Over the Hill

Prepared by Jason Slowbe, San Marcos High School, San Marcos, California

Purpose	Students determine locations on a hillside for a cell phone tower erected to provide a signal to people on the other side of the hill. They identify necessary information, represent the problem with a scale model, and answer questions in context. This task is appropriate for students who have had experience in determining equations of linear functions through two points and in solving systems of linear equations.
Task Overview	A cell phone tower is to be built somewhere on the west side of a hill, as pictured in the diagram on the activity sheet. Find the lowest point on the west side of the hill above which the tower can be based and still provide a signal to anyone east of a lake lying at the foot of the east side of the hill. *An activity sheet that gives students the complete task is included.*

Focus on Reasoning and Sense Making	**Reasoning Habits** ***Focus in High School Mathematics: Reasoning and Sense Making*** Analyzing a problem—identifying relevant concepts, procedures, or representations; making preliminary deductions and conjectures Implementing a strategy—making purposeful use of procedures; monitoring progress toward a solution Reflecting on a solution—interpreting a solution; revisiting initial assumptions **Process Standards** ***Principles and Standards for School Mathematics*** Problem Solving—monitor and reflect on the process of mathematical problem solving Connections—recognize and apply mathematics in contexts outside of mathematics Representation—use representations to model and interpret physical, social, and mathematical phenomena	**Standards for Mathematical Practice** **Common Core State Standards for Mathematics** 1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure.
Focus on Mathematical Content	**Key Elements** ***Focus in High School Mathematics: Reasoning and Sense Making*** Reasoning with algebraic symbols—connecting algebra with geometry Reasoning with functions—using multiple representations of functions Reasoning with geometry—geometric connections and modeling	**Standards for Mathematical Content** **Common Core State Standards for Mathematics** A-CED.2. Create equations in two or more variables to represent relationships between quantities. A-REI.6. Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. F-BF.1b. Combine standard function types using arithmetic operations.
Materials and Technology	• Over the Hill activity sheet • Straightedge or ruler • Online applet (optional) at www.MathRSM.net/applets/hill	

Use in the Classroom

Working with an applet available at www.MathRSM.net/applets/hill might help students to make sense of the problem before solving it, to monitor their progress toward a solution, or to check the reasonableness of their solutions after solving the problem.

> The teacher supports students in the appropriate use of models to represent and understand the problem.

Distribute part 1 of the activity sheet, which presents the task and a not-to-scale diagram of the hillside. Students can work on the activity individually or in small groups. Allow students ample time to think about the problem and what might be a good entry point into it. Question 1 asks students to identify information needed to solve the problem, and question 2 asks them to think about ways to mathematize the problem by using geometry or algebra.

> Allowing sufficient wait time supports students in analyzing and making sense of a problem.

Once students have completed these preliminary tasks, you might lead a class discussion based on their responses. Focus students' thinking on which pieces of information are important to know and which are unimportant, as well as on possibilities for quantifying particular measurements. Don't be concerned if students fail to recognize the need for a particular measurement right away—as they monitor their progress toward a solution, they will discover that more information is necessary. Support students as they grapple with decisions about what information is important, since this process is central to the formulation stage of mathematical modeling.

> Discerning relevant information and ways to quantify and represent measurements helps students formulate a mathematical model as well as select and implement an appropriate strategy.

Distribute part 2 of the activity sheet. As students create their scale models in question 3, you might discuss how coordinates are useful in representing their solutions. You might ask, "How can we describe locations in the diagram mathematically?" or, "Are some places better than others to draw x- and y-axes to create a useful coordinate system? Why?"

In question 4, students predict how far up the west side of the hill the tower must be based to send a signal to every point on the east side of the lake. Once they have determined their solution, you might ask students how they could describe that point on the hillside to others. You might also ask students to describe to one another the key pieces of information that they considered in making their predictions and how those considerations guided their thinking.

> Making a prediction helps students analyze a problem and provides a basis for monitoring their progress toward a solution.

As students discuss solution methods, either as a whole class or in small groups, you could ask questions to focus their attention on auxiliary lines as a way to mathematize the problem further. If students used coordinate or synthetic geometric approaches, you could have them compare one another's approaches for similarities and differences.

> Students often need support in recognizing and using structure.

In question 5, students reflect on their predictions and the reasonableness of their solutions. To support students in understanding their solution point as a lower bound that must be exceeded in the placement of the tower, you might ask the question, "If you were standing exactly at the edge of the lake, would you actually get a signal?" You might pose additional reflection questions, such as, "Which other methods can you use to solve this problem?" and, "How might you check the reasonableness of your solution?"

> Through questioning, the teacher prompts students to interpret their solution in context and interpret its reasonableness.

Focus on Student Thinking

At first, in the absence of any numerical measurements, students might struggle to make sense of the problem—particularly in question 1 as they attempt to determine which pieces of information are necessary, might be helpful, or are not necessary to solve the problem. The following questions can start a class discussion about these issues:

- Is it important to know the units of measure?
- Will using different units change the solution?
- Is it necessary to know a particular piece of information, or can we solve the problem another way without that information?

In question 3, students might use the grid but not automatically construct coordinate axes. To prompt students to think about this issue, you might ask questions such as the following:

- Would constructing axes help us describe locations in the diagram?
- Are some places better than others to place the origin?
- How would the location of the tower on the hillside change if we placed the axes somewhere else?

If students struggle to find an entry point into the problem, you might ask them to mark two points on the west side of the hill—one point where the tower's signal can definitely reach everyone on the other side of the lake, and one point where the tower's signal definitely cannot reach everyone. Asking students to explain why the tower's signal can or cannot reach every point on the eastern side of the lake can help students gain access to the problem and engage in solving it.

Some students might determine the linear functions representing the line up the west side of the hill, $l(x)$, and the line from the eastern edge of the lake through the vertex of the hill, $m(x)$. Then they might reason that the difference of these two functions gives the vertical distance between the lines (see fig. A). Solving the equation $m(x) - l(x) = 200$ gives the x-coordinate of point P—the point at the base of the cell tower on the hill in the diagram below. Other students might translate line $l(x)$ vertically 200 feet for the height of the tower and then solve for the intersection point of $l(x) + 200$ and $m(x)$ (see fig. B). Students who finish early might solve the problem by using both of these methods and then might demonstrate that the two methods are algebraically equivalent.
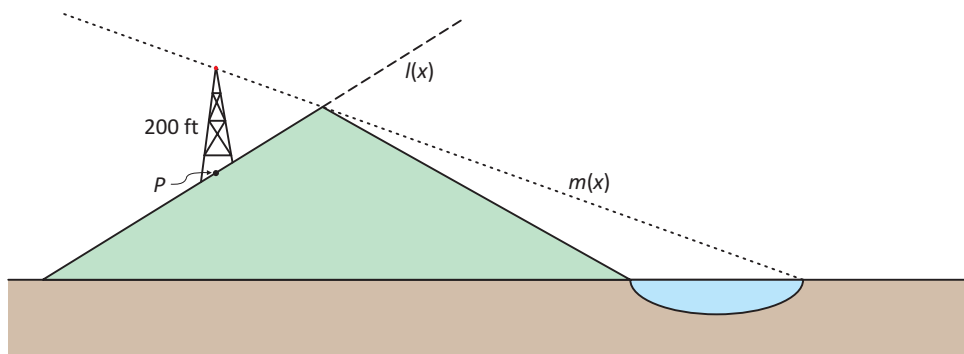
$$m(x) - l(x) = 200$$

Fig. A
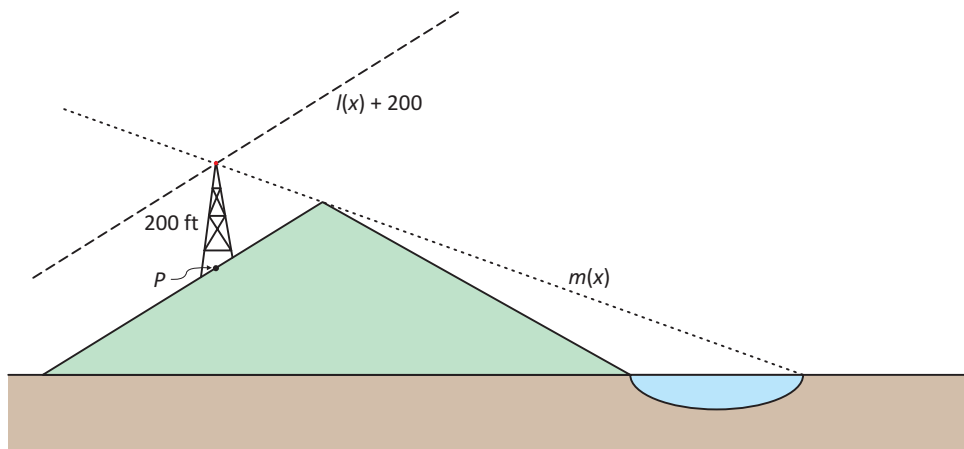
$$l(x) + 200 = m(x)$$

Fig. B

Focus on Student Thinking—*Continued*

Students might also use similar triangles or trigonometry to find the horizontal distance from point *P* to the eastern edge of the lake, as pictured in figure C.
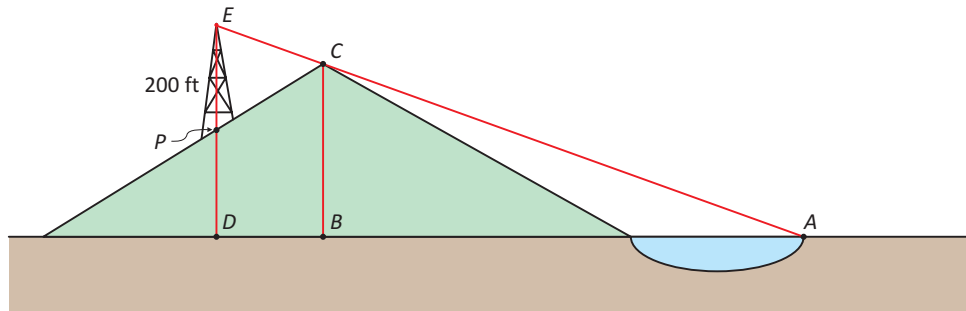

Fig. C

In question 4, some students might reason backward to verify their solutions. If students imagine themselves standing at the eastern edge of the lake, they can determine whether they could look back over the hill and see the top of the tower. Students can make sense of the problem by realizing that if the tower were visible to them, then, equivalently, the tower's signal would be able to reach them.

Assessment

Students should summarize their approaches to solving this problem, including their thought processes throughout the development of their solution.

Invite students to change one of the measurements in this problem (height of the tower, width of the lake, width or height of the hill, or the slope of one or both sides of the hill) and then solve the new problem. Students should explain how changing each measurement affects how far up or down the hillside the tower must be shifted.

Resources

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

———. *Focus in High School Mathematics: Reasoning and Sense Making.* Reston, Va.: NCTM, 2009.

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math).* Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org.
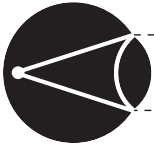
Over the Hill
Student Activity Sheet

A cell phone tower will be built somewhere on the west side of the hill pictured in figure 1. How far up the hill must the tower be placed to provide a signal to anyone on the east side of the lake?
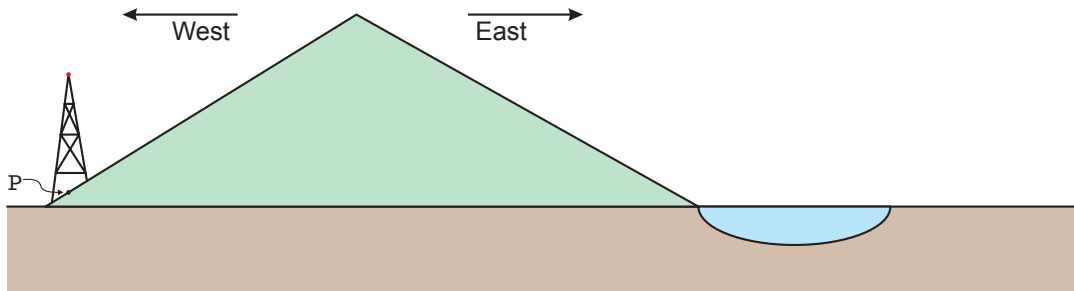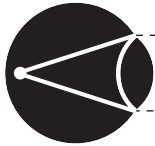

(*Not drawn to scale*)

Fig. 1

Part 1: Preliminary Probing

1. What information is needed to solve the problem? What information is not important to know?

2. Thinking algebraically or geometrically, how can you mathematize the problem?

Over the Hill
Student Activity Sheet (Continued)

Part 2: Down to Details

3. Use the following information to draw an accurate model:

   - The cell tower is 200 feet tall.
   - The hill is 800 feet tall and 2800 feet wide from west to east at the base.
   - The hill has vertical symmetry.
   - The lake starts at the base of the hill and is 600 feet wide.
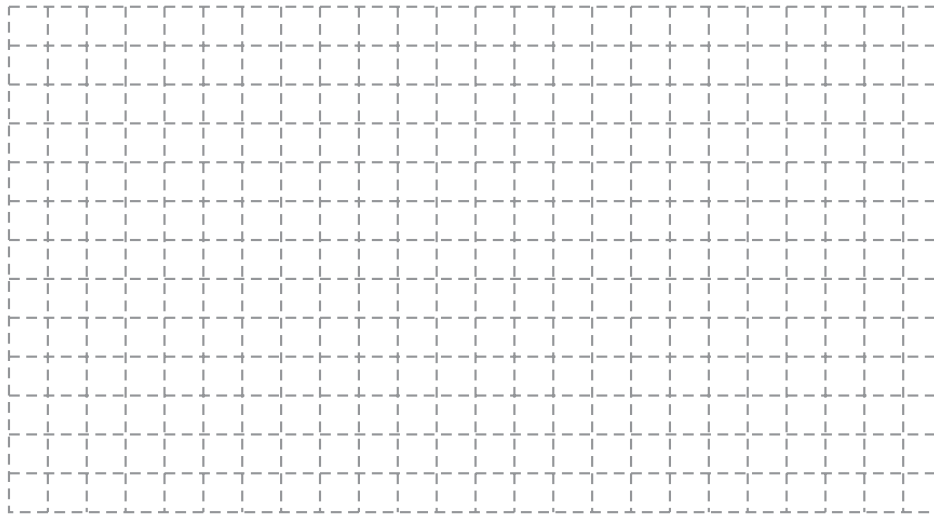
4. First, predict how far up the hillside the tower must be built so that the tower can provide a signal to all points on the other side of the lake. Then determine the exact location on the hillside for the base of the tower. Explain your thinking completely.

5. How close was your prediction to your actual solution? Is your solution reasonable? Explain.