Area Conceptions *Sprout* on Earth Day

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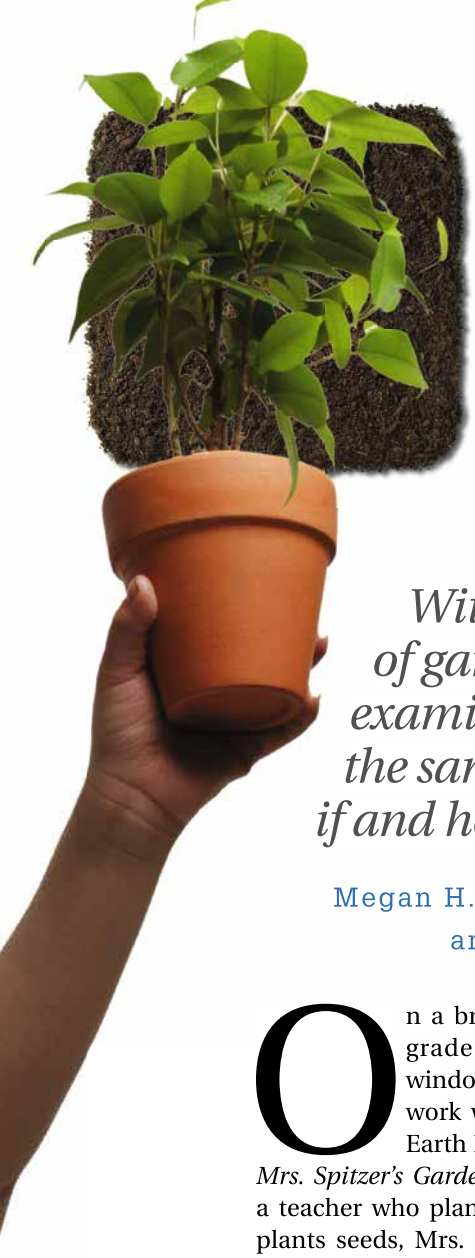


Within the context of gardening, students examine rectangles with the same perimeter to see if and how their areas differ.

Megan H. Wickstrom, Julie Nelson, and Jean Chumbley

On a breezy spring day, our second- and third-grade students looked out their classroom windows and imagined what it might be like to work with their teachers to build a garden for Earth Day. The students had just read the book, *Mrs. Spitzer's Garden* (Pattou 2001), which tells the story of a teacher who plants a garden each school year. When she plants seeds, Mrs. Spitzer knows exactly how to help each plant thrive and grow to its potential. While reading the book, we inserted our names and school into the story so that students could imagine themselves in the plot. Our class had spent several weeks discussing seeds and plant growth, so we ended the story by talking about what is important when planting a garden. Several students suggested that plants need space to grow because they compete for food and water. This concept of space sparked our Gardening and Area Measurement lesson.

SOIL: EASY_ASA/THINKSTOCK; GIRL: XIXINXING/THINKSTOCK

With the adoption of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), many concepts related to area are covered in third grade:

- **Recognizing** area as an attribute of a plane figure
- **Understanding** that a square with a side length of one is a unit square
- **Measuring** area by tiling figures and counting the squares it takes to cover without gaps or overlaps
- **Relating** area to the operations of multiplication and addition

Area concepts take time and experience to work through and understand. Researchers (Battista 2007, Sarama and Clements 2009) have shown that when students first encounter area, they often find difficulty structuring square units in rows and columns. Students will overlap area units or will leave gaps between them as well as create nonuniform units.

In response to these concerns, Miller (1984) and Wolf (1995) have documented that when students are allowed to use physical square units or tools, they are more likely to develop strategies consistent with multiplicative rules and also develop mental imagery. To build spatial awareness, students need to solve problems through mathematical modeling, allowing interaction with and sense making of their world (Lesh and Zawojewski 2007).

SHOWCAKE/THINKSTOCK

The lesson

We wanted to encourage our students to conceptualize the idea of area beyond the formula. On the basis of the research ideas, we decided to create a measurement lesson centered on gardening and plant growth. Students investigated area and perimeter by constructing rectangular gardens using the same perimeter of fencing to see if and how this affected the area. We incorporated estimating, drawing, building, and checking to help students develop and refine their conceptions of area. This lesson took approximately three days of instruction, one day for part 1 of the lesson and two days for part 2.

Part 1

On the first day of the lesson, we spent fifteen minutes reading and discussing the story of *Mrs. Spitzer's Garden* or, in our case, Mrs. N's garden. After discussing the story, we posed the following task and recruited our students to help us design a classroom garden:

> Mrs. N wants to plant a rectangular garden at school for her students. She has 16 feet of fence that she can use to make a garden. Before building, she needs to make a plan. She wonders how many garden possibilities she could make with 16 feet of fence. While getting supplies, Mrs. C tells Mrs. N that it does not matter how she lays the fence down; all the gardens will look the same. What do you think?

- **Draw:** Take five minutes to think about the question. Can you draw a garden that will work?
- **Build:** If you like, grab a bag of Popsicle® sticks and see if you can build any other gardens. Make sure that you record your new garden on a piece of paper.
- **Discuss:** Compare your drawing with another group's. Is it the same or different? How?

Before students began working on the problem, we had a short whole-class discussion about the properties of a rectangle to aid students in constructing their gardens. Then we asked them to think about the first prompt independently for a few minutes: "Can you draw a rectangular garden with sixteen feet of fence for Mrs. N?" They each received a piece of paper to write their ideas and draw a garden possibility. After students had made one garden, we asked, "Is this the only garden Mrs. N can make, or can you draw another?"

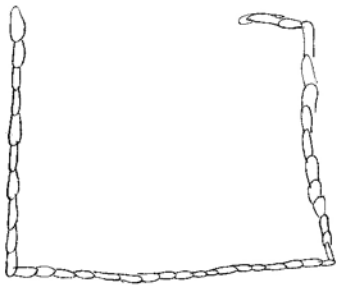
The purpose of the drawing phase was to give us an idea of how our students thought about number and space independently (see **table 1**). Almost all the students were able to draw their interpretation of a garden. Some

TABLE 1

Students' drawings of possible gardens highlighted common strategies that the authors observed.

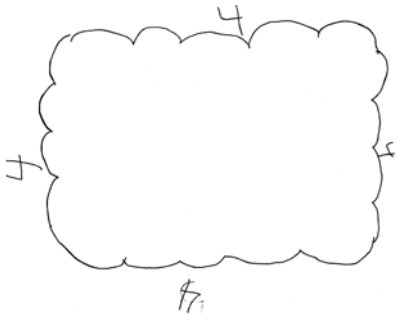
(a) Student response: "I can't make a rectangle that will work. I drew the pieces of fence and counted them, but there are too many."

The student drew the fence by making one-foot lengths and connecting them. This did not always result in a garden with a perimeter of 16 feet.



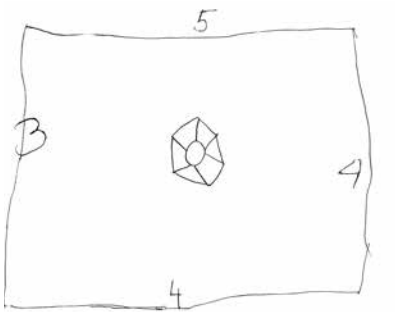
(b) Student response: "I thought of numbers that would make 16 and I know 4 + 4 + 4 + 4 = 16. Each hop represents a piece of fence."

The student thought about possible number combinations that would result in a rectangle with a perimeter of 16 feet, then drew lengths or hops to show the perimeter.



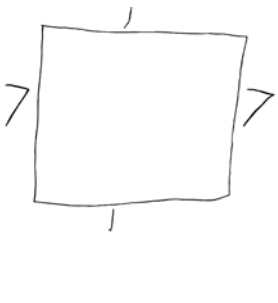
(c) Student response: "I made a garden with a flower in the middle. I thought of four numbers that make 16 for each of the sides because 3 + 5 + 4 + 4 = 16."

The student thought about possible side length combinations that would result in 16 feet of fence but forgot about the properties of a rectangle.



(d) Student response: "I thought of two numbers for the different sides that I could make 16 with. I know this works because 1 + 1 + 7 + 7 = 16."

The student correctly found rectangle side lengths that would result in a perimeter of 16 feet, but did not consider the lengths in relation to one another.



(e) Student response: "I thought of two numbers that could be the side lengths, 6 and 2. I know this because 6 + 6 + 2 + 2 = 16. The 6 is the longer side, and the 2 is the shorter side."

The student was able to correctly find rectangle side lengths that would result in a perimeter of 16 feet as well as draw the side lengths in proportion to one another.

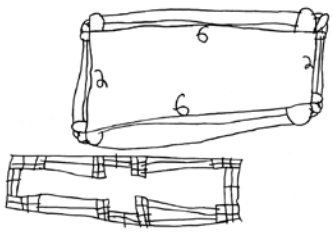


FIGURE 1

Given sticks, many students who had been unable to draw a garden began to draw rectangles to scale and required larger sheets of paper to record their findings.

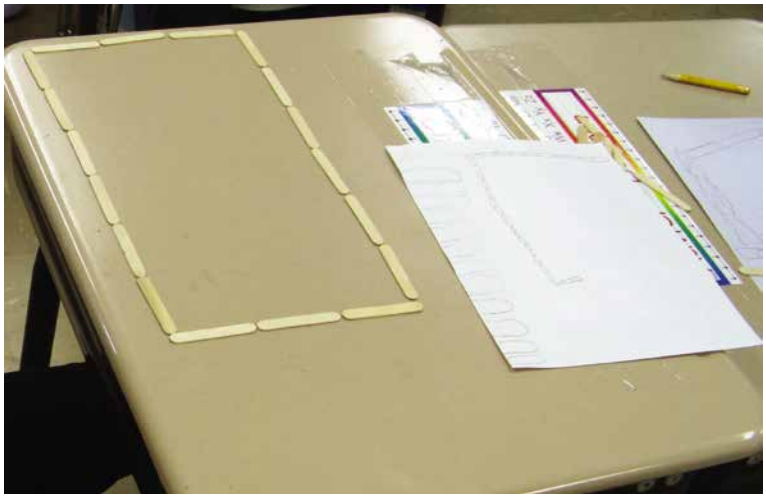
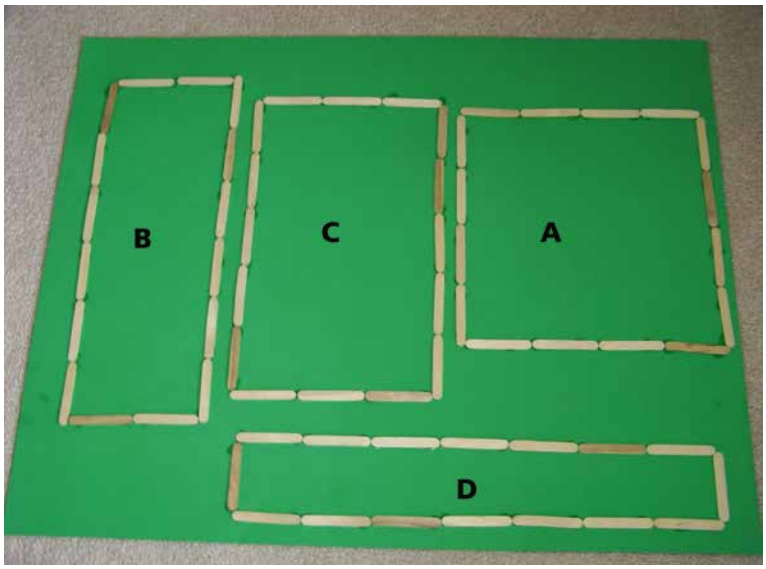


FIGURE 2

Students changed initial estimates of possible gardens with whole-number side lengths from seven to four when they realized that several gardens were the same but rotated.



students began by making lengths of fence and connecting them until they had sixteen or more. This was problematic because students wanted the sides of the rectangle to connect but often needed more than sixteen feet due to their initial construction. Half the class was able to reason about viable side-length combinations. Some students thought about a possible combination and then drew individual units or hops to represent the lengths. Others understood that the side lengths should sum to sixteen feet but either forgot the properties of a rectangle or did not draw the sides in proportion to one another. Only two or three students in the class were able to think of a correct solution as well as draw the side lengths in proportion to one another.

The goal of the lesson was to eventually investigate area, so we thought that visualizing and constructing rectangles that were in proportion to one other were important tasks for students to do. The drawings showed us students' thinking and gave students ideas to work from but could be confusing when comparing areas. To prepare for the second lesson after working independently, students were paired, given sixteen Popsicle sticks to help them generate ideas, and asked to document their findings using the sticks.

With the addition of sticks, the task became easier and more accessible for all students. The student whose drawing was featured in **table 1a** was unable to construct a drawing of a garden composed of sixteen square feet, but when given sticks, he was able to construct and draw a new rectangle (see **fig. 1**).

After students found a solution, they were to glue it to a poster board. To end the first lesson, everyone was asked to reflect on these questions: How many garden choices does Mrs. N have? Have we made all the possible choices? They were to reflect, as well, on the strategies they had used to construct the gardens. Several students discussed what they called the "add-and-take-away" strategy. They found that if they added one to each of the lengths and took away one from each of the widths, then they could generate a new garden. Other students discussed their observation that the length and the width always added to eight, so they had tried to think of number combinations with a sum of eight. When we asked why they thought it added to eight, most were unsure. One student suggested that it was because we made it halfway around and half of sixteen is eight.

Next we asked students, "Do you think we found all the gardens?" The class initially came up with seven gardens that had whole-number side lengths. After some discussion,

this number decreased to four (see **fig. 2**) because students realized that several of the gardens were the same but rotated. Many students thought maybe more could exist but were unsure of construction. A few students suggested that partial feet could be used to construct such gardens as 3.5 feet by 4.5 feet. As a class, we discussed that these numbers still fit the properties of a rectangle as well as added to a perimeter of sixteen. Because this lesson transitioned into area concepts, we discussed rational number solutions for perimeter but told students that for our garden, we would first explore only whole-number side lengths.

Part 2

The second piece of the lesson, which lasted about two days, focused on exploring area concepts. We began our discussion by recalling students' comments about space, sharing that farmers and gardeners often create garden plots so they can ensure that plants will have enough space to grow. We showed students a picture of a garden plot (see **fig. 3**) in relation to their gardens, and we posed the following problem:

> Mrs. N wants to grow as many plants as she can in her garden. She knows that each plant needs a certain amount of space so it can grow well. She wonders which garden would be the best for her to have the largest number of garden plots or whether all the gardens are the same. Another teacher, Mrs. W, thinks all the gardens will have the same amount of space because they all have a perimeter of sixteen feet of fence. What do you think?

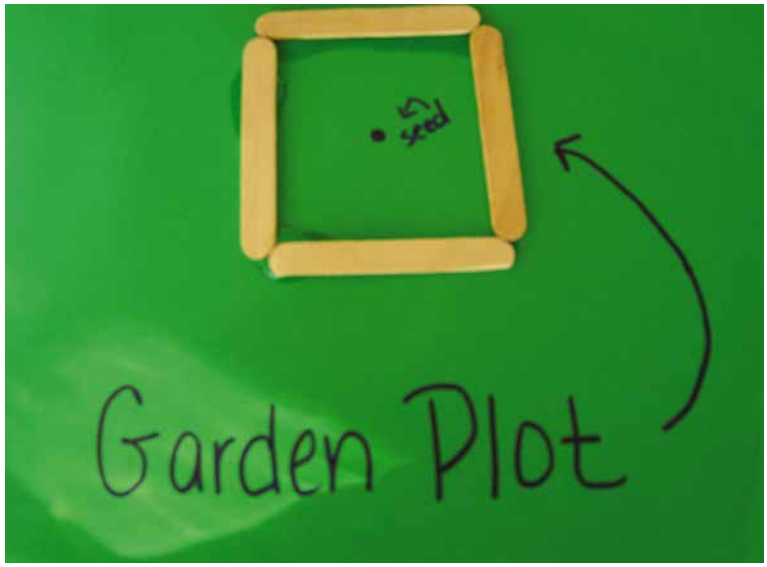
- **Predict!** Just by looking [*at* **fig. 2**], which garden do you think will have the most garden plots? Which one will have the least? Or will they all have the same?
- **Measure.** Find a garden plot and use squares and/or Popsicle sticks to help you measure the number of plots.
- **Record.** Write on the chalkboard the number your group decided on.
- **Discuss.** Talk with someone from another group about your measurements.

Predictions

We asked students initially, "How many garden plots do you think will fit in each garden?" Students were to write their initial predictions on a notecard to share with the class. Surprisingly, class responses were almost evenly distributed. Several students picked garden D (in **fig. 2**) as the biggest because it was the longest, and several others chose garden C because it was the widest. Students who chose garden A or B did so because those gardens were somewhat long and somewhat wide. To our surprise, none of the children thought all the gardens would contain the same number of garden plots. We also asked students to estimate *how many* garden plots would fit into each garden. Some students' estimates were fairly close, such as estimating that nine plots would fit in garden D; and some

FIGURE 3

The second part of the lesson lasted about two days and had students compare their gardens with a picture of a garden plot.



The goal of the lesson was to eventually investigate area.

estimates were high or low, such as estimating that fifty plots would fit in garden C.

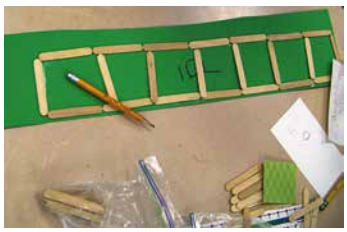
Measuring

We knew that structuring area units is often difficult for young students and that they need tools, such as square tiles, to help them develop conceptions of area and space. To help advance students' thinking, we provided paper garden plots and additional Popsicle sticks. Students were encouraged to use what they needed to help them solve the problem.

As we walked around, we saw four primary strategies implemented equally across the classroom (see **table 2**). Some students began by grabbing as many square units as they could, covering their garden, and counting the total, one by one. This was problematic because students often left gaps between plots or overlapped them. Other students grabbed sticks

TABLE 2

Student strategies to find area included these.

	Strategy 1: "I put as many garden plots in the garden as I could." Students used the green tiles that represent garden plots to completely cover the rectangular space. Sometimes this method was problematic because students overlapped garden plots or left spaces in between garden plots.
	Strategy 2: "I used Popsicle sticks to make garden plots like the one you showed us." Students used Popsicle sticks to partition the rectangle into garden plots. This strategy was almost always effective for the student in finding the correct number of area plots.
	Strategy 3: "I took one square and put it in the corner of the rectangle. Then I put my finger down to remember my spot and put the square on the other side. I did this for all the rows and counted as I went." The student used one garden plot and iterated it or flipped it (using a finger to mark her place) to find the total number of garden plots. This was not always an effective strategy because the student often left large gaps with fingers.
	Strategy 4: "I put down four squares across the bottom. I saw three rows above, so I skip-counted 4, 8, 12, 16." The student found one row or one column of garden plots and repeated it across or down (usually skip counting) to find the total number of garden plots.

and connected them to make garden plots. This strategy was usually effective because students could imagine and construct the outlines of the plots. A couple groups grabbed only one square and moved it down a column and then across, using their finger as a place marker. Students who used this strategy were often off in their measurement as well because they did not consider that fingers take up space. One group of students used squares to measure one row or one column and then skip-counted up or across the rectangle. This particular group was usually effective at finding the correct area measurement.

Recording and discussion

After groups had each measured the gardens, they recorded their findings on the board. We saw that for B (2 × 6) and D (1 × 7), most responses were identical. Students said they could visualize where the garden plots would go. One student said that for garden B, she could see two squares sitting next to each other and then counted by two down the length of the rectangle. We then asked the class, "How do you know where a garden plot ends?" Students stated that they could see that a plot would stop at the end of a stick because it was the same length.

Because garden A (4 × 4) and garden C (3 × 5) (see **fig. 2**) seemed to have a variety of answers, we asked students to explain their strategies. Group members explained and/or physically showed what they had done. As a class, we discussed whether a way exists that we could visualize garden A or garden C as we had garden B and garden D. We asked, "How could we really know what the correct measurement is?"

Students were to check their method by filling the entire garden with plots. Students became unsure when they got to the inner part of the wider gardens and it became harder for them to visualize the squares and to see the rows and columns as they had with the longer gardens. After several minutes of checking, we met for a whole-class discussion.

One group claimed that it laid down only as many squares as could fit in one row and then repeated the row over and over again until it reached the bottom. Other groups discussed that we could do the same thing with columns and skip-count across. A student even exclaimed that either strategy would work for measuring the garden because 5 × 3 is the same as 3 × 5. To visualize their descriptions, we taped square units together so that students could imagine the row or column moving across the garden. The class liked this strategy; students could visualize units in a row and could also visualize the row moving.

Lesson wrap-up and reflection

To end the lesson, we revisited our initial question of which garden held the most plots and, more important, why. By examining their work, students could see that garden A (4 × 4) would hold the most garden plots. Students were asked to think about how they could solve this question for a larger space without laying out tiles. Several students responded that we have to consider the length and width of the garden and look at how many area plots are in each row or column. Some students made the connection to multiplication and said they could see that the area was

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Four primary strategies were implemented equally across the classroom.

FIGURE 4

To end the lesson, students used containers of soil to plant seeds in an indoor garden.

going to be the number of plots in a row multiplied by the number of columns. Our goal for the lesson was for students to begin to visualize row-and-column structure, so we accepted students' thinking about area in either an additive sense or in a multiplicative sense. As the unit progressed, we thought more students would readily make the transition between the two. We ended the lesson by giving students containers with seeds and dirt to plant the garden (see **fig. 4**).

The construct of gardening allowed for fluidity between many mathematical concepts as well as a connection to the sciences. Because of the topic, students were invested and interested in the problem throughout the lesson, and some even replicated the problem at home with their families. We felt that our students left this lesson with a wealth of experiences that will help them conceptualize area far beyond the formula.

Common Core Connections

3.MD.C.5	3.MD.C.7
3.MD.C.5a	3.MD.C.7a
3.MD.C.5b	3.MD.C.7b
3.MD.C.6	3.MD.D.8

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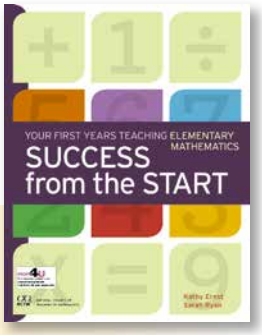

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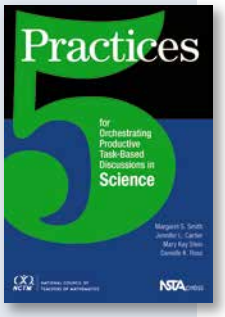

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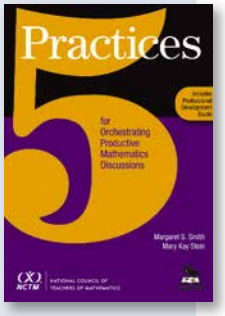

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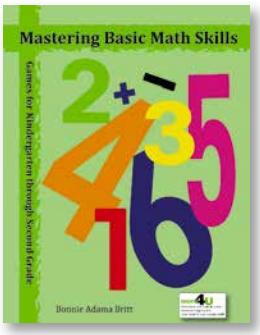

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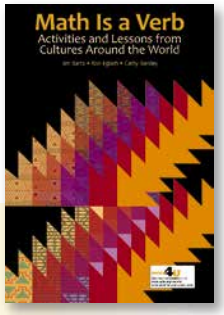

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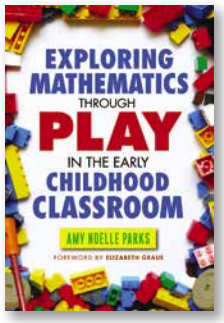

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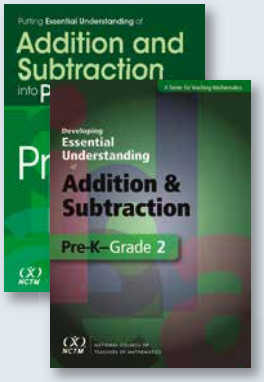

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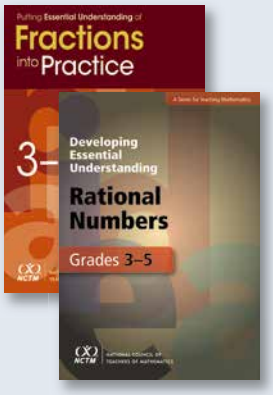

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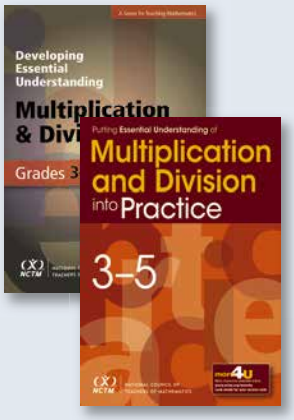

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