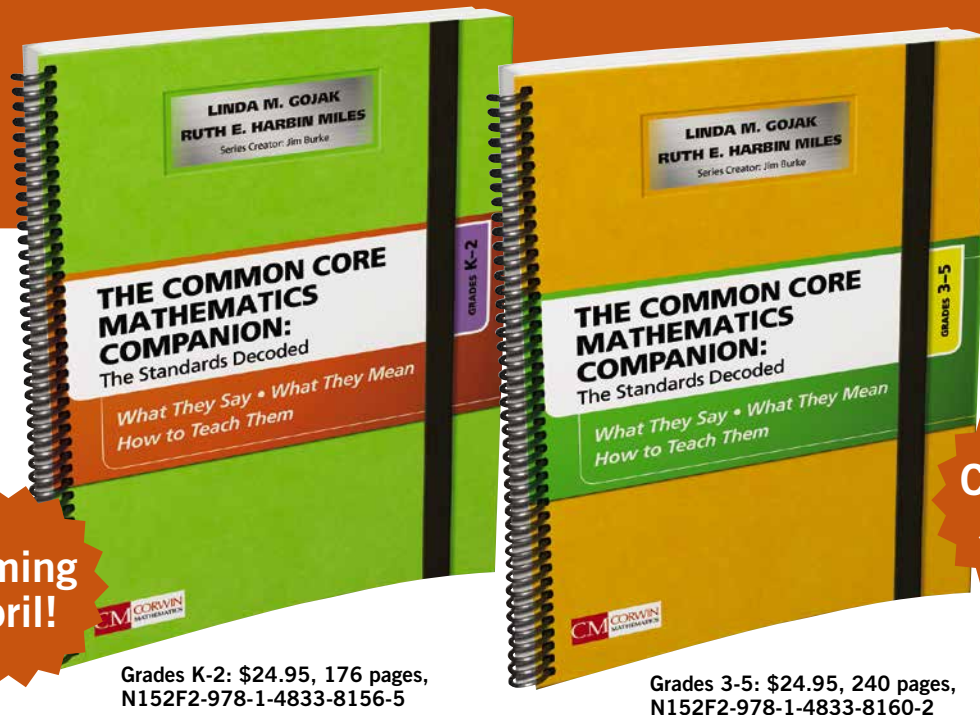


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Teaching Children Mathematics (ISSN 1073-5836) (IPM 1124463) is published monthly except June and July, with a combined December/January issue, by the National Council of Teachers of Mathematics at 1906 Association Drive, Reston, VA 20191-1502. Periodicals postage is paid at Herndon, Virginia, and at additional mailing offices.

POSTMASTER: Send address changes to *Teaching Children Mathematics*, 1906 Association Drive, Reston, VA 20191-1502. Telephone: (703) 620-9840; orders: (800) 235-7566; fax: (703) 476-2970; email: nctm@nctm.org; World Wide Web: www.nctm.org.

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Making formative assessment multidimensional

KYLE T. SCHULTZ AND KATERI THUNDER

The emphasis on standardized testing for more than a decade has caused many teachers to cringe at the mention of the term *assessment*. Although preparation for end-of-year, high-stakes tests can be stressful for students and teachers alike, other types of assessment can enhance mathematics instruction, keeping it harmonious, productive, and efficient. The recently published *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics [NCTM] 2014) names assessment as one of the essential elements of effective school mathematics programs, describing specific beliefs and practices with respect to the assessment necessary to ensure that all students succeed in mathematics. In particular, effective assessment “includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement” (p. 5). Planned, ongoing, and continual data collection during instructional activities, known as *formative assessment*, enables teachers to make decisions about the mathematics they teach as well as how they teach it. When done effectively, formative assessment can drive instruction and result in more efficient use of instructional time.

What effective formative assessment looks like

Whereas summative assessment data, usually in the form of test scores, present

a concise picture of a students’ achievement with respect to district or state standards, formative assessment data can provide a continuous multidimensional view of students’ mathematical thinking and understanding. Van de Walle, Karp, and Bay-Williams (2013) highlighted this distinction using the analogies of summative assessment as a digital snapshot and formative assessment as a streaming video. Much like a video provides additional dimensions of sound and movement, formative assessment can provide additional detail regarding students’ mathematical proficiency, such as understanding and misconceptions with respect to particular concepts, reasoning and problem-solving skills, and attitudes toward mathematics. For example, during a unit of instruction (in this case, a unit on subtracting multidigit whole numbers), a teacher might use the following variety of techniques:

- Contextual performance-based tasks that allow students to use a variety of strategies and ways of thinking about a concept to solve a task: *Mary sold 85 boxes of cookies during the first two weeks of the fundraiser. After the third week, she had sold a total of 119 boxes. How many boxes did she sell during the third week?*
- Writing prompts that elicit student thinking about learned concepts and procedures: *When calculating*

300 – 48, Roberto said you can just take away 1 from both numbers, 299 – 47, to make the problem easier. Will his method always work? Why do you think so?

- Student self-assessments: *How well do you think you understand subtracting numbers with more than one digit? What is the most difficult thing to remember when subtracting?*
- Observations of students during classroom activities and discussions and systematic documentation of these observations: for example, using an observation guide that allows the teacher to quickly note students’ performance on a task as proficient, developing, or novice as well as to record relevant details.

Regardless of the technique used, NCTM urges teachers to ask, “What evidence



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will this provide about students' mathematical knowledge?" (2014, p. 94) when selecting formative assessment tasks.

Good first steps for the classroom

Holding productive beliefs about assessment is crucial to implementing effective practices. These beliefs include viewing assessment as (a) having the primary purpose of improving teaching and learning, (b) an ongoing process embedded in instruction, and (c) requiring a variety of forms to measure mathematical understanding and processes (NCTM 2014). With these beliefs in place, teachers can analyze and reflect on their practices using existing frameworks, such as Wiliam's (2000) four elements of effective formative assessment:

1. Questioning strategies: Do I ask questions that elicit students' current understandings and misconceptions of mathematical content as opposed to questions that elicit simple one-word or yes/no answers?
2. Providing feedback: Do I give students comments that enable them to build on their current understanding as opposed to evaluating their progress in terms of final outcomes ("right or wrong") with respect to the learning goal?
3. Sharing criteria: Do I share and discuss with my students my goals for their learning and the benchmarks I use to evaluate their work and mathematical thinking?

4. Student self-assessment: Do my students have the opportunity to evaluate and reflect on their own progress toward learning goals using these same criteria?

By considering these questions, teachers can begin to think about broadening their formative assessment practices, moving from static snapshots to a richer and more fluid model of their students' learning and mathematical understandings, leading to mathematical success for all.

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Response to the November Coaches' Corner

Your Coaches' Corner in the November issue of *Teaching Children Mathematics* ("A Closer Look at Mathematical Practice 6: Attend to Precision," p. 199) was incredibly helpful! I used this to help me give teachers words and visuals they could use with parents at our upcoming parent/teacher conferences, while also delivering a staff-wide message as to what this mathematical practice really means. I'd love to see you do something with each of the other seven practices. Thank you again for such a wonderful message!

Robin Vechazone

Instructional Specialist
West Maple Elementary School
Bloomfield Hills, Michigan

Ed. note: For "A Closer Look at Mathematical Practice 1" and "A Closer Look at Mathematical Practice 3," see the News & Views department of *TCM* December 2013/January 2014, p. 279, and September 2013, p. 70, respectively.

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Excel as a math coach: 7 surefire strategies

BY ROBYN SILBEY, PD AND CAMPUS CONSULTANT

As educational leaders, coaches teach teachers with the goal of ensuring that all students become proficient in, and enthusiastic users of, mathematics. Meeting this goal can be accomplished more easily if a coach is part of a thriving, cohesive community of learning leaders who value knowledge in all content areas. With this in mind, here are seven strategies to help coaches reach their goals:

- See math as part of the big picture. View the mathematics program objectively from a variety of lenses, optimizing the academics and values of the school. Compromises in time, personnel, and budget can be expected as math takes its part within the total program. Patience and grace are appreciated by colleagues and reciprocated when appropriate.
- Enlist and manage colleagues. Great coaches are effective managers. A clear vision accompanied by a specific implementation plan and committed people—ideally from a cross-section of stakeholders—will bring the vision to fruition.
- Prioritize. A coach helps teachers channel their efforts. With a coach's help, teachers can identify what must be implemented immediately and what can be worked on for the short term. A coach can anticipate and value the long-term, end result and relay that to teachers—as well as to administration—if needed.
- Share professional upgrades. A coach will share opportunities for growth that may be received with mixed feelings. A coach's job is to convey the value of growth; champion the upgrades as occasions to learn; and offer as much support as possible during transition periods.
- Grow those around you. A coach's greatest asset is the ability and willingness to mentor others. Encourage those who are seen as potential leaders to volunteer for special activities, share ideas in public forums, and seek additional growth prospects.
- Celebrate growth for all. A coach is expected to be the expert—clearly articulating and guiding the math program. As obstacles arise, a coach is expected to address them. The staff will appreciate seeing a coach reach out to obtain additional information, data, and answers to the staff's concerns.

We need strong leaders if math education is to improve for all students. Effective coaches make the best of their leadership roles.

Questions? Comments? Contact robyn@robinsilbey.com.