# Fuel for Thought

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## Purpose

Students use mathematical reasoning to determine patterns in fuel consumption. The analysis requires care in using appropriate units of measure while developing a mathematical model to be analyzed.

## Task Overview

Analyze given information on fuel consumption to determine which of two new car options would result in saving more fuel. *An activity sheet that gives students the complete task is included.*

## Focus on Reasoning and Sense Making

### Reasoning Habits

***Focus in High School Mathematics: Reasoning and Sense Making***

Analyzing a problem—seeking patterns and relationships

Reflecting on a solution—interpreting a solution; reconciling different approaches to solving the problem; refining arguments

### Process Standards

***Principles and Standards for School Mathematics***

Problem solving—build new mathematical knowledge through problem solving

Reasoning and proof—develop and evaluate mathematical arguments and proofs

Communication—analyze and evaluate the mathematical thinking and strategies of others

### Standards for Mathematical Practice

**Common Core State Standards for Mathematics**

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

## Focus on Mathematical Content

### Key Elements

***Focus in High School Mathematics: Reasoning and Sense Making***

Reasoning with number and measurement—reasonableness of answers and measurements

Reasoning with functions—using multiple representations of functions

### Standards for Mathematical Content

**Common Core State Standards for Mathematics**

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems.

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

## Materials and Technology

- Fuel for Thought activity sheet
- Applet (optional): Fuel Consumption, http://www.MathRSM.net/applets/fuel
- Newspaper blog article (optional): "The Illusion of Miles per Gallon," by Richard S. Chang, http://wheels.blogs.nytimes.com/2008/06/20/the-illusion-of-miles-per-gallon/

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## Use in the Classroom

Introduce the activity, distribute part 1 of the activity sheet, and ask students to work on this part of the activity in pairs or small groups.

Working in pairs or small groups will allow students to build on one another's knowledge and gain a deeper understanding of the mathematical patterns and relationships that they are seeking.

Note that in this problem, the relationship between miles per gallon (mpg) and fuel consumption is inversely proportional, meaning that doubling the mpg halves the fuel consumption. Although students are not likely to recognize the relationship initially, the activity should help them arrive at that realization.

If necessary, help a group of students get started or think about the relationship more deeply by asking, "What is the question asking us to compare?"

Questioning will help students build new mathematical knowledge through problem solving. Asking questions may help student to realize that they need to analyze the problem more fully.

After several minutes, bring the class together, and have students compare the answers that they have determined. Seek a variety of answers from a range of students.

If no student makes a case for choice (b), the conclusion that in the given situation replacing the SUV by the sedan would save more fuel than replacing the compact car by the hybrid, you might ask (or ask again), "What is the task asking you to compare?" You could then continue by asking the class questions such as, "Is the problem asking which new car would get more miles per gallon?" or, "How could we tell which new car would actually save more fuel?"

After giving the class an opportunity to debate the merits of the two choices, (a) and (b), you might ask the students to work in small groups to explore the relationship of mpg to actual gasoline consumption, perhaps by making a graph or a table or completing the table in part 2 of the activity sheet.

Debate among students requires the students to reflect on possible solutions and analyze and evaluate the mathematical thinking and strategies of others, as well as to develop their own mathematical arguments. Using multiple representations of mathematical ideas allows students to see different approaches to the problem. Students should always be encouraged to generalize a solution.

Ask each small group to develop a written report on its observations and conclusions. In the reports, the students should conclude that as the mpg increases, the increments in the amount of fuel saved become smaller. An online applet can help students explore the relationship (see http://www.MathRSM.net/applets/fuel).

By producing a written group report, students will have to develop and communicate their ideas and arguments more fully than if they are required only to summarize their thinking verbally.

One way to wrap up the exploration is to ask students to discuss the merits of different units used to measure fuel efficiency—for example, mpg vs. gallons per 100 miles. You might note in closing that in many other countries, fuel efficiency is reported in the latter manner, although using liters and kilometers.

## Focus on Student Thinking

Students might conclude that choice (a) saves more fuel, since in this case the mpg increases by 20, whereas with choice (b) it increases only by 10. Note that this comparison of choices (a) and (b) is additive.

Two typical student responses follow:

1. "I think choice (a) saves more fuel, since the change from 34 mpg to 54 is an increase of about 59 percent, but the 18 to 28 mpg change is an increase of only about 56 percent."
2. "It looks to me as though choice (b) is better, since you will save more fuel by switching from the SUV to the sedan:

   For switching from the compact car to the hybrid:

   100 miles/54 mpg = 1.85 gallons used for the hybrid.

   100 miles/34 mpg = 2.94 gallons used for the compact car.

   So switching from a 34-mpg to a 54-mpg car would save 1.09 gallons of gas.

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## Focus on Student Thinking—*Continued*

For switching from the SUV to the sedan:

100 miles/28 mpg = 3.57 gallons used for the sedan.

100 miles/18 mpg = 5.56 gallons used for the SUV.

So switching from an 18-mpg to a 28-mpg car saves 1.99 gallons of gas every 100 miles. That means that you are actually saving more gas by replacing the SUV than by replacing the compact car."

## Assessment

Listen to group presentations and see whether students can articulate and justify the relationships that they found.

Ask students to complete part 2 of the activity sheet. In addition, you might distribute the *New York Times* article on which this task is based (see Resources below). Ask students to read the article (perhaps as homework) and analyze it, along with some of the readers' responses that appear at the website.

## Resources

Chang, Richard S. "The Illusion of Miles per Gallon." *New York Times*, June 20, 2008. http://wheels.blogs.nytimes.com/2008/06/20/the-illusion-of-miles-per-gallon.

Common Core State Standards Initiative. *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math)*. Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010. http://www.corestandards.org.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

———. *Focus in High School Mathematics: Reasoning and Sense Making.* Reston, Va.: NCTM, 2009. Example 2, pp. 23–34, based on "The Illusion of Miles per Gallon" by Richard S. Chang, *New York Times*, June 20, 2008.

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# Fuel for Thought

## Student Activity Sheet

### Part 1: A Fuel-ish Question

1. Which of the following would save more fuel?
    *a*. Replacing a compact car that gets 34 miles per gallon (mpg) with a hybrid that gets 54 mpg.
    *b*. Replacing a sport utility vehicle (SUV) that gets 18 mpg with a sedan that gets 28 mpg.
    *c*. Both changes would save the same amount of fuel.

2. Explain your reasoning for your choice.

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# Fuel for Thought

## Student Activity Sheet—*Continued*

### Part 2: Extending the Discussion—MPG vs. Fuel Consumption

1. Complete the following chart comparing mpg and fuel consumption.

| MPG | Fuel consumed to travel 100 miles |
|---|---|
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2. Use your values to sketch a graph.

3. Develop a written report explaining your observations and conclusions.

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