CAUTION: VENN DIAGRAMS AHEAD!

To highlight potential roadblocks, we contrast how this tool is typically used in mathematics and language arts, then circle back to make recommendations for your teaching practice.

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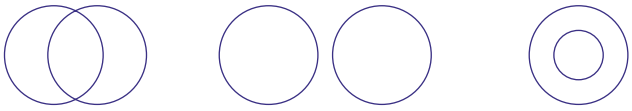
Thirty teachers of mathematics, language arts, social studies, and science in self-contained elementary school classrooms were participating in a professional development session aimed at boosting their mathematical content knowledge. The workshop leader was a college mathematics/mathematics education professor who was using Venn diagrams to assess understanding of the relationships between various types of two-dimensional geometric shapes.

FIGURE 1

When the workshop leader, a mathematics educator, posed questions about Venn diagrams, the responses she got surprised her.

Which of the following Venn diagrams best represents the relationship between the set of rectangles and the set of squares? Explain your choice.

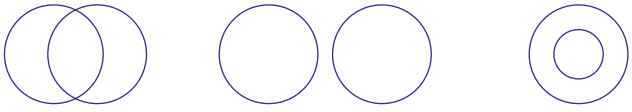
(a) (b) (c)



The intended answer was (c), with the large circle representing the set of rectangles and the small circle representing the set of squares, because every square is a rectangle.

Which of the following Venn diagrams best represents the relationship between the set of right triangles and the set of acute triangles? Explain your choice.

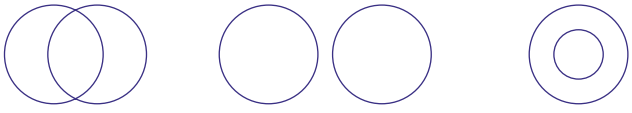
(a) (b) (c)



The intended answer was (b), with one circle representing the set of right triangles and the other circle representing the set of acute triangles, because there is not a triangle that is both a right triangle and an acute triangle. Answer (a) would also have been accepted if the student made clear that the overlapping part in the middle contained no triangles.

Which of the following Venn diagrams best represents the relationship between the set of rectangles and the set of rhombuses? Explain your choice.

(a) (b) (c)



The intended answer was (a), with one circle representing the set of rectangles and the other circle representing the set of rhombuses. Some rectangles are rhombuses, but there are rectangles that are not rhombuses and rhombuses that are not rectangles.

When the workshop leaders posed questions (see **fig. 1**), several teachers gave answers other than those the questions were intended to solicit. The workshop leader was perplexed. Could it be that the teachers understood the mathematics but misunderstood the Venn diagrams that were being used to picture the mathematics? She asked some similar questions involving well-understood nonmathematical entities, such as giraffes, elephants, and other animals, in place of the geometric objects. The in-service teachers did not answer these questions as expected either. The workshop leader was then more perplexed. Why were Venn diagrams being understood differently than intended?

Back on the university campus, the workshop leader mentioned her confusion to a mathematics education colleague who worked closely with language arts education professors and thus was familiar with strategies for teaching reading comprehension. The colleague pointed out that Venn diagrams are used in language arts education as a graphic organizer for recording information to enhance comprehension of a reading passage. The workshop leader realized that this was a different usage of Venn diagrams from that to which she had been exposed in her mathematics training.

Contrasting definitions/ interpretations

Two perspectives of the term *Venn diagram* (see **fig. 2**) reflect the typical differences in the uses of Venn diagrams in the subject areas of mathematics and language arts. These differences are subtle; nevertheless, they can potentially be confusing. In language arts, the circles in a Venn diagram typically represent things that can be compared and contrasted. For example, the items could be characters in a story, philosophies of teaching, or classes of animals.

"COULD IT BE THAT THE TEACHERS UNDERSTOOD THE MATHEMATICS BUT MISUNDERSTOOD THE VENN DIAGRAMS THAT WERE BEING USED TO PICTURE IT?"

In this usage, the *characteristics* are placed in the circles, with characteristics that both share placed in the overlapping portion of the circles and characteristics unique to one placed in the nonoverlapping portion of the appropriate circle. This serves to highlight similarities and dissimilarities of the items being compared. In contrast, in the subject area of mathematics, the circles typically represent not *things* but their attributes or characteristics. The things are typically mathematical objects, such as numbers or geometric figures. The objects themselves, or representations of the objects, are placed in the circles instead of characteristics of the objects being placed in the circles. In this case, the Venn diagram is used to sort the objects into sets, or in other words, to classify the objects according to whether they possess the characteristics represented by the circles.

Illustrations from the elementary school teachers' library

Consider Venn diagrams in two books familiar to many elementary school teachers, *Navigating through Geometry in Grades 3–5* (NCTM 2001) and *Vincent and Viola* (Colarusso 2006). The Ring Labels activity in the first book illustrates a typical mathematical use of a Venn diagram. The circles represent such characteristics of geometric figures as "at least one set of parallel sides" and "at least one obtuse angle." Students place pictures of various quadrilaterals in the regions, thus sorting and classifying the quadrilaterals (see **fig. 3**). In contrast, the book *Vincent and Viola* is often used as an example of children's literature that incorporates mathematics. In the Venn diagrams depicted in this book, one circle represents Viola, and the other circle represents Vincent. Characteristics of Vincent and Viola are placed in their respective circles. For example, among the characteristics shared by both Vincent and Viola are the facts that each is adopted and each has a pet. However, only Viola is nine years old; and only Vincent is five years old (see **fig. 4**).

Illustrations from preservice teachers' comments

To further illustrate the typical use of contrasting definitions in the two subject areas, consider responses from three preservice teachers to the question, "Which of the following Venn

FIGURE 2

One definition below is from *The Reading Teacher* (Camp 2000, p. 402), and it references *The Literacy Dictionary: The Vocabulary of Reading and Writing* (Harris and Hodges 1995, p. 271). The second definition is from the online mathematics resource Wolfram Mathworld.

Venn Diagram
Definition (Camp 2000, p. 402)

"a graphic organizer contructed by 'overlapping circles to indicate features common or unique to two or more concepts' (Harris and Hodges 1995, p. 271). . . . The nonintersecting parts of the circles are used to record information unique to each concept."

Definition (Wolfram Mathworld, Weisstein, http://mathworld.wolfram.com/VennDiagram.html)

"a schematic diagram . . . to depict collections of sets and represent their relationship."

FIGURE 3

This is an illustration of the use of Venn diagrams in the Ring Labels mathematics activity from *Navigating through Geometry in Grades 3–5* (NCTM 2001).

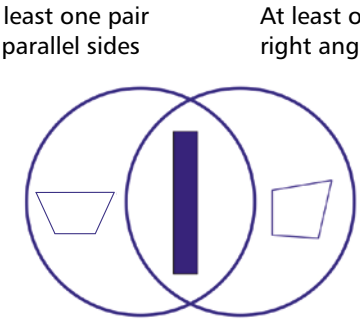


FIGURE 4

This is an illustration of the use of Venn diagrams in the children's literature book *Vincent and Viola* (Colarusso 2006).

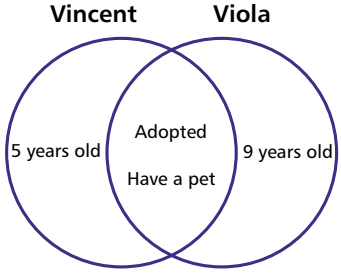


FIGURE 5

Here is a Venn diagram for classifying subsets of rational numbers.

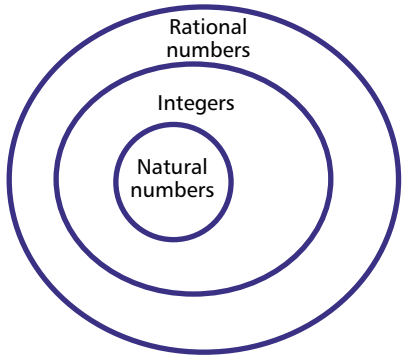
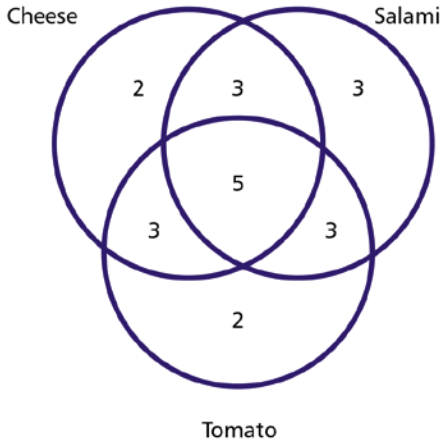


FIGURE 6

This Venn diagram depicts the information in the How Many Sandwiches? problem (Olson, Sakshaug, and Olson 1998).



diagrams—a, b, or c—best represents the relationship between the set of animals and the set of giraffes? Please explain briefly why you chose this answer." (See the three choices presented in **fig. 1**.) One preservice teacher chose (a) because "they share some characteristics but not all." This individual seems to be using the typical language arts definition of Venn diagram. Another preservice teacher chose (c) because "not all animals are giraffes, but all giraffes are animals." This individual is using the mathematical set definition of Venn diagrams. A third preservice teacher answered that "giraffes are animals. However, not all animals have the same traits as giraffes, so it could be (a) or (c)." This person seemed to recognize both definitions and was conflicted about which one to use.

Various uses of Venn diagrams in mathematics

Venn diagrams were invented by the English logician John Venn and were first mentioned in Venn's 1880 publication *On the Diagrammatic and Mechanical Representation of Propositions and Reasonings*. His diagram featured the overlapping circles, in contrast to an Euler diagram, which consists of concentric circles developed by Leonhard Euler (1707–1783) (Edwards 2004). Today mathematicians often use the term *Venn diagram* to refer to all three diagrams in **figure 1**, specifically, overlapping circles, disjoint circles, and concentric circles. Venn diagrams have always been used in mathematics to classify sets of objects. However, the mathematical contexts in which they have been used have expanded since the first use of Venn diagrams. For example, Venn and Euler originally used Venn diagrams to analyze logical arguments (Edwards 2004; Van Dyke 1995). Today Venn diagrams are additionally used as an aid in understanding and analyzing how various sets of mathematical objects are related and as an aid in problem solving.

Examples from elementary school math curriculum

An example of the use of Venn diagrams for classification to understand and analyze how various sets of quadrilaterals are related is their use in Ring Labels (NCTM 2001) (see **fig. 3**). Other examples of the use of Venn diagrams are to demonstrate how various subsets of rational numbers are related (see **fig. 5**) and as an aid in problem solving (see **fig. 6**). In the diagram in **figure 6**, the mathematical definition of Venn diagram is being used, although the number of objects is placed in the region, rather than the objects themselves. An aid in problem solving, this Venn diagram also serves to classify the sandwiches in the problem.

Revisiting the professional development scenario

When the purpose of using Venn diagrams is to compare and contrast, as is typically the purpose in the language arts setting, only overlapping circles are needed. Thus, often in language arts, only the overlapping circles type of Venn diagram is used. (This became apparent when a literacy professor indicated that she had seen

only one of the three representations of Venn diagrams she was shown.) Any two things, even opposites, have similarities and differences. For example, day and night are similar in that they are both parts of a twenty-four–hour cycle but different in that one has daylight and the other, darkness.

This brings us back to the professional development scenario. Most teachers chose the overlapping circles diagram rather than the disjoint or concentric circles diagrams to answer the questions regarding which Venn diagram best represents the relationship between two sets of geometric figures. If one is using the compare-and-contrast definition of Venn diagram that is typically used in a language arts setting, choosing the overlapping circles always makes sense. For example, when Venn diagrams are used to compare and contrast rectangles and squares, such characteristics as "must have four right angles" are placed in the overlapping portion of the circles. Characteristics like "Squares must have equal side lengths" and "Rectangles could have unequal adjacent side lengths" go in the appropriate nonoverlapping portions of the circles (see **fig. 7**). Of course, other characteristics could be included in the circles in **figure 7**. However, using the classification definition of Venn diagram that is typically used in mathematics, the actual rectangles and squares go in the circles rather than characteristics of the rectangles and squares. Because every square is a rectangle, the concentric circles diagram would be most appropriate. Likewise, using the classification definition, because a triangle cannot be both an acute triangle and a right triangle, the disjoint circles diagram best represents the relationship between the set of acute triangles and right triangles. However, using the compare-and-contrast definition, the overlapping circles would be an appropriate answer because acute triangles and right triangles share some characteristics and also have characteristics unique to each, as illustrated in **figure 7**.

It is possible for individuals to experience both uses of Venn diagrams without confusion in contexts that promote sense making and not be cognizant of the fact that they are using the Venn diagram in different ways. For example, in the NCTM publication *How to Use Children's Literature to Teach Mathematics* (1992, p. 69) a Venn diagram is shown in which two overlapping circles are used to classify children's books. One of the circles represents books about food, and the other circle represents books about grandparents. The intersection of the circles represents books about both food and grandparents. Each region of the Venn diagram lists various books. In this example, the Venn diagram is being used in a way consistent with the mathematics definition instead of the language arts definition. When this was made explicit to a language arts teacher educator, she indicated that she was quite comfortable with the usage in this example and would not have thought of this as being different from the typical compare and contrast usage. So, why did the misinterpretation occur in the professional development scenario? Neither adequate context nor an explanation for determining the intended use of the Venn diagram was provided (see **fig. 8**), so both interpretations were plausible.

FIGURE 7

Below are examples of using Venn diagrams to compare and contrast types of geometric figures.

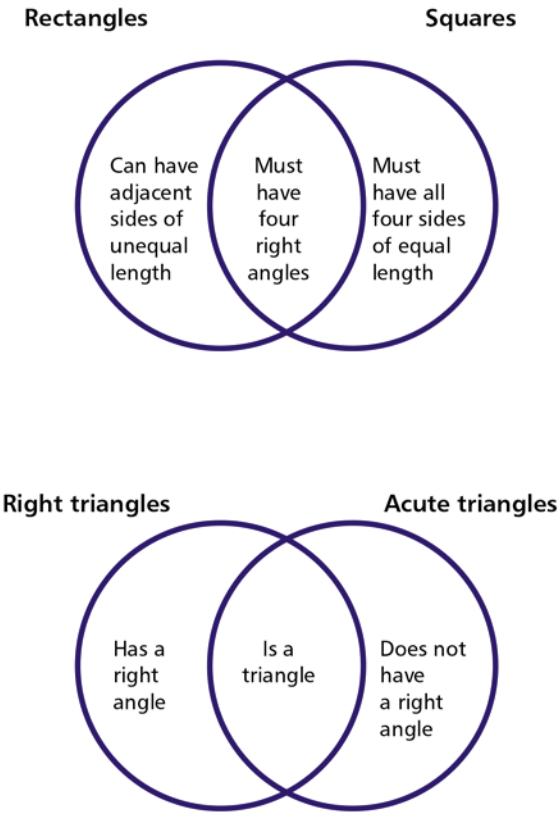
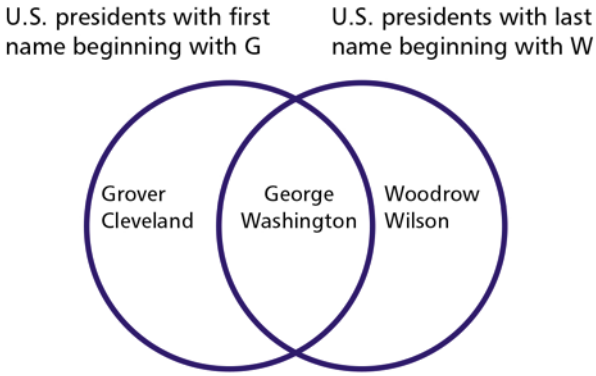


FIGURE 8

This is an example of a Venn diagram for which adequate context is provided, and therefore no explanation of the intended use is needed.



Students' understanding of Venn diagrams

To investigate whether elementary school students have ideas about Venn diagrams similar to those presented in this article, we asked a fourth-grade class four questions involving Venn diagrams and followed these questions with the Ring Labels activity (NCTM 2001). The first two questions were the giraffes/animals and the squares/rectangles questions previously mentioned. The last two questions each presented a Venn diagram with the instructional prompt, "What do you think this diagram means or is trying to illustrate?" One diagram was the overlapping circles Venn diagram in which Vincent and Viola are compared and contrasted (see **fig. 4**), and the other was a concentric circles Venn diagram with the outer circle labeled *gidgets* and the inner circle labeled *widgets*. Earlier in the school year, these students had been exposed to Venn diagrams in a social studies lesson in which they used the overlapping circles diagram to compare and contrast American colonies. In math class, they had classified quadrilaterals using tree diagrams, so they had studied that a square is a rectangle but had not used a Venn diagram in their previous study of squares and rectangles this year.

Students' responses were similar to those of teachers in that both the compare-and-contrast and the classification usages were present, with compare and contrast used the majority of the time. Eight of the nineteen students in the class answered both the giraffe/animal and the square/rectangle questions by choosing overlapping circles and giving explanations that involved comparing and contrasting. Several of these students did speak of classification in that they mentioned that a giraffe is a type of animal and/or that a square is a type of rectangle, yet they still chose the overlapping circles diagram rather than concentric circles and explained their choices in terms of similarities and differences. For example, one student explained, "Giraffes are animals, yet they have lots of differences" and "A square is a rectangle, so they are partly the same."

Another five of the nineteen students picked the overlapping circles for the giraffe/animal question but switched to the concentric circles for the square/rectangle question. One student explained,

> Because to compare and contrast animals and giraffes, you need a place to put the similarities and differences. On (a) [overlapping circles], you have places to compare and contrast animals and giraffes.

The same student explained her choice of concentric circles for the square/rectangle question:

> [I chose concentric circles] because a square is a rectangle. I also know a rectangle isn't a square, though. So the square would be the little circle, and the rectangle would be the big one. The rectangle is a little different from the square, so you would have both similarities and differences.

One possible explanation for these five students' differing choices on the giraffes/animals and square/rectangles questions is a nonmath context versus a math context. Five of the six remaining students choose disjoint circles for the giraffes/animals question, with two of these choosing concentric circles for the squares/rectangles and three choosing overlapping circles. Explanations of the choice of disjoint circles for the giraffes and animals included the following:

- "Because they are two different things"
- "Because they are exactly the same"

- "Because you are just contrasting and [not] comparing"

Although disjoint circles are typically not used as an aid in comparing and contrasting, these students forced compare and contrast meanings on the disjoint circles to explain the relationship between giraffes and animals.

Each of the nineteen students demonstrated understanding of the diagram in **figure 4**, in which Vincent and Viola are compared and contrasted. In contrast, only five students correctly suggested that the concentric circles Venn diagram, with widgets in the inside circle and gidgets in the outside circle, was trying to illustrate that a widget is a type of gidget. One of these five students explained,

> This diagram shows that everything about [a] widget is the same for a gidget, though gidgets have a few more traits that widgets do not have.

From the responses to the widget/gidget question, we found that students have a variety of ideas about the meaning of the concentric circles Venn diagram. One student said, "I think it means that gidgets are totally different than widgets. Gidgets could be bigger than widgets."

In contrast, another student said,

> I don't know exactly what it is, but I think it is trying to compare gidgets and widgets only for similarity, the smaller circle in the

"STUDENTS FORCED COMPARE AND CONTRAST MEANINGS TO EXPLAIN THE RELATIONSHIP."

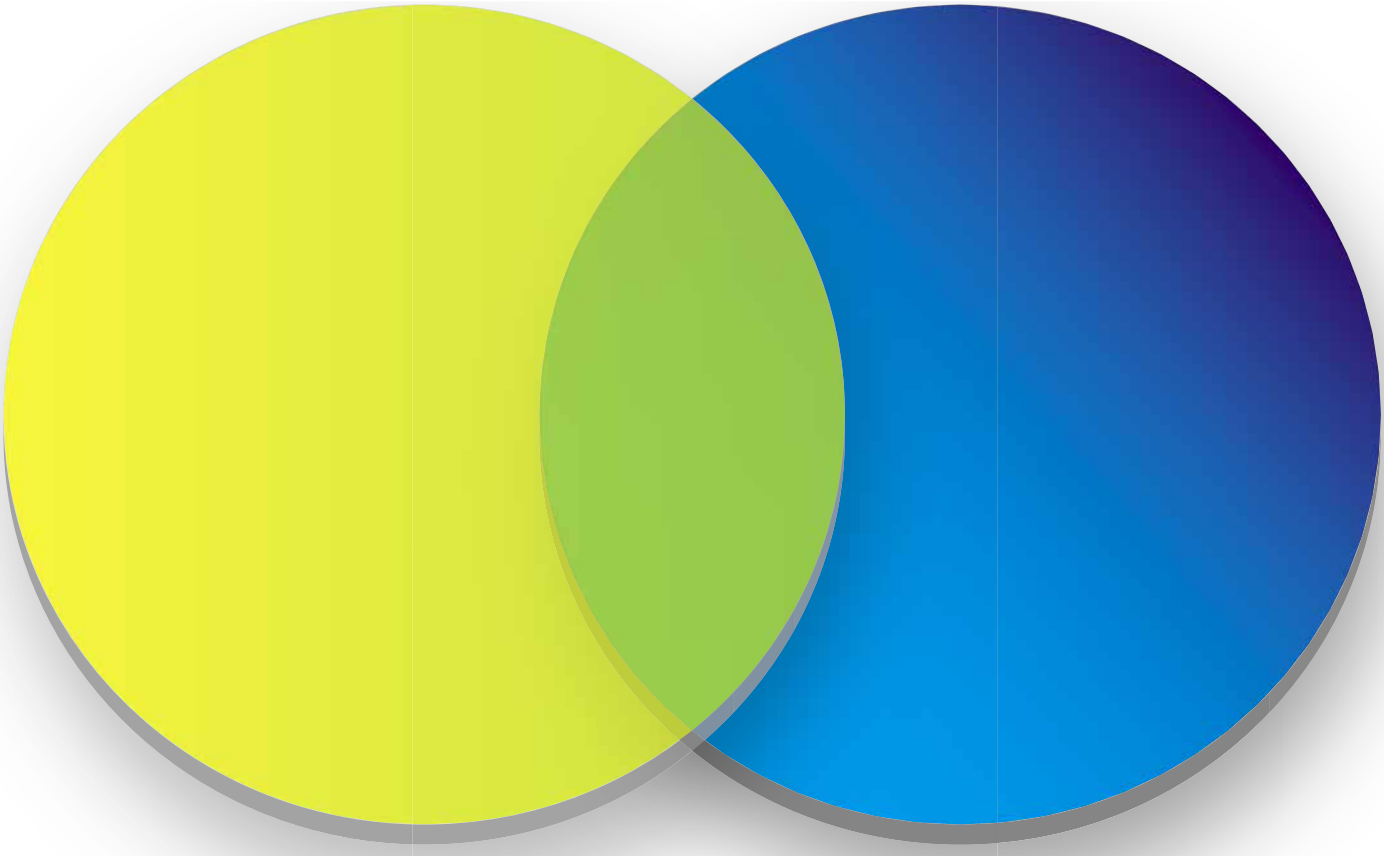


FIGURE 9

The Ring Label activity uses Venn diagrams to classify. A student with a tendency to use Venn diagrams to compare and contrast needed some prompting before successfully sorting quadrilaterals.

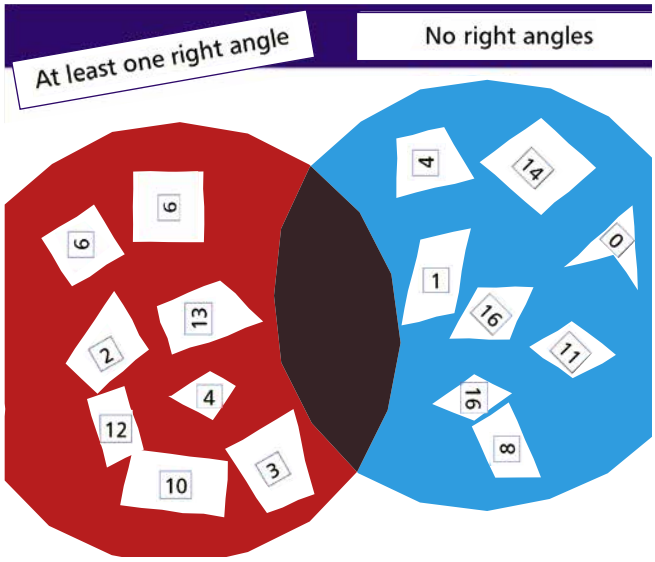
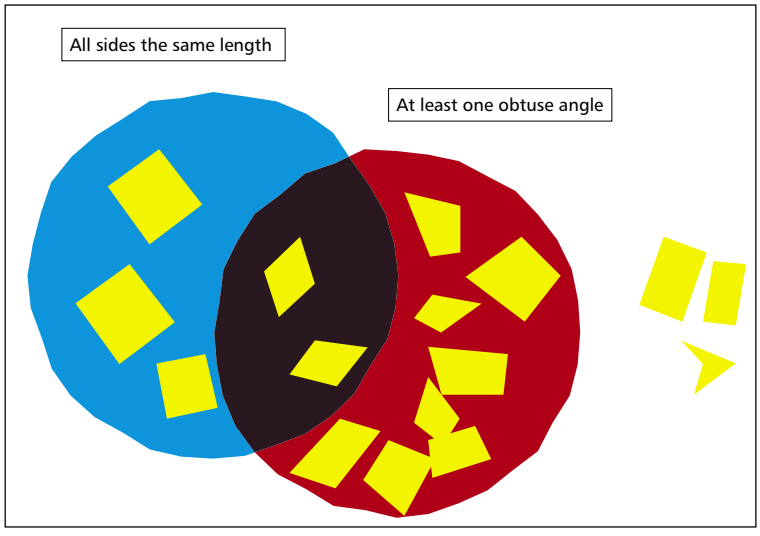


FIGURE 10

When the activity was repeated with a second set of ring labels, the same student did not know where to place two quadrilaterals that did not fit into either circle.



middle meaning "what does widgets have in common with gidgets?"

Another student explained,

I know gidgets and widgets are different but not the same because there is nothing where you can write how they are the same.

One student's work with Venn diagrams

To further investigate the thinking of a student whose responses to the four questions did not include any references to the classification way of thinking about Venn diagrams, we focused on one such student as the class engaged with the Ring Labels activity, an activity in which Venn diagrams are used to classify. The entire class was asked to form a Venn diagram with their rings, label the rings as indicated, and then place a variety of quadrilateral cutouts in the appropriate places in the Venn diagram. The first ring labels were "At least one right angle" and "No right angles." The student began with the overlapping circles and was successful at placing the quadrilaterals in the appropriate places (see **fig. 9**). When asked about the fact that there was nothing in the intersection of the circles, the student's first response was to look for a quadrilateral to put into the intersection. After the student was prompted to think about the ring labels, he indicated that nothing belonged in the intersection. He was then shown how the overlapping circles could be separated to form disjoint circles. When the activity was repeated with a second set of ring labels, the result was that two quadrilaterals did not fit into either circle. The student was perplexed as to where to place these quadrilaterals. We then discussed the meaning of the region outside the circles in a Venn diagram being used for classification (see **fig. 10**). Despite this student's tendency to compare and contrast with Venn diagrams, he was successful in sorting the quadrilaterals in the Ring Label activity, although without further explanation, gaps in his understanding would have persisted.

Implications for the classroom

Our investigation of student understanding of Venn diagrams prompts us to make the following recommendations for elementary school

teachers in using Venn diagrams with their students. We recommend that children be given multiple opportunities to experience all three types of Venn diagrams and both uses of Venn diagrams across subject areas. Using Venn diagrams to compare and contrast mathematical concepts can be helpful to students in understanding the concepts. Students can engage with the classification use of the Venn diagram in reading and language arts, science, and social studies. For example, the book *G Is for Googol: A Math Alphabet Book* (Schwartz 1998, p. 44) shows all three types of Venn diagrams and gives examples of using Venn diagrams in the typical mathematical way although in a language arts setting.

We recommend that teachers always provide adequate context and examples to help illuminate the intended use of the Venn diagram. However, many students will still need instruction as to the meaning of the concentric circles and disjoint circles diagrams. A concentric circle Venn diagram (see **fig. 5**) in which the diagram illustrates that every natural number is an integer and every integer is a rational number may not actually help students understand these relationships if they are viewing the diagram in a compare-and-contrast mode. An excellent way for students to engage with the idea of using concentric and disjoint circles to classify is to do the Ring Labels activity (NCTM 2001), with ring labels chosen so that the circles are either disjoint or concentric.

Furthermore, we suggest that where possible the Venn diagram picture be used as one of several representations not in isolation. Finally, as we were enlightened by the fourth-grade students we observed, much can be learned by having students write about their thinking. This is particularly true in conjunction with the use of Venn diagrams. Doing so can help teachers use Venn diagrams more meaningfully with students in all subject areas.

Common Core Connections

K.MD.B.3
3.G.A.1
5.G.B.3

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