solutions to palette

(Continued from pages 466–67)

(Alternative approaches to those suggested here are encouraged.)

ANSWERS

1. 4.47 miles
2. 0.215 inches
3. 20 feet
4. 13 arrangements
5. Deriving in Runs
6. 2.97 feet
7. 8 goals
8. 249.38 yd.^{3}
9. About $19,508.00
10. A 10-month-old on Jupiter
11. $14.89 per hour
12. 1,094,995 more revolutions
13. $\sqrt{2} : 1$
14. Approximately $7.46, but many predictions are possible
15. 2/9
16. 2520

Table 1 Solution to problem 3

Dimension 1	Dimension 2	Sum	Difference
1	60	61	59
2	30	32	28
3	20	23	17
4	15	19	11
5	12	17	7
6	10	16	4

SOLUTIONS

1. The submarine descends:

$$\frac{45 \text{ m}}{1 \text{ min.}} \times \frac{60 \text{ min.}}{1 \text{ hr.}} \times \frac{8}{3} \text{hr.} = 2700 \text{ m}$$

There are approximately 1609.34 m in 1 mile, thus the submarine descended

$$\frac{7200}{1609.34} \approx 4.47 \text{ mi.}$$

2. Assuming universal thickness, the chunk is

$$(0.25 \cdot \pi \cdot 2^2) + (3 \cdot 2) \approx 9.14 \text{ in.}^2.$$

Half the area would be

$$9.14/2 \approx 4.4.57 \text{ in.}^2,$$

which means the cake should be cut 4.57 in.^{2}/2 in. ≈ 2.285 in. from the right edge (the rectangular edge) of the cake. Because Joel cut the cake 2.5 in. from the edge of the cake, he was 2.5 – 2.285 ≈ 0.215 in. away from cutting the cake into two equal pieces.

3. **Table 1** shows that the only overlap in sums and differences is 17, which means that the plots are 20 ft. × 3 ft. and 5 ft. × 12 ft. The largest of those dimensions is 20 ft.

4. To maximize the number of arrangements, add the numbers of tulips starting with 1 + 2 + 3 + 4 + · · · + until the number of tulips is met (100):

$$1 + 2 + 3 + 4 + \cdots + 12 + 13 + 14 = 105.$$

By eliminating the arrangement with 5 tulips, the number of tulips used would not exceed 100, which means that Sammie can create 14 – 1 = 13 arrangements.

5. Using the clues, Pascal's Pop Ups could not have finished 1st, 3rd, or 4th, so that team must have finished 2nd. Deriving in Runs did not finish 1st or 3rd and could not have finished 2nd (because Pascal's Pop Ups did). Thus, Deriving in Runs finished 4th.

6. Choosing the circular design of fence with an area of 150 ft.^{2} means that the area formula for a circle, $150 = \pi r^2$, must be used to solve for r, the radius, which is about 6.91 ft. Therefore, the length and width (diameter) of the circle will be about 2(6.91) = 13.82 ft. Since both designs have the same length, the width of the rectangle can be calculated by dividing 150 ft.^{2} by the length of 13.82 ft., which gives a width of about 10.85 ft. The difference between the lengths is 13.82 ft. – 10.85 ft. = 2.97 ft.

7. If the Altoona team averaged 4 goals over the 5 games, then members scored 5 × 4, or 20, total goals over the 5 games. If the median is 5, then one goal total is 5, two goal totals will be above 5, and two goal totals will be below 5. To maximize the largest possible goal total, minimize the two goal totals below 5, which can be 0 and 1, and the minimum number of the goals above 5 can be 6. Thus, the maximum possible goal total for 1 of the 5 games can be 20 – 6 – 5 – 1 – 0, or 8, goals.

8. There are

(280 × 2) + (720 × 2) + (5 × 4 for the corners) = 2,020 linear ft.

of sidewalk to be poured. The total pour is then equivalent to the volume

of a rectangular prism of length 2,020 ft., width 5 ft., and depth 8/12 ft. The total volume is calculated as

$$2{,}020 \times 5 \times 8/12 = 6{,}733\ 1/3\ \text{ft.}^3,$$

or

$$6{,}733\ 1/3 \div 27 \approx 249.38\ \text{yd.}^3.$$

9. The total number of yards ordered will be $1.05 \times 249.83 = 261.849$ yd. The cost of the concrete is then $261.849 \times \$74.50 = \$19{,}507.7505$, or about \$19,508.00.

10. Each resident's age in days is calculated by multiplying the age in years by the length, in days, in a year on each respective planet. A 15-year-old on Venus is

$$15 \times 225 = 3{,}375 \text{ days old.}$$

A 9-year-old on Earth is

$$9 \times 365 = 3{,}285 \text{ days old.}$$

A 4-year-old on Mars is

$$4 \times 687 = 2{,}748 \text{ days old.}$$

A 10-month-old on Jupiter is

$$10/12 \times 4{,}329 = 3{,}607.5 \text{ days old.}$$

11. Let s stand for the current salary. The union employees' new salary is $s + 0.46$. The nonunion employees' new salary is $1.0225s + 0.125$. Setting these two expressions equal and solving for s yields \$14.89 per hour.

12. The 16-in. trailer wheel has a circumference of $2\pi(4/3 \text{ ft.}) = 8\pi/3$ ft. The wheel makes a total of

$$4{,}264.5 \text{ mi.} \times 5{,}280 \text{ ft./mi.} \div (8\pi/3 \text{ ft.}) \approx 2{,}687{,}716 \text{ revolutions.}$$

The 27-in. bicycle wheel has a circumference of $2\pi(9/4 \text{ ft.}) = 9\pi/2$ ft. The wheel makes a total of

$$4{,}264.5 \text{ mi.} \approx 5{,}280 \text{ ft./mi.} \div (9\pi/2 \text{ ft.}) \approx 1{,}592{,}721 \text{ revolutions.}$$

The smaller wheel makes 1,094,995 more revolutions than the larger wheel.

13. The ratio of length to width, or $L{:}W$, in the original must be equal to $W{:}(1/2)L$ in the smaller rectangles. So we have

$$\frac{L}{W} = \frac{W}{\frac{1}{2}L},$$

which can be rewritten and simplified as follows:

$$\frac{L}{W} = \frac{W}{\frac{1}{2}L}$$

$$\frac{L}{W}\left(\frac{L}{W}\right) = \left(\frac{W}{\frac{1}{2}L}\right)\frac{L}{W}$$

$$\frac{L^2}{W^2} = \frac{1}{\frac{1}{2}}$$

$$\frac{L^2}{W^2} = 2$$

Taking the square root of both sides, we find

$$\frac{L}{W} = \frac{\sqrt{2}}{1}.$$

This special property of the "root 2" rectangle is the reason that it has been chosen as the aspect ratio for all paper sizes in the European DIN (German Institute for Standardization) paper sizing system.

14. Many approaches are possible. One possible solution is to find the average increase per year in the price of honey. This average is calculated as

$$(0.11 + 0.15 + 0.36 + 0.32 + 0.12 + 0.32 + 0.41)/7 = 0.2557.$$

To predict the price in 2020, multiply the average increase per year times 7 and add it to the price in 2013:

$$0.2557 \times 7 + 5.67 \approx \$7.46.$$

15. See **table 2**'s outcomes for the experiment. There are 36 equally likely outcomes, 8 of which are favorable. Therefore, the probability is $8/36 = 4/18 = 2/9$.

Table 2 Solution to problem 15

		Die 1					
		1	**2**	**3**	**4**	**5**	**6**
Die 2	**1**	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

16. The smallest integer divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 can be constructed using the minimal number of prime factors that guarantee division for 1 through 10. This number must have factors of 2, 3, 5, and 7 to ensure divisibility by these prime numbers. An extra factor of 2 is required to ensure divisibility by 4 (as 2^2). Another extra factor of 2 is required to ensure divisibility by 8 (as 2^3). An extra factor of 3 (as 3^2) is required to ensure divisibility by 9. No extra factors are needed to ensure divisibility by 6 because a 2 and a 3 are already included. The same logic applies for 10. Therefore, the number is $2 \times 3 \times 5 \times 7 \times 2 \times 2 \times 3 = 2{,}520$.