### Purpose
Students analyze the structure of algebraic expressions and a graph to determine what information each expression readily contributes about the flight of a horseshoe. This task is particularly relevant to students who are studying (or have studied) various quadratic expressions (or functions). The task also illustrates a step in the mathematical modeling process that involves interpreting mathematical results in a real-world context.

### Task Overview
The height of a thrown horseshoe depends on the time that has elapsed since its release. Derive information about the flight of a horseshoe from a graph and four given equivalent algebraic expressions that describe its flight. *An activity sheet that gives students the complete task is included.*

### Focus on Reasoning and Sense Making

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### Process Standards

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### Focus on Mathematical Content

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### Materials and Technology
Horseshoes in Flight activity sheet
Use in the Classroom

After introducing the task and distributing the activity sheet, you might begin by having students work individually or in groups to come up with their own conclusions.

If students have difficulty getting started on the task, you might ask, “Just by looking at the four expressions, (a)–(d), without doing any computation, can you tell me one thing that you know about the flight of the horseshoe?”

A class discussion of possible responses might follow, encompassing the questions on the activity sheet. Question 1 asks which expression is most useful for finding the maximum height of the horseshoe. You might have different students present their solutions and the reasoning behind them. Question 2 probes other conclusions that students can draw about the horseshoe’s flight from the other expressions. If the students are having trouble with question 1, discussing question 2 first might be easier and more productive.

After the students have discussed what they can conclude from the other expressions, ask questions to focus their attention on the relationships among their conclusions about the values of the expressions, where those values show up on the graph, and what those values mean in the context.

You might conclude by having the class observe that although these expressions are equivalent, each one provides different insights into the situation—an observation that will naturally raise the question, “Are there other situations like this one?”

Focus on Student Thinking

In responding to question 1, students might note that since the term \(-16(t - 9/16)^2\) is always negative or zero, expression (d) makes it clear immediately that the height can never go above 100/16, or 6 1/4, feet, and it reaches this height at \(t = 9/16\) seconds, which is when that first term has the value 0.

Alternatively, if students look at the graph, they might notice that the maximum height is a little more than 6 feet and is reached after just slightly more than 0.5 seconds have elapsed. They could then use this graphical information to help narrow the group of expressions to determine which one would be most useful for finding the maximum height.

In considering question 2, students might note that substituting in \(t = 0\) in expression (a) reveals that the initial height of the horseshoe is 1 3/16 feet, which would be where the thrower held the horseshoe at the start of the toss. This is also the \(y\)-intercept of the graph.

Students might note that expressions (b) and (c) are essentially the same, since they are factored forms of the original expression. Such forms are useful in finding the zeros, or \(x\)-intercepts, which in this case are \(-1/16\) and \(19/16\). The negative zero isn’t useful in considering the flight of the horseshoe, since the flight begins at \(t = 0\). However, the positive zero tells when the height of the horseshoe returns to 0 feet, which is when it hits the ground. In looking at the graph, this will be at about 1.2 seconds, which is close to 19/16 seconds.

In looking at the factored forms, (b) and (c), students might also note that since the graph is a parabola and parabolas are symmetric, the maximum height should occur at the midpoint of the two zeros, which would be 9/16 seconds, the same value as shown in expression (d).
Assessment

To gain insight into issues that you might need to address in the discussion, walk around and observe what the students initially do with the task.

For homework or as a summative assessment, you might assign one of the following tasks:

- Ask students to write up their conclusions about the task. Alternatively, give them an expression with different coefficients and ask them to use that expression to analyze the path of the horseshoe (or some other object).
- Ask students to write an expression giving the profit that Crumbly Cookies will make in the situation described below and then analyze what they can conclude from the original expression or equivalent expressions:
  
  A survey showed that if Crumbly Cookies sells a dozen cookies for \( x \) dollars per dozen, it will sell
  
  \[ 1000 - 200x \] dozens.
  
  Their cost to make a dozen cookies is about $1.

- Give the students an expression and ask them first to come up with a situation that that it might describe and then to find out what they can about that situation by analyzing the expression and equivalent forms of it.

Resources


As shown in the graph, the height of a thrown horseshoe depends on the time that has elapsed since its release. (Note that this graph of the horseshoe’s height is parabolic, but it is not the same as the graph of the horseshoe’s flight path.)

The height of the horseshoe (measured in feet) as a function of time (measured in seconds and represented by the variable $t$) from the instant of release is

$$1 \frac{3}{16} + 18t - 16t^2.$$ 

The expressions (a)–(d) below are equivalent:

(a) $1 \frac{3}{16} + 18t - 16t^2$
(b) $-16(t - \frac{19}{16})(t + \frac{1}{16})$
(c) $\frac{1}{16}(19 - 16t)(16t + 1)$
(d) $-16(t - \frac{9}{16})^2 + \frac{100}{16}$

1. Which expression is the most useful for finding the maximum height of the horseshoe, and why is it the most useful expression?

2. What information can you conclude about the horseshoe’s flight from other equivalent expressions? Explain your answers.