

Tidal Waves

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Purpose	Students analyze a problem faced by the captain of a shipping vessel. They use a range of functions to model the situation and reflect on their usefulness. Because trigonometric functions can be useful, this task would be particularly appropriate for students who have had an introduction to graphing sine and cosine functions.	
Task Overview	The captain of a shipping vessel must consider the effects of tides on water depth when entering a seaport. On the basis of given information about water depth at high and low tides in a certain seaport, develop a mathematical model of water depth in port as a function of the elapsed time since midnight. Use your model to suggest times when the captain could safely enter the seaport if the minimum depth of water the ship needs is 9 meters. <i>An activity sheet that gives students the complete task is included.</i>	
Focus on Reasoning and Sense Making	Reasoning Habits <i>Focus in High School Mathematics: Reasoning and Sense Making</i> Analyzing a problem—applying previously learned concepts; making preliminary deductions and conjectures Reflecting on a solution—considering the reasonableness of a solution; generalizing a solution to a broader class of problems Process Standards <i>Principles and Standards for School Mathematics</i> Problem solving—solve problems that arise in mathematics and in other contexts Reasoning and proof—develop and evaluate mathematical arguments and proofs Representation—use representations to model and interpret physical, social, and mathematical phenomena	Standards for Mathematical Practice Common Core State Standards for Mathematics 1. Make sense of problems and persevere in solving them. 7. Look for and make use of structure.
Focus on Mathematical Content	Key Elements <i>Focus in High School Mathematics: Reasoning and Sense Making</i> Reasoning with functions—modeling by using families of functions; analyzing the effects of parameters	Standards for Mathematical Content Common Core State Standards for Mathematics F-BF-3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. F-TF-5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
Materials and Technology	<ul style="list-style-type: none">• Tidal Waves activity sheet• Graphing calculators or computer graphing software• Online applet (optional; http://MathRSM.net/applets/tidal)	





Use in the Classroom

After having your students read the description of the situation and the modeling task (question 1) on the activity sheet, you might give the students some time to brainstorm about functions that might be useful in modeling the situation.

You might encourage students to pursue several different functions and evaluate their usefulness prior to focusing the students' attention on the cosine function as potentially very useful. (See "Focus on Student Thinking" for examples.)

For students who have experience with the effects of parameters on families of functions—particularly, the cosine and sine functions—the task may be an application or extension of their current knowledge. In this case, you might ask the students first to work individually to solve the problem (either in class or for homework) and then to compare their answers with those of other students.

If the students are less familiar with the cosine function, encouraging them to use technology to "fit" the function to the constraints of the problem could be useful. For example, students might experiment with an online applet that has sliders for the parameters (see <http://www.MathRSM.net/applets/tidal>). You might then use their experiences with this problem to introduce families of trigonometric functions.

You might ask students to interpret the parameters of the function rule in the context of the problem. For example, the constant term represents the average depth of the water, and the coefficient of cosine represents the variation in the depth of the water from the average to the maximum (or minimum) depth.

After the students have finalized their model, challenge them to use it to explain when the vessel can safely enter the port (question 2 on the activity sheet). In this case, technology will be useful—for assisting in calculating the inverse function, using trial and error, or reading the answer from the graph. You might also ask students to analyze their answers in terms of the periodicity of the function.

You might conclude the discussion by having your students explore what times work for other drafts (defined in question 2 as "the minimum depth of water in which a ship or boat can safely navigate") as well as other aspects and issues that the model might address (question 3 on the activity sheet).

Inviting students to share their own ideas will help them identify previously learned, relevant concepts in analyzing the problem.

Evaluating multiple functions encourages making sense of the problem and persevering without jumping to a premature conclusion.

Students can select appropriate technology tools to differentiate for their learning needs.

Asking students to connect effects of changing parameters on graphical representations helps them see the mathematical structure of the function family in context.

Students can use technology to draw conclusions from the model and reflect on their reasonableness.

Students should always be encouraged to generalize a solution.



Focus on Student Thinking

Students might initially want to use a line to model the situation, since two points are given. Or they might suggest a parabola, since the water level goes up and back down. Help them to see the limitations of those models by considering what the depth would be at various times.

Other students might want to use a "triangle wave" graph that goes linearly from maximum to minimum. They could find the equation between successive maxima and minima to model the problem, but you could point out how messy this would become. Advanced students might explore the triangle wave function, which has the following form, with the brackets indicating the "floor" function:

$$f(x) = \left\lfloor x - \left\lfloor x + \frac{1}{2} \right\rfloor \right\rfloor$$

Because the phenomenon that the students are modeling is periodic, they might recognize that a trigonometric function could be useful. Furthermore, since the given data go from a maximum to a minimum, the cosine function would be a good choice, since its maximum is at 0. The sine graph could also be useful.

Students might use reasoning like the following to find the parameters of the cosine function where x is the time in hours and y is the depth of the water in meters:



Focus on Student Thinking—Continued

- The vertical shift of the function is the average of the minimum and maximum depths, in this case 8.55 meters, since the cosine is centered about the x -axis, where $y = 0$. This is the constant term of the function, which controls its vertical shift.
- The amplitude is half of the difference between the maximum and minimum—in this case, 2.05 meters—since it is half of the total height of the curve. This is the coefficient of the function, since it adjusts the vertical stretch of the curve.
- A full period of the function is 12 hours, since the function goes from the maximum to the minimum in 6 hours. The coefficient of x controls the period, since it adjusts the “speed” at which the function progresses. In radians, the coefficient is $2\pi/12$, since the period of the cosine graph is 2π . In degrees, the coefficient is $360/12$.
- The horizontal shift of the function is 5 hours, since the maximum of the function needs to be at 5:00 a.m., but the maximum of the cosine function is at 0. Subtracting 5 from x in the function makes this adjustment.
- Putting this all together, the function rule (using radians) is $f(x) = 2.05 \cos((\pi/6)(x - 5)) + 8.55$.

(Note that the reasoning for the sine function would be similar, although with a different horizontal shift.)

Students might also use a graphing utility to approximate this function, as suggested previously.

To find when the ship can enter the seaport, the students need to find the value of x when $f(x) = 9$. Students could find this value by using \cos^{-1} on the calculator, by trial and error, or by approximating the answer from a graph. This value is the time at which the ship can enter the harbor—in this case, about 2.42 hours.

Students might read 2.42 as 2:42. To help them reason about this, ask them what time 2.75 would be. They should see that .75 is $\frac{3}{4}$ of an hour, or 45 minutes, so 2.75 would correspond to 2:45. Likewise, the number of minutes in 2.42 would be 0.42×60 , or about 25 minutes. So the ship could safely enter the harbor at 2:25 a.m.

However, the water will come back down, so the students need to find the next value where $f(x) = 9$ to see at what time the vessel can no longer safely enter the harbor. Students could again find this value by using trial and error or approximating the answer from a graph, determining that this value is about 7.58 hours, or 7:35. They could also reason that since the graph is symmetric about the maximum, the vessel can first enter the port 2 hours and 35 minutes before 5:00. Therefore, the last time when it can safely enter is 2 hours and 35 minutes after 5:00, which is also 7:35. Because this is a periodic function with a period of 12 hours, these times could be a.m. or p.m.

In thinking about other questions that might be addressed, students may explore entry times for drafts other than 9 meters. By looking at the graph or thinking about the range of values, they might recognize that if the draft is less than 6.5 meters, the ship can enter at will; if the draft is more than 10.6 meters, it can never enter the port; and in other cases, the beginning and ending times will be symmetric about the time of high tide.

They may also explore what happens when the conditions of the problem, such as the minimum, maximum, or period, are changed.



Assessment

As your students are working on the task, ask questions to gain a better understanding of their thinking.

You could ask the students to write up a report for the sea captain on the times when he can safely enter the port, depending on the draft of his vessel. Students should include the reasoning behind their recommendations.

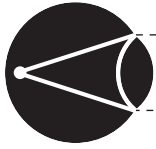


Resources

Common Core State Standards Initiative (CCSSI). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math)*. Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010. <http://www.corestandards.org>.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

———. *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va.: NCTM, 2009. Example 13, pp. 52–53.

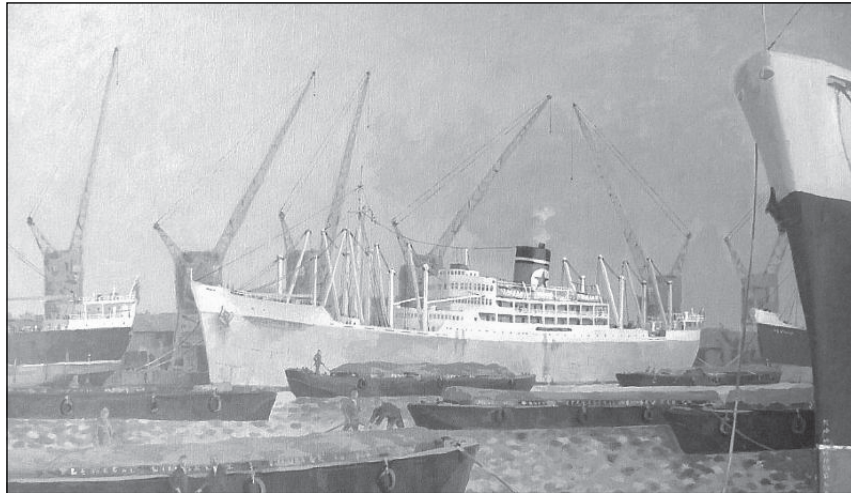


Tidal Waves

Student Activity Sheet

The captain of a shipping vessel must consider the tides when entering a seaport because the depth of the water can vary greatly from one time of day to another.

Suppose that high tide in a certain port occurs at 5:00 a.m., when the water is 10.6 meters deep, and the next low tide occurs at 11:00 a.m., when the water is 6.5 meters deep.



1. Develop a mathematical model that will predict the depth of the water as a function of the elapsed time since midnight.
2. Use your model to suggest at what times the captain could safely enter the seaport if the *draft* (the minimum depth of water in which a ship or boat can safely navigate) of his vessel is 9 meters.
3. What other questions could this model answer?