Cash or Gas?

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Purpose

State lotteries in Florida and other states give winners a choice between cash and another prize, such as free gas for life. In this task, students will evaluate two prize options by discussing and making reasonable assumptions to simplify a complex decision. This task is appropriate for students who can extrapolate quantities over time and are able to make conversions among different units of measure.

Task Overview

Suppose that you have just won the state lottery and are given a choice between $250,000 in cash or free gas for life. Which is the best prize for you? *An activity sheet that gives students the complete task is included.*

Focus on Reasoning and Sense Making

Reasoning Habits

Focus in High School Mathematics: Reasoning and Sense Making

Analyzing a problem—defining relevant variables and conditions; making preliminary deductions and conjectures

Reflecting on a solution—interpreting a solution; revisiting initial assumptions; reconciling different approaches

Process Standards

Principles and Standards for School Mathematics

Problem Solving—solve problems that arise in mathematics and in other contexts

Reasoning and Proof—develop and evaluate mathematical arguments

Communication—analyze and evaluate the mathematical thinking and strategies of others

Connections—recognize and apply mathematics in contexts outside of mathematics

Standards for Mathematical Practice

Common Core State Standards for Mathematics

1 Make sense of problems and persevere in solving them.

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

Focus on Mathematical Content

Key Elements

Focus in High School Mathematics: Reasoning and Sense Making

Reasoning with number and measurement—reasonableness of answers and measurements; approximations and error

Reasoning with statistics and probability—data analysis

Standards for Mathematical Content

Common Core State Standards for Mathematics

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

S-MD.5b. Evaluate and compare strategies on the basis of expected values.

Materials and Technology

- Cash or Gas? activity sheet
- Internet access (optional) for researching assumptions
- Newspaper article (optional): "Lottery Adds to Prizes: Now Gas as Well as Cash," by Carmen Gentile, http://www.nytimes.com/2008/07/08/us/08prize.html.

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Use in the Classroom

Although the lottery has natural interest for many high school students, the idea of determining how much money one is likely to spend on gas in a lifetime will probably be new. You might introduce this task with a brief class discussion, perhaps building on current gas prices in your area, to support students in understanding this task as a real-world problem worth considering.

A group discussion can remove barriers to understanding as students make sense of the problem.

Question 1 on the activity sheet asks students to describe a person who is likely to choose each option and then to make an initial prediction about which option they would choose for themselves if they won the lottery. You might want to facilitate a discussion to help students understand why people might disagree about the best option.

Making an initial prediction prompts students to think about relevant information and processes involved in solving the problem. Discussing multiple perspectives encourages them to make sense of the problem and persevere without jumping prematurely to conclusions.

Students begin a deeper mathematical analysis of the task in question 2. You might have students discuss ways to mathematize this problem in groups. If students need help in getting started, you could pose a variety of questions:

- What factors must be considered to answer this question?
- For which of these factors could mathematics be useful?
- Are there multiple ways to define the "best" option?
- What additional information would be helpful to know, and how could you obtain that information?

Identifying critical factors or conditions is an important first step in understanding a problem.

Students construct and critique different approaches to solving the problem.

Encourage students to challenge one another's answers to these questions and to request evidence that supports their responses. As necessary, focus the discussion on mathematical justifications, not on personal preferences.

When students assign numbers to factors such as the average price that they would expect to pay for a gallon of gas over their lifetime, the number of miles that they would expect to drive in their lifetime, or the average gas mileage that they would expect their cars to get, you might discuss the reasonableness of these numbers and make sure that students recognize the numbers as assumptions.

Modeling involves making assumptions and approximations to simplify a complicated situation.

As an optional avenue of investigation, you might give students access to a library or computer lab where they could research missing information that they need to answer this question more precisely.

Students should have opportunities to make appropriate use of relevant tools to support their analyses.

Question 3 asks students to change one or more of their assumptions to see whether or how the change affects their choice between cash or gas. You might pose additional reflection questions that require students to be critical of their solutions—for example, "What cash amount could the lottery offer that would lead you to change the prize you choose?"

Students need to revisit their initial assumptions to ensure that their solutions are appropriate and robust.

The second page of the activity sheet provides an excerpt from the *New York Times* article (Gentile 2008) on which this task is based. To bring the activity to a close, you might distribute the excerpt for students to read and discuss in their groups. In particular, you might ask students to consider the sentence, "But with a gallon of unleaded regular in South Florida costing an average of about $4.30, some players are ready to forgo the math." Why would some players feel that way?

Focus on Student Thinking

One issue that may arise in class discussions is how to define the "best" option. Some students will want the prize with the largest monetary value. Other students may prefer the certainty of receiving a specific amount of cash to the uncertain value associated with fluctuating gas prices or the possibility that future cars will use electricity or other energy sources. Alternatively, some students may think the gas prize is best almost regardless of the amount of the cash prize, out of fear that gas prices will rise much higher in the future. Encourage students to explain the reasoning behind their thoughts and to speculate how they could use mathematics to support different definitions of the "best" option.

Naturally, students will depend heavily on their own experiences and circumstances to make the necessary assumptions for solving this problem. Students should realize that life circumstances vary among individuals, and that those circumstances may change the assumptions about the miles that one expects to drive, the gas mileage of one's cars, and so forth. For example, someone who is 90 years old should take the cash prize since he or she is not likely to be driving much longer, whereas someone who is 18 years old and driving 100 miles each way to work every day for the rest of his or her life would almost certainly choose the free gas for life. If some students resist thinking beyond their own circumstances or fail to consider the benefits of the other prize option, you might ask them to revisit questions 1(*a*) and 1(*b*) on the activity sheet and then narrow these questions to give only one exaggerated aspect of a person's lifestyle that would lead him or her to choose the other prize.

A 16-year-old male student, after either taking part in a class discussion or doing some research, might assume that with a life expectancy of 76 years, he would have 60 years of driving. Further, he might assume that his cars would average 32 miles per gallon, gas would average \$8 per gallon over his lifetime, and he would drive an average of 170 miles per week. With these assumptions, a lifetime of gas would cost

$$\frac{\$8}{1 \text{ gallon}} \cdot \frac{1 \text{ gallon}}{32 \text{ miles}} \cdot \frac{170 \text{ miles}}{1 \text{ week}} \cdot \frac{52 \text{ weeks}}{1 \text{ year}} \cdot \frac{60 \text{ driving years}}{1 \text{ lifetime}} = \frac{\$132{,}600}{\text{lifetime}}$$

Students may recognize that the number of miles that people drive could vary dramatically, depending on their lifestyles. Some fundamental questions can make differences obvious:

- Will I be living in a city where I can take public transportation?
- Will my job have a long commute that I have to make each day by car?
- What if I want to take a long trip each summer?

Encourage students to make simplifying assumptions about their lifestyles, as well as to consider alternative assumptions. Students might also reason that if the price of gas continues to climb, they might alter their lifestyle to drive less. In this case, the flexibility of the lump-sum option would become more valuable, since they could use the cash to pay for public transportation and other transportation options, whereas the gas prize would be only for gas.

Some students might try to predict how various factors would change in the future. Some of the changes that students could incorporate in their calculations include fluctuations in inflation, changes in gas prices per gallon, increases in life expectancies, variations in life expectancies between males and females, changes in fuel economy standards, and innovations in vehicle technology (alternative fuels, hybrid and electric vehicles), as well as interest earned by investing the cash prize. Asking students to explain how changing each of these factors might change their ultimate choice of lottery prize can lead to a very fruitful discussion.

For instance, increasing the assumed cost per gallon of gas, the number of miles driven per week, or the number of years spent driving (women typically live several years longer than men) will increase the amount of money expended on gas in a lifetime, since these quantities appear in the numerators of the unit conversion fractions, thereby increasing the value of the gas-for-life prize. On the other hand, increasing the fuel economy of one's cars, perhaps through hybrid or electric vehicle technologies, means dividing by a larger number in the denominator, thereby reducing the value of this prize.

Some students might focus their analyses on the actual value of the cash prize. For example, the cash prize would be subject to state and federal taxes, which would reduce its value, perhaps by 1/3 or more, depending on tax rates. On the other hand, prudent investment might increase the future value of this prize.

Students might use the Internet to research these and other variables, discovering information that could lead to interesting extensions. For example, by using data from the past decade, could they make better projections of future gas prices? What rate of return might they expect to get if they invested the lump sum from the cash prize in the stock market?

Focus on Student Thinking—*Continued*

Students should realize that their final answers depend heavily on the assumptions that they have made and that modifying one or more of these assumptions could change their choice of prize. Focus students' thinking on the uncertainty associated with predicting future gas prices and the impossibility of determining other factors right now. Students should also appreciate the complex nature of this problem and recognize that giving a more accurate answer would require improving the quality of one's assumptions. Considering an increase in life expectancies with new medical technologies, projecting the prospects of telecommuting instead of driving to work, defining a step function for discrete increases in fuel economy standards every several years, or making countless refinements in other factors could improve the accuracy of students' calculations, but only to the extent that those assumptions accurately forecast one's future lifestyle. Other factors, such as inflation and rates of return on investments, could be added to the mathematical model. Moreover, using strategies that define yearly changes rather than average changes over a lifetime might improve precision further.

Assessment

You might ask students to advise you personally on which prize would be best for you. If you assign this task as homework, allow time before the end of class for students to ask you questions that they believe will elicit important and relevant information for answering this question. Then, for homework, have the students write a detailed analysis as your financial advisor, explaining which option they think you should choose, including all relevant calculations to support their recommendation.

You might also ask students to come up with lottery prizes that would ensure that winners would choose each prize roughly half the time. Students should discuss their assumptions about the lifestyles of people who typically play the lottery, the public's overvaluation of particular prizes, and other considerations, before showing mathematically that the prizes have roughly equal perceived value to typical lottery players.

Resources

Gentile, Carmen. "Lottery Adds to Prizes: Now Gas as Well as Cash." *New York Times*, July 8, 2008. http://www.nytimes.com/2008/07/08/us/08prize.html.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

———. *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va.: NCTM, 2009.

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math)*. Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org.

Cash or Gas?

Student Activity Sheet

Congratulations! You have just won the state lottery, and now you must choose your prize—either $250,000 in cash or free gas for life.

1. Complete parts (*a*) and (*b*) without making any calculations, and then answer the question in part (*c*):

   a. Describe the lifestyle of a person for whom the $250,000 prize is likely to be the best option.

   b. Describe the lifestyle of a person for whom free gas for life is likely to be the best option.

   c. Which prize do you think would be likely to be best for you? Explain why.

2. Calculate which prize would be likely to have the greatest monetary value for you. Support your answer mathematically. Discuss your reasoning and any assumptions that you make.

3. Show in detail how changing one or more of your assumptions in question 2 could change the prize that is best for you to pick. Include specific calculations to support your answer.

Cash or Gas?

Student Activity Sheet (Continued)

Read all about it! The following excerpt is from an article by Carmen Gentile that appeared in the *New York Times* on July 8, 2008:

> Rising gasoline prices have brought a new twist on the state lottery in Florida.
>
> Once a week for the next two months, the second-prize winner in the latest lottery promotion, Summer Cash, will win free gasoline for life. And some people think that is a better deal than the game's first prize, a quarter of a million dollars.
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> "If gas keeps going up and up — and I expect it will — then I'd rather have free gas for life," said Robert Acosta, who spends about $50 a week on fuel for his four-cylinder Toyota Scion and bought a $5 Summer Cash ticket in anticipation of the first drawing, this Wednesday.
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> Not that a winner's right to free gasoline is unlimited. Rather, each winner will be awarded 26 prepaid gas cards, each worth $100, every year until death. Were the 44-year-old Mr. Acosta to win, and live to be twice his current age, the total payout to him in free gasoline would be $114,400. That is far short of the first prize, particularly since virtually all the gas prize would be paid in future dollars.
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> But with a gallon of unleaded regular in South Florida costing an average of about $4.30, some players are ready to forgo the math.
>
> "Gas has become more precious than cash now," Bernard Feldman said.

Gentile, Carmen. "Lottery Adds to Prizes: Now Gas as Well as Cash." *New York Times,* July 8, 2008. http://www.nytimes.com/2008/07/08/us/08prize.html.