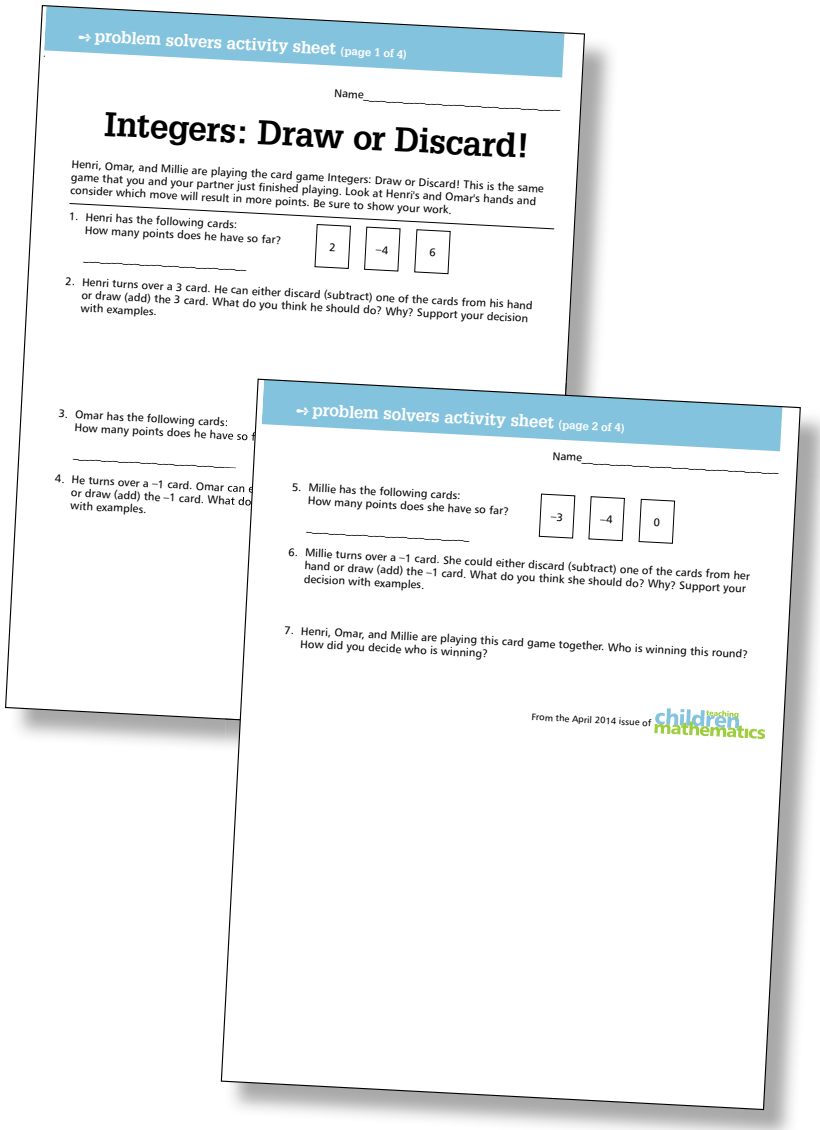
Laura Bofferding and Nicole Wessman-Enzinger

Integers: Draw or Discard!

In the April 2014 problem scenario, games are used to promote the learning of integers. Math games create a friendly environment for classroom discourse and negotiation around the meaning of operations with negative integers. To access the full-size activity sheet, go to **www.nctm.org/tcm**, Back Issues.



problem solvers activity sheet (page 1 of 4)

Name_____

Integers: Draw or Discard!

Henri, Omar, and Millie are playing the card game Integers: Draw or Discard! This is the same game that you and your partner just finished playing. Look at Henri's and Omar's hands and consider which move will result in more points. Be sure to show your work.

1. Henri has the following cards:
How many points does he have so far?

2. Henri turns over a 3 card. He can either discard (subtract) one of the cards from his hand or draw (add) the 3 card. What do you think he should do? Why? Support your decision with examples.

3. Omar has the following cards:
How many points does he have so f

4. He turns over a −1 card. Omar can e
or draw (add) the −1 card. What do
with examples.

problem solvers activity sheet (page 2 of 4)

Name_____

5. Millie has the following cards:
How many points does she have so far?

6. Millie turns over a −1 card. She could either discard (subtract) one of the cards from her hand or draw (add) the −1 card. What do you think she should do? Why? Support your decision with examples.

7. Henri, Omar, and Millie are playing this card game together. Who is winning this round? How did you decide who is winning?

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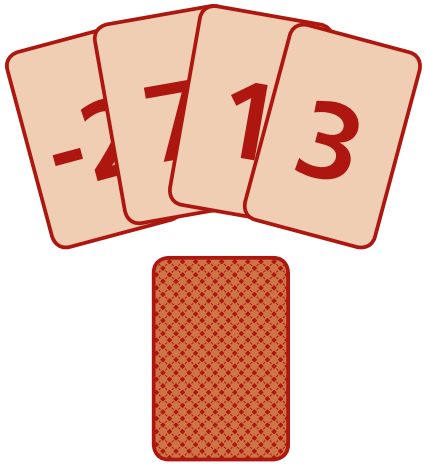

The inclusion of negative numbers in the April 2014 Problem Solvers card game encourages students to consider multiple meanings of the minus sign and the relation of negative numbers to positive numbers so they can make strategic moves.

Walter Stark, a fifth-grade teacher at St. Pancras School in Glendale, New York, used the integer game and wrote, "This was a great lesson that will lead into a much deeper understanding of integers."

Nicole Wessman-Enzinger also facilitated this activity with third graders at Herscher Grade School in Herscher, Illinois. She was impressed by the sophisticated approaches of the students, despite having had no previous instruction on integers. In response to the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), many states introduce integers to students in sixth grade. Yet the NCTM (2000) Standards for grades 3–5 advocate that students "explore numbers less than 0" (p. 148). Students who have no experience with negative numbers may incorrectly apply the commutative property to subtraction problems (e.g., trying to solve 2 – 7 as 7 – 2) (Bofferding 2011), especially when regrouping is required (e.g., trying to solve 22 – 17 as 27 – 12) (Fuson 2003). During the game, as

students debated whether they should draw or discard, they used mathematical practices suggested in CCSSM, such as constructing viable arguments and critiquing the reasoning of others (Standards for Mathematical Practice [SMP] 3, CCSSI 2010).

Introducing the problem and playing the game

Stark began by playing a miniround of the game with his class and modeling how to keep score. When subsequently playing the game in small groups, students quickly began developing strategies and operating with integers. Stark reflects,

> It was amazing that many students already had developed a strategy [for] which cards to keep and which to play…. They were adding and subtracting positive and negative numbers with ease and getting correct answers.

Wessman-Enzinger also played a miniround of the game with her class but used only whole numbers. This led to debate when students encountered negative numbers in their small groups. The students played a few rounds, presented their ideas regarding the use of "new numbers" and symbols, and then played again. Wessman-Enzinger noticed that some students treated negative numbers as zero, whereas others ignored the negative signs and treated the numbers as positive.

Both teachers had class discussions about students' strategies before moving on to the problem-solving sheet. The students' explanations illustrated that they were working with deep mathematical ideas related to the three meanings of the minus sign (Gallardo and Rojano 1994) as well as integer values and order (see **table 1**).

TABLE 1

Some students treated negative numbers as zero, whereas others ignored the negative signs and treated the numbers as positive. Nevertheless, students' explanations of their strategies illustrated that they were working with deep mathematical ideas related to the three meanings of the minus sign as well as integer values and order.

Students' strategies and statements	Integer ideas addressed
One third grader treated −4 just like "minus four."	A student identified the minus sign as designating subtraction.
Third and fifth graders alike referred to the opposite nature of positive and negative numbers, and one third grader used language similar to the additive inverse, indicating that −4 would cancel with 4.	Students identified the minus sign as designating the opposite.
One fifth grader, Matthew, explained, "If I had to keep a negative card, I wanted one that was not that high. Like −1 is better to have than −3." Darwin, another fifth grader added, "That is because the negatives go farther away from the positive as they get bigger."	Students identified the minus sign as designating a negative number. Students discussed the order of negative numbers in terms of which are farther away from the positive numbers.
Third and fifth graders also indicated that they tried to get rid of negatives because they are not worth as much as positives. Further, both groups added that larger or bigger negatives equal more points lost.	Students flexibly interpreted how integer order relates to integers' values; many students recognized −3 as having a larger magnitude than −1, at the same time realizing that −4 corresponds to a smaller score.

Solutions to the activity sheet

The third- and fifth-grade classes displayed interesting differences in how students reasoned about each problem.

Henri's card hand of 2, −4, 6 (add 3 or discard)

Most third graders argued that Henri should discard the −4 to get a total of 8 points. One-third of the fifth graders suggested that Henri should take the 3 because it would increase his score to 7. Darwin reasoned, "He will have more points than 4," which was Henri's initial total. This reasoning considers only the direct impact of adding 3 to Henri's hand.

Another third of the fifth-grade class claimed that Henri should discard the −4. Students argued that Henri would have all positive numbers or that he would not have any points taken away. These arguments demonstrate insight that positive numbers are greater than negative numbers and that adding a negative number leads to smaller point totals.

FIGURE 1

Sophie S. explained Omar's situation. Some of the fifth graders' language suggested that they did not interpret negatives merely as positive numbers to be subtracted.

If Omar takes and keeps the negative 1 card he will have 11 points. If he discards the least value card with is 2 he will have 10. The fist choice ends with more points. Now he has 11 points

FIGURE 2

Third graders Jaxon and Xander justified that the total of a card hand with –3, –4, and 0 is –7.

Two students took a sophisticated approach and compared scores from different moves to determine the best move: Henri would have 7 points if he took the 3, and 8 points if he discarded the –4.

Omar's card hand of 3, 7, 2 (add –1 or discard)

Several third graders wanted Omar to discard the 2 because they did not want him to take a negative card, suggesting that they were not considering the alternate scenario. The third graders disagreed about which strategy Omar should take, but the groups agreed on adding –1 after students took turns presenting arguments for each move. Almost all the fifth graders, in contrast, suggested that he take the –1 card. They demonstrated that if Omar discarded the 2, he would have only 10 points, but if he took the –1, he would have 11 points (see **fig. 1**). One fifth grader added that subtracting 1 is preferable to subtracting 2. Although students used subtraction language when operating with the negatives, the fifth graders frequently referred to the numbers as *negative*. For example, Hayley discussed "adding the negative 1 card," suggesting that she (and others) did not merely interpret negatives as positive numbers to be subtracted.

Millie's card hand of –3, –4, 0 (add –1 or discard)

Millie's problem led the third graders to debate her initial point total: –1 or –7. Some children used discrete representations (e.g., dots) and argued that you could think about –3 + –4 as 4 – 3 = 1, only the answer would be negative. This explanation indicates that students relied on whole-number reasoning to make sense of the situation; however, they knew that the negative sign had meaning and incorporated it into their reasoning. Those who used a number line (see **fig. 2**) argued that the answer should be –7 because you keep "moving down" or "getting farther from 0." Those who interpreted the relative effect of a negative on the score, "the bigger the negative, the worse it is," decided to discard the –4 card.

The fifth graders almost unanimously agreed that Millie should discard the –4, referring to her resulting score; she would "increase her points to –3." Five students compared the effect of discarding the –4 with taking –1, which would leave her with –8 points. Ryan explained, "Because adding the [–1] card subtracts points." Ryan's statement highlights that despite using negatives as subtraction, students interpreted negatives as a new category of numbers.

Reflections

Overall, students made progress in thinking about the distinction between magnitude (distance from zero) and directed magnitude (distance from zero in a particular direction). Several students simultaneously thought about relative negativity, where –5 is more negative than –3, and relative values in terms of points, where –3 is a higher score than –5. This is a sophisticated level of integer value understanding (Bofferding 2014) not addressed until sixth grade in CCSSM (CCSSI 2010). Some third graders continued to operate as if negative numbers were positive numbers and then made the answers negative. Making analogies to the positive numbers can be helpful when adding numbers with like signs (e.g., –5 + –3). However, analogies can break down when summing addends with opposite signs (e.g., –5 + 3),

and having students provide justifications for their reasoning pushed them to consider the integer values in new ways. Discussions that incorporated justification of students' strategies supported students' advancement toward conceptualizing negative values and operations with them. Stark reflected on this:

> Many of the students were debating answers. . . . My students were using arguments and mathematical reasoning to answer these questions. . . . All of this knowledge was obtained from the game and from their prior understanding of integers.

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Laura Bofferding, lbofferd@purdue.edu, is an assistant professor of mathematics education at Purdue University in West Lafayette, Indiana. She investigates young children's numerical reasoning, especially in relation to integers, and is interested in the identification and discussion of children's diverse solution strategies in lessons. **Nicole Wessman-Enzinger**, **nmenzinger@gmail.com**, is a PhD candidate in mathematics education at Illinois State University in Normal, Illinois. She is interested in how children think about negative integers and how they learn to use them in various contexts. Edited by **Erin Moss**, **erin.moss@millersville.edu**, an associate professor in the mathematics department at Millersville University of Pennsylvania, and by **Signe E. Kastberg**, **skastber@purdue.edu**, a teacher of prospective elementary school teachers at Purdue University in West Lafayette, Indiana. Each month, this section of the department showcases classroom results of using problems presented in previous issues of *Teaching Children Mathematics*. Go to **www.nctm.org/tcmdepartments** to find detailed submission guidelines for all departments.