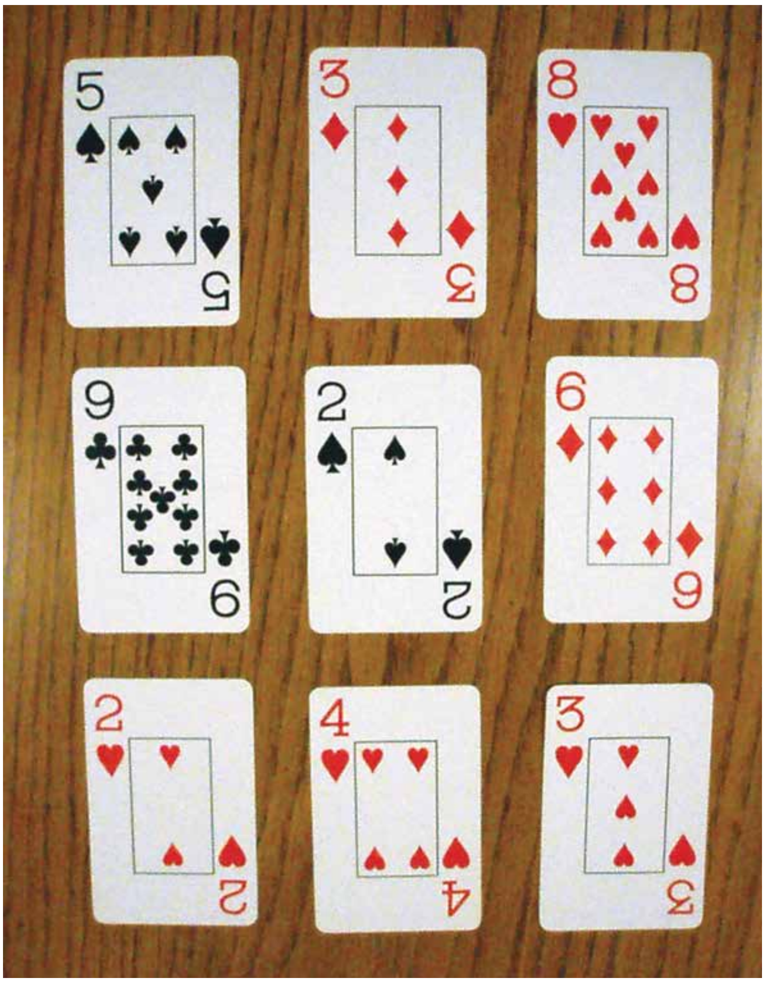
Why counting on is important

Counting all and counting on are distinct counting strategies that can be used to compute such quantities as the total number of objects in two sets (Wright, Martland, and Stafford 2010). Given five objects and three more objects, for example, children who use counting all to determine quantity will count both collections; that is, they count (starting with one set): "One, two, three, four, five" [*move to other set of objects*] "six, seven, eight." (Starting with the larger or smaller set does not matter.) Children who count on do not need to count both collections; that is, given knowledge that one set has five objects, they count on the second collection, "six, seven, eight," to determine the total number of objects (or they start with the smaller set and count on the larger set). A child who relies on counting all uses the objects in each set to determine the total quantity. Children who can use counting on do not rely on physical objects, because they are using the counting sequence (which number name comes next in the sequence) to determine total quantity (Wright, Martland, and Stafford 2010). To count on, given the quantity of the first set, a child could use one of several strategies to keep track of the second set (e.g., fingers, verbalization cues, head bobbing).

Children who count on have developed a more sophisticated sense of quantity (Wright, Martland, and Stafford 2010). The known set is already counted and does not need to be counted again; and the second set is in addition to the first (Baroody 1998). Thus, children who count all see a single collection of objects, whereas those who count on see both a single collection and a decomposition of that collection into two parts. Decomposition is a key concept underlying the development of mastery of basic math facts (Baroody 2006). Children who decompose sums are more likely to use known relationships and reasoning strategies, rather than counting strategies, to calculate unknown basic math facts (Baroody 2006). Thus, children who are confident in using counting on in a variety of contexts are ready for computations that are more sophisticated.

FIGURE 1

Counting all and counting on are possible strategies for such number games as Make Ten. Players count all the "dots" on the cards that they choose from the array and then make adjustments accordingly. The dealer replaces the cards before the next turn.



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Helping children who persistently count all

I have tried several teaching strategies intended to encourage young school-age children who struggle to learn math to progress from persistent use of counting all to proficient use of counting on—with mixed results. These children persist with counting all even after observing many of their classmates progress to proficient use of counting on. I enter the scene as an extra resource in the school, using a pull-out program to help these children.

I have played number games, such as Make Ten (see **fig. 1**), where counting all and counting on are possible strategies. During the game, a child selects cards whose sum is ten from a common 3 × 3 array (cards are replaced by the dealer

before the next turn). Children can count all the dots on the cards that they think will work and make adjustments accordingly. For example, in **figure 1**, a 6 and a 5 are available. If a player selects these, he or she can count the total number of dots and determine to choose smaller cards. The example illustrates the potency of the game to trigger magnitude comparisons. But I have played this game on a weekly basis for seven or eight weeks without observing the use of counting on by children who persist with counting all. The enjoyment of playing a game and feeling successful at mathematics wore off before counting on could be observed. A disposition of patiently waiting for children who struggle to learn math to invent counting on while playing Make Ten lacks potency.

I have also tried modeling counting on with the playing cards and then encouraging children to use this strategy while playing Make Ten. Covering the first card seems to help children practice counting on. For example, if the first card is a 4, cover it with your hands and say *four*, and then count "Five, . . ." while counting the dots on the second card. Children often need to be reminded of the next number while pointing at the first dot on the second card. The children must coordinate several actions, but they become more confident and proficient with practice. That they have learned a procedure is the crux of the matter; has their proficiency with counting been enhanced, or are they merely parroting the procedure? Although I have seen evidence from some children who struggle with math that they have shifted from persistently counting all to proficiently counting on, this is not the case for others. For example, if the context for counting is changed from cards to dice, children who have used counting on with cards can shift back to and persist with using counting all with the dice.

Other approaches

The strategies described in the previous section do not seem to help all children. It seems to me that standard reform-based approaches are inadequate to help all children and that children who persist with counting all—and who struggle with mathematics—need a special kind of intervention. The number board game described below was developed by Siegler and Ramani (2008, 2009). The game board is one or more rows of a standard hundred chart (I used a game board numbered 1–20). Children start at the 1 and move their token to the end of the board, based on a die toss or a spinner. Furthermore, while moving the token the required number of spaces, a player must name each number that he or she lands on. Suppose a token is at the 13 and a player rolls a 3. Normally, the player counts, "One, two, three," to keep track of moving the token the required number of spaces. In this game, the player would move the token one space and say *fourteen*, move another space and say *fifteen*, and move another space and say *sixteen*. The player must find another way to track when he or she has moved the required three spaces (see **fig. 2**). Thus, a player must simultaneously coordinate naming the number of each space landed on and how far his or her token has moved.

FIGURE 2

Siegler and Ramani (2008, 2009) suggested a game board of one or more rows of a standard hundred chart. While moving a token the number of spaces indicated by a die toss or a spinner, a player names each number she lands on, simultaneously coordinating how far her token moves.

PAUL BETTS

While playing the game

I played the game with four children from grade 2 or 3 who were identified by their teachers as struggling with mathematics. Before we started to play the game, I had observed persistent use of counting all in various contexts, but I had not observed counting on by any of these

children. We played the game together weekly for about twenty minutes for four weeks.

Modeling strategies

All the children initially struggled to simultaneously coordinate naming the number of each space landed on and how far their token had moved. I modeled the procedure for taking a turn, which was insufficient for any of the children to successfully enact a move independently. The children focused on saying the number name of each space landed on; they could not simultaneously keep track of how many spaces they had moved. I introduced the idea of using our fingers: If you roll a 3, for example, display one finger at a time for each space, stopping when three fingers are displayed. I modeled this strategy, and we practiced together while playing the game. At first, all four children needed my help; that is, I kept track with my fingers while they moved the token and named the number on each space. At each turn, I encouraged the children to use their own fingers. I would help them display their fingers one at a time while they moved their token, and I would say the game board numbers with them. By the end of the first session, three of the four children, Tina, Sara, and John, had successfully moved their token (kept track with their fingers while simultaneously saying the appropriate game board numbers) independently at least once. Nathan, on the other hand, always needed help coordinating his fingers to keep track. I would hold his hand and display his fingers as he moved his token, or he would use my fingers as he moved his token. It seemed to be overtaxing for Nathan to coordinate both looking at the game board to move his token and using fingers to track the spaces he had moved.

A roll of a 6 proved problematic for all the children during the first session. This is because one hand is needed to move their token and the other hand has only five fingers. I helped the children deal with this dilemma by suggesting that they use my hand as well and allowing them to do so.

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Nathan seemed unable to coordinate moving his token on the game board while using his fingers to count the number of spaces he had moved.

Finding their own new strategies

During the second session, the first strategies emerged. After reminders, modeling, and practicing how to move a token and keep track with fingers of how many spaces they had moved, Tina, Sara, and John quickly shifted to successful and independent use of their fingers to track moves. Then Tina rolled a 2, bobbed her head twice while moving her token two spaces, and did not use her fingers. I asked why she had not used her fingers to keep track, and she said she just knew when to stop. Later, Sara rolled a 1, and said, "This will be easy."

All four children agreed that a 1 is an easy roll; according to John, "It is just the next space."

Near the end of session 2, Tina rolled a 4. Until this point, she had always used her fingers except when rolling a 1 and one other instance of rolling a 2. To move four spaces, she bobbed her head twice, paused, and bobbed her head twice again. "What did you just do?" I asked.

Tina responded, "Two and two is four!" She had a big grin on her face, knowing she had done something special. I had her repeat the strategy for the other children, to consolidate her thinking and to facilitate the others noting and perhaps adopting Tina's strategy.

During session 2, John developed a novel approach to the problem of rolling a 6. Near the beginning of session 2, when John rolled a 6, instead of using one finger of my hand and five fingers of his hand, he spontaneously used a new strategy. After reaching five by using his

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Three of the four children used head bobs to quickly develop a decomposition strategy.

Teacher modeling mitigates against frustration.

fingers to track, he hesitated and then moved one more space.

Decomposing and subitizing

At the beginning of session 3—after I had given some reminders and had modeled how to play the game and keep track of spaces moved using fingers—I asked Tina to recall how she had moved four spaces without using her fingers. Tina remembered that she had decomposed a 4 during the last session and had used the game board to demonstrate for everyone. During session 3, Tina's use of a decomposition strategy quickly developed. She did rolls of 1, 2, and 3 quickly and easily, as if the amounts were subitized (quantity known without counting). She handled a roll of a 4 as first observed in session 2. Rolls of a 5 and a 6 were initially done using fingers, but both were eventually done by decomposing: a 6 as three head bobs, a pause, and three more head bobs; a 5 as two head bobs, a pause, two head bobs, a pause, and one more. Tina appeared to decompose on the basis of dice patterns, although she was unable to articulate how she decided to decompose.

Sara's and John's ability to use a decomposition strategy also emerged, but more slowly than it had for Tina. Sara and John eventually consistently subitized rolls of a 1 and a 2, and they decomposed higher dice rolls using their fingers or head bobs. John, for example, used three head bobs, a pause, and two more for a roll of a 5; Sara preferred to use her fingers for rolls of a 5 and a 6.

The fourth child, Nathan, continued to struggle to independently keep track of how far a token had moved, and he used his fingers during all four sessions. He required hand-over-hand help, or he would use my fingers. He eventually used his own fingers independently. At the end of week 3, he could use his fingers for die rolls of 1, 2, and 3 and needed help for die rolls of 4, 5, and 6, but this progress was forgotten at the beginning of week 4. Each week, Nathan progressed toward independence but then regressed by the beginning of the next session.

The emergence of counting on

After playing the game for four weeks, I directly taught the students how to count on, using the second method described in a previous section. Tina, John, and Sara seemed to immediately develop proficiency at counting on. They accurately used the strategy while playing

Make Ten. The following week, we played a dice game for which only a reminder was needed for these children to continue accurately and confidently using counting on. Several weeks later, the teachers of these children reported independent use of counting on during math class. Tina, John, and Sara appeared to have developed proficiency using counting on because they seemed to be consistently choosing the counting-on strategy to accurately and confidently determine total quantity in various contexts (i.e., with dice, counters, and playing cards as well as on a number line).

Nathan, who had struggled to develop consistency with using his fingers to keep track of spaces he had moved while playing the board game, did not develop proficiency with counting on. His pattern each week seemed to be some progression followed by regression at the beginning of the next session. Nathan's inconsistency suggests deeper cognitive difficulties, which I am still trying to understand.

From counting all to counting on and beyond

The potential of Siegler and Ramani's number board game, in terms of preparing students to progress from persistent use of counting all to proficient use of counting on, appears to be the emergence of a strategy to keep track of spaces landed on during a game move. In this case, three children developed the use of a rhythmic head-bobbing strategy to keep track. These children eventually used a heuristic and multiple strategies based on number size: If you make a small die roll, then keep track by subitizing; and if you make a larger die roll, then keep track by decomposing into smaller numbers that can be dealt with by subitizing. In their rhythms and head bobs, these children are experiencing and practicing decomposition. This is not a metacognitive awareness, but it does seem to prepare these students for progressing from persistent use of counting all to proficient use of counting on.

This special number board game could have potency beyond counting on. For example, the board could be in rows of 10 from 1 to 50 (or higher), with a die from 1 to 10 (or higher). A child could develop the same strategies for decomposing numbers 6–10 into numbers 1–5. A child may become strategic in decomposing a die roll by making a multiple of 10 (e.g., if you are at the 38 and you roll a 7, move 2 spaces to the 40 and add 5 more to get to the 45). Thus, the game is an opportunity for children to develop strategies that are more sophisticated, progressing from finger use and/or counting to mental computation. The special requirement for a game move is critical, so that children do not persist with counting spaces, thereby developing other strategies.

When working with children who struggle with mathematics, a balance between teacher modeling of a strategy and opportunities for students to invent a strategy is important. Teacher modeling mitigates against frustration and encourages progress, but it needs to be balanced against the temptation to do the work for children who struggle to invent their own strategies.

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