Mathematics Education in the United States 2004

A Capsule Summary Fact Book

written for

The Tenth International Congress on Mathematical Education (ICME-10)

Copenhagen, Denmark
July 2004

by

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National Council of Teachers of Mathematics
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PREFACE

On 1 July 2002, the population of the United States was about 288 000 000. Approximately 89% of the population aged 5–17 was formally enrolled in an elementary or secondary school. In the entire population, about 65% were age 25 or over, and of those adults, 88% had some postsecondary education (OECD 2001) and 27% had at least a college bachelor's degree, a record high (Kincannon 2004). (Educational Testing Service [ETS] Web site)

No single government agency controls grades K–12 public education in the United States. Rather, authority for most educational decisions lies with education agencies in the 50 individual states, which in turn share decision making with the 14 559 [in 2001; 14 883 in 1996] individual districts within them. In addition, 11.4% of students in grades K–12 were enrolled in private (non-government-controlled) schools in 1999–2000 [projection through 2009–2010: 11.1%] (National Center for Education Statistics [NCES], as cited in *The World Almanac and Book of Facts 2004*).

Similarly, both public and private institutions exist at the college and university level, with ultimate authority at the state level for public institutions and at the institutional level for most private institutions. Of the approximately 4100 [1995: 3700] colleges and universities, one source (*World Almanac and Book of Facts 2004)* lists 1442 [2000:1315] four-year colleges (those that award a bachelor's degree) with current enrollments of more than 1000 each. A total of about 14 million students attend these and smaller colleges. The nine largest of the four-year colleges are state universities with enrollments between 40 000 and 53 000.

Determining what is happening in such a large and complex arena is quite difficult even for those in the United States and others familiar with education in the United States. Furthermore, many attendees at ICME conferences lack familiarity with education in the United States. Consequently, in 1999 the U.S. National Commission on Mathematics Instruction recommended that the National Council of Teachers of Mathematics request funds from the National Science Foundation to bring together available data about mathematics education in the United States for a document to be distributed at the Ninth International Congress on Mathematical Education (ICME-9) to provide mathematics educators throughout the world with information about this complex system. The current document, written for ICME-10, updates the 2000 version with the latest information available as of early 2004.

We begin this document with some general information about education in the United States. We then describe the three kinds of curriculum identified in the Second International Mathematics Study and Third International Mathematics and Science Study—intended, implemented, and attained. We include a section on programs for high-achieving and other special students because of the interest of this topic to those in other countries. Finally, we provide some information about mathematics teacher education and about doctoral programs in mathematics education. One message that comes through repeatedly in these descriptions is the variety of available programs and thus the inability to characterize them adequately in a brief document like this one. Another message is that great flux occurs at all levels of the educational system, and even though we have attempted to provide the latest available information, the information presented here will quickly become dated. By listing our sources, we hope we enable the interested reader to be able to obtain updated information.

We would like to acknowledge the efforts of Gail Burrill, who wrote the grant under which the funding for this publication was obtained; the advice of USNCMI past and current members Daniel Goroff, Glenda Lappan, Marilyn Mays, Dan Teague, Joan Garfield, Skip Fennell, Hyman Bass, Herbert Clemens, and Karen Dee Michalowicz in helping to plan and overseeing this document; the help of Jim Bohan, Jennifer Griffin, David Moore, Erik Nagel, Carol Siegel, and Iris Weiss in obtaining or checking the information used here; and the fine work of Harry Tunis, Ann Butterfield, and Nick Abrash at NCTM in editing and producing this document. We have tried to be as accurate as we could be and apologize for any errors.

Zalman Usiskin

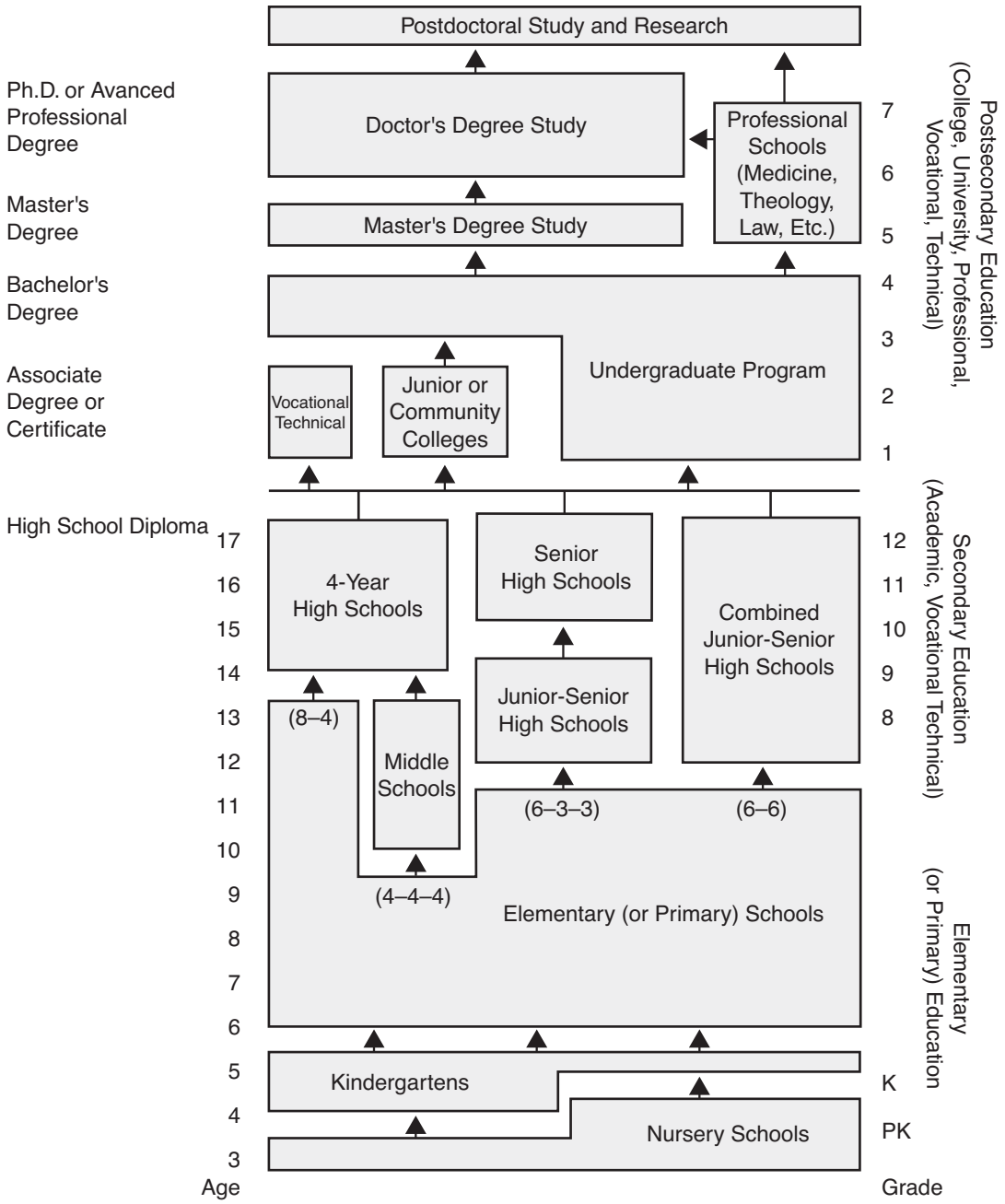
John Dossey

June 2004

PART I: GENERAL INFORMATION

Overall Organization of Education in the United States

Figure 1 shows the general framework of education in U.S. schools. This system can be thought of as having four broadly defined levels: elementary school (grades K–5 or K–6, corresponding to ages 5–10); middle school or junior high school (grades 6–8 or 7–8, ages 11–13 or 12–13); senior high school (grades 9–12, ages 14–17); and postsecondary or tertiary education (grades 13 and above, ages 18 and above).



NOTE—Adult education programs, while not separately delineated above, may provide instruction at the elementary, secondary, or higher education level. Chart reflects typical patterns of progression rather than all possible variations.

Fig. 1. The structure of education in the United States (NCES 2003a, figure 1, p. 7)

After graduating from high school, students may leave the educational system or choose to attend a technical or vocational institution, a two-year or community college, or a four-year college or university. The two-year and community colleges usually offer a mixture of the first two years of a four-year college's curriculum along with a number of courses also found in the technical colleges and high schools. In these colleges, an Associate of Arts (A.A.) degree can usually be earned through the equivalent of two years of full-time study. The four-year colleges and universities offer Bachelor of Science and Bachelor of Arts degrees (B.S. and B.A.), which can usually be completed in four years of full-time study. In addition, the universities offer graduate programs leading to master's and doctoral degrees (M.S., Ed.D., and Ph.D.). Programs leading to professional degrees (law, medicine, business, etc.) exist both within universities and in institutions that offer no other degree programs. Time to complete postbachelor degrees varies with the field and the institution.

Enrollments

Education is compulsory by law in most states from age 6 or 7 through at least age 16 (Council of Chief State School Officers [CCSSO] 1998, p. 47), but most students stay in school longer. National data are not kept on the percentage of students completing secondary school, but this percentage seems to have remained nearly constant in recent years. Of those who entered high school as 9th graders in the fall of 1997, 67% graduated (finished 12th grade) *in the same state* in the spring of 2001 (National Center for Education Statistics 2003a). Many who do not complete high school with their class earn equivalent diplomas later. In the 25–29 age group in 1999, 88% of females and 85% of males had a high school diploma or its equivalent, with major differences among racial/ethnic subgroups: 92% of whites, 87% of blacks; 62% of Hispanics (Education Testing Service [ETS] Web site).

As table 1 suggests, the number of students in U.S. schools has risen steadily since 1985. Projections through 2012 show the total number of students in public K–12 schools remaining nearly constant, with elementary- and middle-grades enrollments decreasing through 2008 and then increasing, whereas secondary school enrollments are shown as increasing through 2007 and then decreasing. The proportion of students in public and private education has changed very little since 1985. At the same time, the percent of students continuing after secondary school to some form of postsecondary education has increased dramatically. From 1979 to 2001, the percent of children living with a parent who had completed college increased steadily from 19% to 31%. At the same time, the percent of children living in a two-parent household decreased steadily from 75% to 68% (NCES 2003d, p. 121).

TABLE 1
School and College Enrollments over Time (in Millions)

	Year			
Level	1985	1999	2003	2008 (projected)
K-12 public	39.4	47.1	47.6	48.2
K-12 private*	5.6	6.0	6.0	6.1
Postsecondary	12.2	14.8	15.6	16.0

*Nongovernmental, including parochial schools (governed by religious bodies)
Source: NCES, 2003a, table 1

A recent phenomenon, for which little hard data exist, is the home schooling of children in the United States. Such children may be taught by their parents, by hired tutors, or by a teacher serving a cluster of families. This approach is most prevalent with children of elementary school age, but in places the practice extends to the secondary school level. The U.S. Department of Education estimated that the number of home-schooled students in grades K–12 in 1997–98 was between 800000 and 1000000 children (NCES 1999).

College Admission

Graduates of public or private senior high schools may matriculate into the nation's colleges, but they must apply to individual schools to be considered for admission. Many two-year colleges will accept any student from their geo-

graphic area. Other two-year colleges and most four-year colleges require that certain numbers of courses be taken in English, mathematics, science, social studies, and foreign language. Many state-supported institutions have formulas for admission that may take into consideration the intended field of study, senior high school course grades, percentile rank in class, scores on college entrance examinations, letters of recommendation, sports and other extracurricular activities, and other information supplied by the high school or the student. Private colleges use some of the same criteria as public institutions but may take other factors, such as family variables, into consideration. The more select schools also consider the difficulty of courses taken in high school and scores on Advanced Placement examinations.

The costs of college attendance range from tuitions of less than $100 per course at some colleges to over $35,000 for tuition and fees at some select universities. Many students receive scholarships from various sources, including the college they attend, government sources, or private foundations. The College Board scholarship database has more than 2300 sources of funding with nearly $3 billion in available aid. Government loans and loans from private sources constituted 54% of the aid students received in 2001–2002 (The College Board 2002), and many college students hold part-time jobs. The federal government offers tax credits for tuition and certain other education expenses.

Matriculation into college does not have to occur immediately after high school graduation, nor does attendance in college need to be full time. For financial and other reasons, many students delay college work. From 1970 to 1999 (the latest data available), proportionally more college students became enrolled part time (39% in 1999 vs. 28% in 1970) and at two-year colleges (44% vs. 31%). Women have overtaken men as the majority of college attendees (56% vs. 42%), and more students are age 25 or older (39% vs. 28%). (NCES 2002a)

In 1999–2000, 8% of all undergraduates participated in distance education at the institution in which they were enrolled or at both the institution at which they were enrolled and somewhere else. Among those who participated, 29% were enrolled in programs available entirely through distance education. Among students who participated in distance education, 60% participated through the Internet, 39% through prerecorded television program or audio transcription, and 37% through live television program or audio session (NCES 2002b). We do not have data specifically covering the use of distance education in mathematics classes.

Governance of Education in Mathematics

Because the Constitution of the United States does not claim education as a responsibility of the federal government, individual states are considered to have responsibility for the education of citizens within them. Legislation at the state levels provides most of the structure for schooling. State laws define the ages for compulsory education for all students; outline the general framework for required studies in reading, writing, mathematics, science, social science, physical education, and other subjects; define the minimum number of days of school attendance per year; and define requirements for teacher certification. State laws also provide the mechanisms by which local schools are recognized by the state government, and state statutes are enacted to establish and accredit private schools.

Nevertheless, the U.S. government has a Department of Education that sets certain standards for schooling and provides funding for special programs, such as school lunch programs for students in poverty and compensatory programs for students needing special educational assistance. The role of the federal government has increased markedly in the past few years as a result of the No Child Left Behind Act (NCLB) passed by Congress in 2001. NCLB authorizes the Department to undertake a program offering financial incentives for good performance of schools and imposing penalties for poor performance; such a program is unprecedented in the nation's history (see pp. 4–5).

The description of mathematics education as an enterprise in U.S. schools is more difficult than the description of education as a whole, because education in mathematics is, in most school districts, left to the control of a locally elected board of education. Each district, operating under its own authority and various state laws, sets standards, designs delivery programs, and provides financial support for its own mathematics education program. Because the United States has approximately 15 000 different school districts, many views of mathematics and its goals are represented, and much variance occurs in the amount of resources expended toward mathematics.

The guidance that states provide to schools within them also varies. Forty-nine states (all but Iowa) provide curricular frameworks for mathematics in required programs of study. These frameworks for mathematics are suggestions for school programs, not binding curricular programs of study, but they may define boundaries for a state assessment program in mathematics, outlining what mathematical knowledge is expected by a certain grade level. At least 46 states also employ resource individuals (state mathematics supervisors or consultants) who assist schools on questions concerning the classroom teaching and learning of mathematics and on issues concerning statewide assessment programs. However, in the end, the decisions made in a local school district determine the actual content of the school mathematics program within the district. Research studies have shown that school curricular decisions are almost always determined by the textbook series adopted for use in the district's schools (Porter et al. 1988), although in recent years more influence of external assessments has been observed and perhaps more locally written curricula have been adopted.

The picture is much the same at the college and university level. Great variance occurs in the programs of study that U.S. students complete under the name of mathematics. Yet the curricula offered by schools and universities exhibit a great deal of similarity. Part of this similarity is the result of core recommendations for study in mathematics issued explicitly by state governments and professional societies and implicitly by commercial textbooks and examinations.

"No Child Left Behind" Legislation

Three days after taking office in January, 2001, President George W. Bush announced the No Child Left Behind program, his framework for education reform that he described as "the cornerstone of my Administration." Less than a year later, the No Child Left Behind Act of 2001 (NCLB Act) was passed by Congress.

The NCLB Act has four main thrusts: increased accountability for states, school districts, and schools; greater choice for parents and students, particularly those attending low-performing schools; more flexibility for states and local educational agencies (LEAs) in the use of Federal education dollars; and a stronger emphasis on reading, especially for the youngest children (NCLB Web site). The summary here is taken from the Executive Summary written by the Department of Education (http://www.ed.gov/nclb/overview/intro/execsumm.html).

Increased accountability

The NCLB Act requires all 50 states to implement statewide accountability systems covering all public schools and students. These systems must be based on challenging state standards in reading and mathematics, annual testing for all students in grades 3–8, and annual statewide progress objectives ensuring that all groups of students reach proficiency within 12 years. Assessment results and state progress objectives must be broken out by identifiable groups of students on the basis of poverty, race, ethnicity, disability, and limited English proficiency to ensure that no group is left behind. School districts and schools that fail to make adequate yearly progress (AYP) toward statewide proficiency goals will, over time, be subject to improvement, corrective action, and restructuring measures aimed at getting them back on course to meet state standards. Schools that meet or exceed AYP objectives or close achievement gaps will be eligible for state Academic Achievement Awards.

More choices for parents and students

LEAs must give students attending schools identified for improvement, corrective action, or restructuring the opportunity to attend a better public school, which may include a public charter school within the school district. The district must provide transportation to the new school. For students attending persistently failing schools (those that have failed to meet state standards for at least three of the four preceding years), LEAs must permit low-income students to use Title I funds to obtain supplemental educational services from the public- or private-sector provider selected by the students and their parents. Providers must meet state standards and offer services tailored to help participating students meet challenging state academic standards. Schools that want to avoid losing students—along with the portion of their annual budgets typically associated with those students—must improve or, if they fail to make adequate yearly progress for five years, run the risk of reconstitution under a restructuring plan.

Greater flexibility for states, school districts, and schools

New flexibility provisions in the NCLB Act include authority for states and LEAs to transfer up to 50% of the funding they receive under four major state grant programs to any one of the programs or to Title I. The covered programs include Teacher Quality State Grants, Educational Technology, Innovative Programs, and Safe and Drug-Free Schools.

Putting reading first

No Child Left Behind stated President Bush's unequivocal commitment to ensuring that every child can read by the end of 3rd grade. To accomplish this goal, the new Reading First initiative significantly increases the Federal investment in scientifically based reading instruction programs in the early grades. This new initiative makes competitive six-year awards to LEAs to support early language, literacy, and prereading development of preschool-age children, particularly those from low-income families. Recipients are to use instructional strategies and professional development drawn from scientifically based reading research to help young children attain the fundamental knowledge and skills they will need for optimal reading development in kindergarten and beyond.

Other major program changes

The No Child Left Behind Act of 2001 combines the Eisenhower Professional Development and Class Size Reduction programs into a new Improving Teacher Quality State Grants program that focuses on using practices grounded in scientifically based research to prepare, train, and recruit high-quality teachers. The new program gives states and LEAs flexibility to select the strategies that best meet their particular needs for improved teaching that will help them raise student achievement in the core academic subjects. In return for this flexibility, LEAs are required to demonstrate annual progress in ensuring that all teachers teaching in core academic subjects within the state are highly qualified.

PART II: INTENDED CURRICULUM

Historical List of Major Documents—Grades K–12

The 1890s saw major increases in enrollments and pressures on schools to adjust to those increases. Thus began more than a century of meetings and recommendations of professional groups of national scope attempting to define, in varying amounts of detail, a strong mathematics program.

1894:	Committee of Ten on the Secondary School Syllabus
1899:	NEA Committee on College Entrance Requirements
1911–1918:	International Commission on the Teaching of Mathematics
1916–1923:	National Committee on Mathematical Requirements
1933–1940:	Joint MAA-NCTM Commission, *The Place of Mathematics in Secondary Education*
1938–1940:	Progressive Education Association, *Mathematics in General Education*
1943:	*Pre-Induction Courses: Essential Mathematics for Minimum Army Needs*
1944–1947:	Commission on Post War Plans
1959–1960:	Commission on Mathematics, College Entrance Examination Board
1963:	Cambridge Conference on School Mathematics, *Goals for School Mathematics*
1964:	IEA (First) International Study of Mathematics Achievement (report in 1967)
1975:	National Advisory Committee on Mathematical Education (NACOME), *Overview and Analysis of School Mathematics K–12*
1975:	National Institute of Education Conference on Basic Mathematics Skills and Learning
1976:	National Council of Supervisors of Mathematics, *Position Paper on Basic Mathematical Skills*
1978:	NCTM-MAA, *Recommendations for the Preparation of High School Students for College Mathematics Courses*
1980:	NCTM, *An Agenda for Action*
1982:	Conference Board of Mathematical Sciences, *The Mathematical Sciences Curriculum K–12: What Is Still Fundamental and What Is Not*
1983:	National Commission on Excellence in Education, *A Nation at Risk*
1983:	College Board, *Academic Preparation for College*
1987:	SIMS, *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (using data collected in 1981–1982)
1989:	National Research Council, *Everybody Counts*
1989:	NCTM, *Curriculum and Evaluation Standards for School Mathematics*
1990:	National Research Council, *Reshaping School Mathematics*
1991:	NCTM, *Professional Standards for Teaching Mathematics*
1995:	NCTM, *Assessment Standards for School Mathematics*
1995–1996:	TIMSS Third International Mathematics and Science Study (reports from 1997 on)
2000:	NCTM, *Principles and Standards for School Mathematics*
2001:	CBMS, *The Mathematical Education of Teachers*
2004:	CUPM, *Undergraduate Programs and Courses in the Mathematical Sciences*

National Council of Teachers of Mathematics (NCTM) *Standards* Documents

In 1985, building on recommendations within the mathematics education community starting in 1975 and the wider consensus of a need for raising expectations in a contemporary mathematics curriculum, the NCTM began the process that led to standards documents on curriculum and evaluation (National Council of Teachers of Mathematics [NCTM] 1989), teaching (NCTM 1991), and assessment (NCTM 1995).

Although throughout the 20th century various groups had made suggestions relative to what the U.S. school mathematics curriculum should be, none combined the scope and detail of these three documents. *Curriculum and Evaluation Standards* provided a listing, by grade-level bands (K–4, 5–8, 9–12), of the mathematics that students should know about problem solving, reasoning, communication, connections, and various content aspects of mathematics relevant to those grade levels.

None had described teaching in as much detail as *Professional Standards for Teaching Mathematics*. The teaching standards took as a basic principle that what students know about mathematics is a product of how they learned it. They asserted that concepts, procedures, and relationships are often best developed in contexts that ask students to develop knowledge themselves under the guidance and watchful eye of a teacher and to engage in discourse about that knowledge. They called on teachers to create conditions that would allow learners to focus on important aspects of content and on the connections between mathematics and other subject areas and among various areas within mathematics. *Assessment Standards for School Mathematics* asked for assessments that reflected the deeper understanding called for in the curriculum and teaching standards.

The existence of the *Standards* also influenced other school subjects. By the end of the 1990s, professional organizations in virtually every other major school discipline had published its own standards. The visibility of the *Standards* also led to a greater interest among groups within mathematics and mathematics education in the revision that began in 1995 and culminated in the release of *Principles and Standards for School Mathematics (PSSM)* in April 2000. Realizing that not everyone agreed with the recommendations in the *Standards*, NCTM took the approach of developing a draft of *PSSM* and then holding a nationwide year of discussion of the draft that allowed individuals and organizations to comment on the recommendations so that they might be revised on the basis of that feedback.

The *PSSM* are guided by six principles (NCTM 2000), listed here. (1) Equity: Excellence in mathematics education requires equity—high expectations and strong support for all students. (2) Curriculum: A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades. (3) Teaching: Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well. (4) Learning: Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. (5) Assessment: Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. (6) Technology: Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

At each of four grade levels—pre-K–2, 3–5, 6–8, and 9–12—*PSSM* contains a comprehensive body of mathematical understandings and competencies organized under five content areas—number and operations, algebra, geometry, measurement, and data analysis and probability—and five ways of acquiring and using that content—problem solving, reasoning and proof, communication, connections, and representation. Despite its potential influence, in the U.S. educational system, *PSSM* is only a resource and guide; it carries no legal

weight.

State Standards and Textbook-Adoption Guidelines

During the mid-1980s many states raised their high school graduation requirements in mathematics in reaction to the findings of the National Commission on Excellence in Education's report *A Nation at Risk* (National Commission on Excellence in Education 1983). As of 2002 (the latest data available), of the 50 states, 44 have statewide credit requirements for high school graduation, 5 transfer this power to local school districts, and only 1 (North Dakota) has no written requirement. Four states require four credits (years) in mathematics, 23 require three credits, 16 require two credits, and 1 requires a total of five credits from science and mathematics. Eighteen states identify one or more specific mathematics courses that must be taken for graduation, typically first-year algebra. Twenty states have high school exit examinations. The items on these examinations are mostly applied arithmetic with small amounts of algebra and geometry such as can be found in middle school textbooks.

All states but Iowa have content standards in mathematics. The changes in expectations signaled by the NCTM *Standards* led 46 states to revise their mathematics expectations between 1985 and 1996. Since 1996, 42 states have revised either their frameworks or their mathematics standards again (CCSSO 2003), and further revisions are presently being undertaken in many states because of the No Child Left Behind legislation.

Within states, local school districts make the decisions regarding which materials to use. For textbooks, this decision process is called *textbook adoption* and may be subject to formal regulations. In 22 states, mainly in the South and West, certain state funds are earmarked for textbooks selected or recommended by a statewide committee for use in that state in accordance with the state's content standards. In these states, all adoptions for a given course or level may take place in the same school year. State textbook-adoption practices have tended to become more lenient over the years, allowing schools a greater selection of materials. In the other 28 states, school districts typically adopt textbooks under more relaxed rules; for example, they may be required to adopt every five years but may use any available materials.

College-Level Documents

Although public K–12 schools receive some guidance in terms of mandated subjects, time allotments, and graduation requirements, few restrictions are placed on college or university programs, public or private.

At the community college level, the American Mathematical Association of Two-Year Colleges developed and released *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus* (American Mathematical Association of Two-Year Colleges [AMATYC] 1995), a document consistent with the spirit and content of the NCTM *Standards*. This document gave recommendations from the AMATYC regarding the teaching and learning of collegiate courses at the precalculus level. It is currently being updated.

Perhaps the strongest force shaping the collegiate mathematics programs taken by prospective teachers of mathematics in any state are the certification requirements for grades K–12 teaching positions in that state. A set of national recommendations for such programs, the first in a generation, was recently prepared through a project of the Conference Board of the Mathematical Sciences (2001). (For some details, see Part VI of this document.) Previous sets of recommendations were developed first in the late 1960s and early 1970s by the Mathematical Association of America (1968) and the NCTM (1972). These publications were both revised in the early 1980s (MAA 1983; NCTM 1981) and discussed once again with the release of the NCTM *Professional Teaching Standards* in the early 1990s (Leitzel 1991). Past initiatives by the mathematics community in the area of stating recommended courses of study for preservice teachers of mathematics have been successful in helping shape state requirements for certification. However, in many states, individuals are allowed to teach mathematics with less than the full requirements for certification as a mathematics teacher if they are teaching mathematics for less than one-half their teaching load or if they are hired in a situation of teacher shortage. This and other loopholes lead to situations in which many of the nation's students in middle school and high school are being taught mathematics by someone with less than the recommended full slate of courses required for certification.

Relating to collegiate programs more broadly are the recommendations of the Committee on the Undergraduate

Program in Mathematics (CUPM) of the MAA. In the 1990s, CUPM made recommendations on the general program for undergraduate mathematics majors and for the teaching of calculus (CUPM 1991; Tucker 1995; Roberts 1996).

The initiatives to make changes in the teaching of calculus began with a meeting at Tulane University in 1986 (Douglas 1986). Following a national convocation on calculus reform held at the National Academy of Sciences (Steen 1988), textbooks began to appear. The calculus reform focused on three issues: (1) revising the content of the course to enhance students' understanding of the basic concepts using linked graphical, numerical, and symbolic approaches; (2) emphasizing the importance of instituting new modes of instruction—particularly laboratory-based project work and applications—and using technology to engage students in active visual explorations; and (3) attempting to make calculus more inclusive rather than exclusive. Results from a national survey in 1994 indicated that about 44% of two-year colleges and universities had made no changes in their calculus sequences, 45% had made modest changes, and 11% had made major changes. These results were fairly consistent across institution types, from major research universities to community colleges (Tucker and Leitzel 1995).

Within the past year, CUPM has released a report (CUPM 2003) dealing with the entire collegiate mathematics curriculum for all students, including those who take just one course. Six recommendations are offered to assist mathematics departments in designing and teaching their courses and programs, with supplementary recommendations for particular student audiences. The six recommendations, detailed in the report (available at www.maa.org/cupm), fall under the following broad guidelines: (1) understand the student population, and evaluate courses and programs; (2) develop mathematical thinking and communication skills; (3) communicate the breadth and the interconnections of the mathematical sciences; (4) promote interdisciplinary cooperation; (5) use computer technology to support problem solving and to promote understanding; and (6) provide support to faculty for instructional improvement.

Mathematicians and Mathematics Education

With the advent of the NCTM Curriculum and Evaluation Standards in the late 1980s, many individuals in the mathematics community became active in programs focusing on helping teachers and schools update their curricula and improve their delivery of quality mathematics to their students. In some situations, this work was consultation, in others it involved leading professional development activities, and in still others it involved policy work to assist schools in garnering more resources and support for or against the changes taking place.

Foremost among the early efforts of mathematicians in support of K–12 mathematics education activities was the founding of the Mathematicians and Education Reform network (http://www.math.uic.edu/MER/pages), which built on earlier activities that individual members of the pure mathematics community had been involved in over the years. A second major effort came through the Education Forum established by the leaders of the Conference Board of the Mathematical Sciences. The constituent organizations of the CBMS established a working group that provided support information for schools on careers in mathematics, supported the development of the proposal that culminated in the NSF-funded project that developed the *Mathematical Education of Teachers* report (CBMS 2001), and coordinated cross-organization efforts to support mathematics education (e.g., reactions to governmental proposals, such as the Voluntary National Mathematics Test for 8th graders proposed by the Clinton administration, changes in support for mathematics education by federal and state agencies, and other issues). At the time of this writing, the Mathematical Science Research Institute at the University of California at Berkeley is planning a series of workshops on topics central to mathematics education. The first of these was on the topic of assessment and was held in mid-March 2004. Other special-topic workshops will follow. These workshops bring mathematicians and mathematics educators together to discuss and further the work of providing quality mathematics education to all children.

PART III: IMPLEMENTED CURRICULUM

No publicly available national data are routinely collected regarding the specific textbooks sold or in use in the United States in grades K–12. Furthermore, grades K–12 textbooks occupy a position different from virtually any other published books; they are not found in the periodical *Books in Print.* As a consequence, one must contact publishers to determine which materials are available. (See pp. 45–46 for contact information.)

Mathematics Study in Elementary Schools (Grades K–5 or K–6)

Mathematics in elementary schools is nearly always taught by a teacher who teaches the same students reading, science, and social studies and is with them almost the entire day. These teachers do not have the time to create lessons for all these subjects and tend to use those in the mathematics materials purchased by their school district. According to an industry survey, more than 90% of schools in the country reported in the school year 2002–03 using a basal or core mathematics series that they either follow very closely or from which they pick and choose as needed [2000–01: 94%]. In the past, most publishers would market elementary school curricula to cover all the grades K–8. In the past decade, these curricula have been split into two parts, a K–5 or K–6 elementary school series and a 6–8 middle school series. The four most-used elementary school series in 2002–03 included two mainstream series (Scott Foresman–Addison Wesley *Mathematics* and Harcourt-Reed's *Math Advantage*) and two other series representing opposite ends of the spectrum (Saxon *Mathematics* and Wright Group/McGraw-Hill's *Everyday Mathematics*). Each of these four series was used by between 11% and 17% of students, and together they account for a little over half of the textbooks used.

Mainstream series are those whose publishers market nationally with large sales forces and which have evolved from decades of earlier series by the same publishers. These series try to meet published guidelines in virtually all the states and, where they exist, in large city or county school districts. We estimate that about 70% of classrooms in the United States are using mainstream series. Mainstream series usually have some activities for calculators, but most lessons are meant to be done without them.

Teachers in grades 4–6 report spending an average of five hours a week on mathematics. Most programs, whether mainstream or not, include teacher's editions with many ideas for teaching; kits of manipulative materials; collections of resource materials for teachers with remediation, extension, and computer activities; and a wide variety of assessment and other materials to assist teachers. Programs furnished by large publishers undergo a copyright revision about every three years and a major revision about every six years.

A listing of contents cannot substitute for a more detailed look at any of these materials. Here is a summary of the contents of grades K–5 student books from the 2002 Harcourt-Reed *Math Advantage* series. Except at the kindergarten level, each book in this series contains six to nine units split into a smaller number of chapters. To save space, we have indicated only the unit titles preceded by the numbers of the chapters within them.

Kindergarten

Sorting and Classifying
Patterns and Movement
Matching and Counting
Numbers 0–5 and Graphing
Numbers 6–10
Geometry and Equal Parts
Numbers 10–30
Money
Measurement
Time
Exploring Addition
Exploring Subtraction

Grade 1

Getting Ready for Grade 1

1–4	Addition and Subtraction Concepts
5–8	Addition and Subtraction Facts to 10
9–14	Numbers to 100 and Addition and Subtraction to 12
15–19	Money, Time, and Graphing
20–25	Geometry, Measurement, and Fractions
26–30	Addition and Subtraction to 20 and with 2-Digit Numbers

Grade 2

Getting Ready for Grade 2
1–4 Numbers, Operations, and Data
7–10 Money and Time
11–16 2-Digit Addition and Subtraction
17–20 Geometry and Measurement
21–24 Number Sense and Fractions
25–30 3-Digit Addition and Subtraction, Multiplication, and Division

Grade 3

1–4 Understand Numbers and Operations
5–6 Money and Time
7–10 Multiplication Concepts and Facts
11–13 Division Concepts and Facts
14–16 Data, Graphing, and Probability
17–19 Geometry
20–22 Measurement
23–26 Fractions and Decimals
27–30 Multiply and Divide by 1-Digit Numbers

Grade 4

1–4 Understand Numbers and Operations
5–7 Data, Graphing, and Time
8–9 Multiplication and Division Facts
10–12 Multiply and Divide by 1-Digit Numbers
13–16 Divide by 1- and 2-Digit Divisors
17–18 Geometry
19–22 Fractions and Decimals
23–26 Measurement and Geometry
27–30 Probability, Algebra, and Graphing

Grade 5

1–4 Use Whole Numbers and Decimals
5–8 Algebra, Data, and Graphing
9–10 Multiply Whole Numbers and Decimals
11–14 Divide Whole Numbers and Decimals
15–16 Number Theory and Fractions
17–20 Operations with Fractions
21–24 Algebra and Geometry
25–27 Measurement
28–30 Ratio, Percent, and Probability

Here are the chapters in grades K–5 from the Scott Foresman–Addison Wesley *Mathematics* series.

Kindergarten

Position and Sorting
Graphing and Patterns
Numbers through 5
Numbers through 10
Numbers through 31
Measurement
Time and Money
Geometry and Fractions
Readiness for Addition and Subtraction
Understanding Addition
Understanding Subtraction
Counting and Number Patterns to 100

Grade 1

Patterns and Readiness for Addition and Subtraction
Understanding Addition and Subtraction
Strategies for Addition Facts to 12
Strategies for Subtraction Facts to 12
Geometry and Fractions
Time
Counting to 100
Place Value, Data and Graphs
Money
Measurement and Probability
Addition and Subtraction Facts to 18
Two-Digit Addition and Subtraction

Grade 2

Understanding Addition and Subtraction
Fact Strategies for Addition and Subtraction
Place Value to 100 and Money
Mental Math: Addition and Subtraction
Two-Digit Addition
Two-Digit Subtraction
Geometry and Fractions
Time, Data, and Graphs
Measurement and Probability
Numbers to 1000
Addition and Subtraction of Three-Digit Numbers
Understanding Multiplication and Division

Grade 3

Place Value and Money
Addition and Subtraction Number Sense
Adding and Subtracting
Time, Data, and Graphs
Multiplication Concepts and Facts
More Multiplication Facts
Division Concepts and Facts
Geometry and Measurement
Fractions and Measurement
Decimals and Measurement
Multiplying and Dividing Greater Numbers
Measurement and Probability

Grade 4
Place Value and Money
Addition and Subtracting Whole Numbers and Money
Multiplication and Division Concepts and Facts
Time, Data, and Graphs
Multiplying by One-Digit Numbers
Multiplying by Two-Digit Numbers
Dividing
Geometry and Measurement
Fraction Concepts
Fraction Operations and Customary Measurement
Decimals and Metric Measurement
Graphing and Probability

Grade 5
Place Value, Adding and Subtracting
Multiplying Whole Numbers and Decimals
Dividing with One-Digit Divisors
Dividing with Two-Digit Divisors
Data, Graphs, and Probability
Geometry
Fraction Concepts
Fraction Operations
Measurement
Measuring Solids
Ratio, Proportion, and Percent
Algebra: Integers, Equations, and Graphing

The third of the four most used series is a program with a special concentration on skills published by Saxon. These materials discourage the use of calculators. Unlike the other series, Saxon *Mathematics* is not organized into units or chapters but is arranged by consecutive lessons. Topics are not dropped as the year progresses but increase in complexity and are practiced every day. The books, with the number of lessons in each, are as follows:

Math K	24	Meetings followed by 112 Lessons
Math 1	130	Lessons
Math 2	132	Lessons
Math 3	140	Lessons
Math 54	141	Lessons
Math 65	140	Lessons

Insufficient space is available here to identify individual lessons, so the interested reader can examine the publisher's information at http://www .saxonpublishers.com.

About 25% of elementary schools are using one of the newer, more conceptually based series, such as those developed with funds from the National Science Foundation. These series tend to allow calculators almost always. Here is a listing of the strands in the kindergarten curriculum and the units in *Everyday Mathematics* published by Wright Group/McGraw-Hill for grades 1–5.

Grade K (strands)
Numeration
Measurement
Geometry
Operations
Patterns and Functions
Money
Clocks and Calendars
Data and Chance

Grade 2
Routines and Assessments
Addition and Subtraction Facts
Place Value, Money, and Time
Addition and Subtraction
3-D and 2-D Shapes
Whole-Number Operations and Number Stories
Patterns and Rules
Fractions
Measurement
Decimals and Place Value
Whole-Number Operations Revisited
Year-End Reviews and Extensions

Grade 1
Establishing Routines
Everyday Uses of Numbers
Visual Patterns and Number Patterns
Measurement and Basic Facts
Place Value, Number Stories, and Basic Facts
Developing Fact Power
Geometry and Attributes
Mental Arithmetic, Money, and Fractions
Place Value and Fractions
Year-End Review and Assessment

Grade 3
Routines, Reviews, and Assessments
Adding and Subtracting Whole Numbers
Linear Measures and Area
Multiplication and Division
Place Value in Whole Numbers and Decimals
Geometry
Multiplication and Division
Fractions
Multiplication and Division
Measurement and Data
Probability; End-of-Year Review

Grade 5
Number Theory
Estimation and Computation
Geometry Explorations and the American Tour
Division
Fractions, Decimals, and Percents
Using Data; Addition and Subtraction of Fractions
Exponents and Negative Numbers
Fractions and Ratios
Coordinates, Area, Volume, and Capacity
Algebra Concepts and Skills
Volume
Probability, Ratios, and Rates

Grade 4
Naming and Constructing Geometric Figures
Using Numbers and Organizing Data
Multiplication and Division: Number Sentences and Algebra
Decimals and Their Uses
Division; Map Reference Frames; Measures of Angles
Fractions and Their Uses; Chances and Probability
Perimeter and Area
Percents
Reflections and Symmetry
Mammals: An Investigation
3-D Shapes, Weight, Volume, and Capacity
Rates

Mathematics Study in Middle School or Junior High School (Grades 6–8 or 7–8)

Students of ages 11–13 may attend elementary schools (K–8), middle schools (commonly grades 6–8) and junior high schools (most commonly grades 7–8). In most school districts, separation of students into different levels begins in these grades. In some schools, this separation means some students follow a course of study that others will follow the next year. In other schools, a more demanding course of study over similar content is given to some students.

Because of the National Assessment of Educational Progress work in mathematics and various international comparative studies about K–12 mathematics, more data exist relating to the curriculum at 8th grade than at any other grade. These studies show that schools have offered courses ranging from remedial courses with a curriculum primarily in whole-number arithmetic to courses in algebra and beyond equivalent to their high school counterparts or a prealgebra course that reviews arithmetic and introduces algebra. The plurality of students take a typical or regular course that contains an extensive review of arithmetic studied in previous years, with some geometry, measurement, probability, and an introduction to algebra—what has been termed a "mile wide, inch deep" curriculum (Schmidt, McKnight, and Raizen 1997). Most remaining students take courses designated prealgebra or first-year algebra (in some form). As table 2 shows, a consistent shift has occurred over the past two decades toward placing a greater percentage of students in more demanding courses and a smaller percentage in remedial courses.

TABLE 2

Percents of 8th Graders Studying Various Types of Mathematics Curricula (Crosswhite et al. 1986; Braswell et al. 2001; NCES 2003c)

Year	Study*	Type of Course				
1981	SIMS		Algebra	Enriched	Typical	Remedial
			13	11	66	10
1986	NAEP		Algebra	Prealgebra	Regular Math	Other
			16	19	61	5
1996	NAEP		Algebra	Prealgebra	Regular Math	Other
			20	36	39	5
2000	NAEP	Geom or Adv. Alg.	Algebra	Prealgebra	Regular Math	Other
		3	25	31	37	5
2003	NAEP	Geom or Adv. Alg.	Algebra	Prealgebra	Regular Math	Other
		5	28	29	33	5

*SIMS = Second International Mathematics Study; NAEP = National Assessment of Educational Progress.

The most widely used middle school materials are from three-year series. These series are relatively new; none existed at the time of the TIMSS curriculum study 10 years ago. A picture of today's mainstream curriculum for grades 6–8 is given by the table of contents of the most widely used series, published by Glencoe/McGraw-Hill. We also show here the Pre-Algebra course from this company because it has more usage than Course 3.

Mathematics: Applications and Connections, Course 1

Problem Solving, Numbers, and Algebra
Statistics: Graphing Data
Adding and Subtracting Decimals
Multiplying and Dividing Decimals
Using Number Patterns, Fractions, and Ratios
Adding and Subtracting Fractions
Multiplying and Dividing Fractions
Exploring Ratio, Proportion, and Percent
Geometry: Investigating Patterns
Geometry: Understanding Area and Volume
Algebra: Investigating Integers
Algebra: Exploring Equations
Using Probability

Mathematics: Applications and Connections, Course 2

Problem Solving, Algebra, and Geometry
Applying Decimals
Statistics: Analyzing Data
Using Number Patterns, Fractions, and Percents
Algebra: Using Integers
Algebra: Exploring Equations and Functions
Applying Fractions
Using Proportional Reasoning
Geometry: Investigating Patterns
Geometry: Exploring Area
Applying Percents
Geometry: Finding Volume and Surface Area
Exploring Discrete Math and Probability

Mathematics: Applications and Connections, Course 3

Problem Solving and Algebra
Algebra: Using Integers
Using Proportion and Percent
Statistics: Analyzing Data
Geometry: Investigating Patterns
Exploring Number Patterns
Algebra: Using Rational Numbers
Applying Proportional Reasoning
Algebra: Exploring Real Numbers
Algebra: Graphing Functions
Geometry: Using Area and Volume
Investigating Discrete Math and Probability
Algebra: Exploring Polynomials

Pre-Algebra

The Tools of Algebra
Integers
Equations
Factors and Fractions
Rational Numbers
Ratio, Proportion, and Percent
Equations and Inequalities
Functions and Graphing
Real Numbers and Right Triangles
Two-Dimensional Figures
Three-Dimensional Figures
More Statistics and Probability
Polynomials and Nonlinear Functions

At present nationally at the 8th-grade level, approximately 15% of students are studying from some form of one of the NSF curricula developed in response to the NCTM Standards (Reys 2003; NCTM 2000). The second–most widely used series is a three-year series especially written for middle schools by the Connected Mathematics Project (CMP) with support from the NSF and published by Prentice-Hall. The units of this three-year program are published separately, and some units do not have to be used in the order shown here. Below are the eight units for each grade from the 2002 edition. As the titles suggest, the CMP curriculum is built around general themes. Instruction emphasizes inquiry and discovery of mathematical ideas through investigation of structurally rich problem situations. Students and teachers are expected to use the information-processing capabilities of calculators and computers—and the fundamental changes that such tools are making in the ways people learn mathematics—and to apply their knowledge to problem-solving tasks.

Grade 6
Prime Time
Data about Us
Shapes and Designs
Bits and Pieces I
Covering and Surrounding
How Likely Is It?
Bits and Pieces II
Ruins of Montarek

Grade 7
Variables and Patterns
Stretching and Shrinking
Comparing and Scaling
Accentuate the Negative
Moving Straight Ahead
Filling and Wrapping
What Do You Expect?
Data around Us

Grade 8
Thinking with Mathematical Models
Looking for Pythagoras
Growing, Growing, Growing
Frogs, Fleas, and Painted Cubes
Say It with Symbols
Kaleidoscopes, Hubcaps, and Mirrors
Samples and Populations

Clever Counting

Like their elementary school series counterparts, virtually all middle school textbooks come with a plethora of related instructional materials for the teacher.

Although in the past eight years only minor changes have occurred in the frequency of calculator use in 8th-grade classes, a significant change has occurred in the percent of 8th-grade students using graphing calculators, as table 3 shows. The frequency of calculator use varies by the type of course, with students in algebra and higher courses more likely to use graphing and other calculators than students in prealgebra or regular courses.

TABLE 3

Percents of 8th-Grade Students in NAEP Studies Reporting Types of Calculator Use in Mathematics Class (Braswell, et al. 2001; NCES 2003b)

Year	**Levels of Usage**		
Frequency of Use	Daily or 1–2 Times/Week	1–2 Times Monthly	Never/Hardly Ever
1996	73	15	13
2000	73	14	14
2003	70	14	15
Use on Tests/Quizzes	Always	Sometimes	Never
2000	23	45	32
2003	22	48	30
Use of Graphing Calculators	Yes	No	
1996	11	89	
2000	18	82	
2003	26	74	

Mathematics Study in High School (Grades 9–12)

At the high school level, the mainstream curriculum is built around a sequence of three full-year courses, algebra-geometry-algebra or algebra-algebra-geometry, beginning in 8th, 9th, or 10th grade, followed by a year of precalculus mathematics, usually giving strong attention to functions and trigonometry. Almost all schools in the country follow this curriculum for their better students and slow it down for lower-performing students.

During the past 25 years, high school graduation requirements and college admission requirements have increased and many four-year colleges and universities now require two years of algebra and a year of geometry for admission. Also, as a result of the impact of technology on everyday lives and worries about lackluster student performance in international studies, the public appears to have become more aware of the role mathematics can play in the future lives and careers of secondary school students. Further, as the data in table 2 indicate, more students are taking courses in algebra before high school. These factors have contributed to a steady and significant increase over time in the percents of students taking higher level mathematics courses in high school (see table 4).

TABLE 4

Highest Mathematics Course Taken by High School Graduates in the United States, from Transcript Studies in Selected Years (NCES 2002c, pp. 157, 215–16)

Course‡	1982	1987	1990	1994	1998
Prealgebra or General Math*	25.0	20.6	17.9	13.1	10.1
Alg. 1 or Geom. or Unified 1, 2	30.6	26.8	25.4	22.4	20.8
Algebra 2 or Unified 3	18.2	23.1	26.2	26.4	27.7
Algebra/trigonometry	15.5	12.9	12.9	16.3	14.4
Precalculus	4.8	9.0	10.4	11.6	15.2
Calculus	5.9	7.6	7.2	10.2	11.8

‡ These studies considered more descriptors of the courses than the most commonly used descriptors named here.
* Entries in this row were found by subtracting the sum of other entries in each column from 100.

The data in table 4 also show that far fewer students are discontinuing their studies of mathematics short of the completion of a course equivalent to Algebra 1. Confirming these data, another source (Blank and Langesen 1999, p. 55) reports that the percent of all high school students enrolled in general or consumer mathematics (courses that are considered more elementary than first-year algebra) decreased nationally from 20% in 1990 to 7% in 1998. Data from the 2000 Conference Board of the Mathematical Sciences (CBMS) study of mathematics programs in two- and four-year colleges, as well as from a study by the National Center of Education Statistics (Parsad and Lewis 2003), also indicate a decrease in the number of students enrolling in remedial courses in mathematics covering high school content prior to precalculus. Concurrent with these changes has been an increase in the numbers of students taking advanced placement courses in mathematics (Lutzer, Maxwell, and Rodi 2002; Gollub, et al. 2002). (See pp. 29–31.)

Through the era from 1965 to 1985, perhaps a majority of books in use in all high school grades were those of the Dolciani series published by Houghton Mifflin. Now no such domination exists. At the high school level the mainstream books of Glencoe/McGraw-Hill, Prentice Hall, McDougal Littel, and Holt, Rinehart and Winston; the conservative books of Saxon; and the progressive and technology oriented books of Key Curriculum Press have significant sales. We include here the chapter titles from the Glencoe series, the most-used high school series.

Algebra 1
The Language of Algebra
Real Numbers
Solving Linear Equations
Graphing Relations and Functions
Analyzing Linear Equations
Solving Linear Inequalities
Solving Systems of Linear Equations and Inequalities
Polynomials
Factoring
Quadratics and Exponential Functions
Radical Expressions and Triangles
Rational Expressions and Equations
Statistics
Probability

Algebra 2
Solving Equations and Inequalities
Linear Relations and Functions
Systems of Linear Equations and Inequalities

Matrices
Polynomials
Quadratic Functions and Inequalities
Polynomial Functions
Conic Sections
Rational Expressions and Equations
Exponential and Logarithmic Functions
Sequences and Series
Probability and Statistics
Trigonometric Functions
Trigonometric Graphs and Identities

Geometry
Discovering Points, Lines, Planes, and Angles
Connecting Reasoning and Proof
Using Perpendicular and Parallel Lines
Identifying Congruent Triangles
Applying Congruent Triangles
Exploring Quadrilaterals
Connecting Proportion and Similarity
Applying Right Triangles and Trigonometry
Analyzing Circles
Exploring Polygons and Area
Investigating Surface Area and Volume
Continuing Coordinate Geometry
Investigating Loci and Coordinate Transformations

Advanced Mathematical Concepts
Linear Relations and Functions
Systems of Linear Equations and Inequalities
The Nature of Graphs
Polynomial and Rational Functions
The Trigonometric Functions
Graphs of Trigonometric Functions
Trigonometric Identities and Equations
Vectors and Parametric Equations
Polar Coordinates and Complex Numbers
Conics
Exponential and Logarithmic Functions
Sequences and Series
Iteration and Fractals
Combinatorics and Probability
Statistics and Data Analysis
Introduction to Calculus

None of the high school series developed with support from NSF has the breadth of use that is enjoyed by their elementary and middle school counterparts. Together with the NCTM *Standards*, they have influenced mainstream textbooks to include more applications and more work with technology. At the same time, pressure from colleges has influenced these textbooks to maintain if not increase skill work with algebra and functions.

Mathematics Study at the Postsecondary Level

At the postsecondary level, students have a wide variety of options for studying mathematics. Coursework is available through community colleges, universities, and a variety of vocational schools, work-based educational programs, and commercial outlets. The data collected every five years by the Conference Board of the Mathematical Sciences provide the best trend data for curricular programs and enrollments in two- and four-year colleges.

Mathematics courses at these institutions range from arithmetic, particularly widespread at two-year colleges, through advanced courses available only at four-year institutions. Tables 5 and 6 demonstrate this wide range and the change in enrollments over time, using data collected every five years by the Conference Board of the Mathematical Sciences. In these tables, remedial courses include arithmetic and elementary and intermediate algebra. Precalculus courses include college algebra and trigonometry as well as finite mathematics, non-calculus-based business mathematics, mathematics for prospective elementary school teachers, and other courses for non–science majors. These tables do not include mathematics courses taught outside mathematics departments. Enrollments are for the fall quarter or semester of the year (Lutzer, Maxwell, and Rodi 2002).

TABLE 5
Estimated Enrollment (in Thousands) in Mathematics Courses in Two-Year Colleges

	Year					
Course	1970	1980	1985	1990	1995	2000
Remedial	191	441	482	724	800	763
Precalculus	134	180	188	245	295	274
Calculus	59	86	97	128	129	106
Statistics	16	28	36	54	72	74
Other	171	218	133	144	160	130
Total	555	953	936	1295	1456	1347

Table 5 shows that since 1985, more than half of the mathematics enrollments in two-year colleges have been at the remedial level. The overall increase in number of mathematics courses in two-year colleges is partially a function of the overall increase in enrollments at these institutions.

TABLE 6
Estimated Enrollment (in Thousands) in Undergraduate Mathematics Courses in Four-Year Colleges

	Year					
Course	1970	1980	1985	1990	1995	2000
Remedial	101	242	251	261	222	219
Precalculus	538	602	593	592	613	723
Calculus	414	590	637	647	538	570
Statistics	na	na	na	125	143	171
Advanced	135	91	138	119	96	102
Total	1188	1525	1619	1744	1612	1785

When the enrollments in mathematics courses in two- and four-year college mathematics departments are added, the sums show a consistent increase in total enrollment over time. However, trends for the various types of courses are mixed. One might expect that the remedial and precalculus enrollments are mainly from students who expect to major in fields other than science and engineering and do not realize how much mathematics is needed in those fields. However, in 2000, 20.8% [1995, 22%] of all first-year college students who intended to major in science or engineering reported a need for remedial work in mathematics (National Science Board 2002). This self-reporting of "remedial work" by students does not necessarily match the coursework titled "remedial" in tables 5 and 6.

Table 7 contains data on enrollments in the first two semesters of calculus and the two beginning non-calculus-based statistics courses from 1990 to 2000.

An examination in table 7 of the enrollments for mainstream calculus (that for mathematics, engineering, and physical science students) shows a 4% decline from 1990 to 2000. Enrollments in nonmainstream calculus showed a more precipitous drop over the same time period, with declines of 32% and 39% for the first two courses. The enrollment in first-year, non-calculus-based statistics courses was quite different. Here, nationally, a net gain of 42% was realized over the ten-year period for the elementary statistics and the probability and statistics enrollments combined. The data indicate that a significant change was made in the offerings within two- and four-year institutions and in both mathematics and statistics departments to an introductory course that focuses on statistics from the combined treatment of probability and statistics as the introductory course.

TABLE 7

Estimated Enrollments (in Thousands) in Calculus and Statistics Courses by Level and by Department Teaching Course (Source: Lutzer, Maxwell, and Rodi 2002)

	Enrollments by Years		
CALCULUS	1990	1995	2000
Four-Year Institutions			
Mainstream Calculus I	201	192	192
Mainstream Calculus II	88	83	87
Two-Year Institutions			
Mainstream Calculus I	53	58	53
Mainstream Calculus II	23	23	20
Total Mainstream Calculus I	**254**	**250**	**245**
Total Mainstream Calculus II	**111**	**106**	**107**
Four-Year Institutions			
Non-mainstream Calculus I	148	98	105
Non-mainstream Calculus II	15	14	10
Two-Year Institutions			
Non-mainstream Calculus I	31	26	16
Non-mainstream Calculus II	3	1	1
Total Non-mainstream Calculus I	**179**	**124**	**121**
Total Non-mainstream Calculus II	**18**	**15**	**11**
STATISTICS			
Four-Year Institutions			
Mathematics Departments			
Elementary Statistics	61	97	114
Probability and Statistics	25	18	13
Departments of Statistics			
Elementary Statistics	22	35	40
Probability and Statistics	10	8	4
Two-Year Institutions			
Elementary Statistics	47	69	71
Probability and Statistics	7	3	3
Total Elementary Statistics	**130**	**201**	**225**
Total Probability and Statistics	**42**	**29**	**20**
Total Calculus and Statistics I	**563**	**575**	**591**

When one combines the enrollments of mainstream and nonmainstream first-year calculus with those of the two statistics courses over the ten-year period, one finds an overall 5% growth in first-course enrollment in the decade. This growth was characterized by a shifting of students from the calculus sequence to offerings in statistics. This shift was most prominent in the shift of students from nonmainstream calculus courses to courses in probability and statistics (Lutzer, Maxwell, and Rodi 2002).

Total enrollments in advanced mathematics courses declined from 1985 to 1995 and then remained flat, at about half the 1970 enrollment, from 1995 to 2000, as seen in table 8. Although enrollments in advanced geometry courses remained steady from 1995 to 2000, the percent of mathematics departments offering these courses declined from 69% in 1995 to 56% in 2000.

TABLE 8
Fall Enrollments (in Thousands) in Selected Undergraduate Courses, 1970–2000 (CBMS 1995; CBMS 2000)

	Year					
Course	1970	1980	198:	0	1995	2000
Calculus level	413	590	637	647	539	570
Discrete mathematics (intro)	na	na	14	17	16	20
Modern algebra I, II	23	10	13	12	13	11
Discrete structures	na	na	7	3	3	5
Geometry	13	4	7	8	6	6
Total advanced mathematics		**91**	**138**	**120**	**96**	**102**
Total all mathematics		**1525**	**1619**	**1621**	**1471**	**1614**

According to CBMS data, the total enrollment in statistics courses in four-year colleges and universities increased from 169 000 in 1990 to 208 000 in 1995, with about 25% of these students taught in statistics departments separate from mathematics departments. These numbers do not include statistics courses taught outside statistics or mathematics departments or in two-year colleges. Including two-year colleges, enrollments in elementary statistics courses increased from 236 000 in 1995 to 274 000 in 2000 (Moore 2001).

Bachelor's Degrees Awarded

Table 9 shows the number of bachelor's degrees in the mathematical sciences awarded from mathematics or statistics departments in five-year increments since 1975. An 8% decline occurred in both the number of bachelor's degrees in mathematics and in the total number of degrees awarded in the mathematical sciences between 1990 and 2000.

TABLE 9
Bachelor's Degrees Awarded in the Mathematical Sciences, 1975 to 2000 (Loftsgaarden, Rung, and Watkins 1997, p. 9; Lutzer, Maxwell, and Rodi 2002)

	Year					
Area	1975	1980	1985	1990	1995	2000
Mathematics (including actuarial science, operations research, & joint degree)	18833	11687	16123	14852	13792	13664
Statistics	570	467	538	618	1031	644
Computer science	n.a.	n.a.	8691	5075	2741	3315
Mathematics education	4778	1752	2567	3116	4829	4991
Totals	**24181**	**13906**	**27928**	**24455**	**22895**	**22614**

Over the past half-century a significant increase has occurred in the number of women in the mathematical sciences. In 1950, less than 25% of the bachelor's degrees in the mathematical sciences went to women. By 1970, the numbers had increased to 37% (Madison and Hart 1990). The percents of degrees granted to women in 1995 were 44% in mathematics (including actuarial mathematics and operations research), 49% in both mathematics education and statistics, and 22% in computer science. In 2000, these numbers were 42% in mathematics, 49% in statistics, 59% in mathematics education, and 24% in computer science. These increases gave an overall 43% of degrees granted to women for the mathematical sciences combined (Lutzer, Maxwell, and Rodi 2002).

Data from *The American Freshman*, taken at five-year intervals, indicate the percent of entering college freshmen intending to major in mathematics or statistics: 4.5% (1966), 1.0% (1976), 0.7% (1986), 0.5% (1996), and 0.7%

(2001) (Sax et al. 2001; Astin et al. 1997). On the one hand, the percent for 2001 is significantly lower than that in the 1960s, even when computer science majors are included. On the other hand, the mathematical requirements of majors outside the physical sciences have increased significantly in the same time period. Although some of the mathematics needed to fulfill these requirements is taught outside departments of mathematics and statistics, the increases in these requirements are a major factor in the overall increase in number of courses taken in departments of mathematics and statistics.

Instructional Practices

NAEP studies indicate that 79% of 4th-grade students in 2003 had access to school-owned calculators in their mathematics class, a significant increase over the 59% in 1992. Varied studies in the 1990s indicated that more than 85% of 8th-grade students had access to calculators in their mathematics classes. When asked about use of calculators in various settings, 9% of 4th-grade teachers in 2003 and 65% of 8th-grade teachers in 2000 reported allowing their

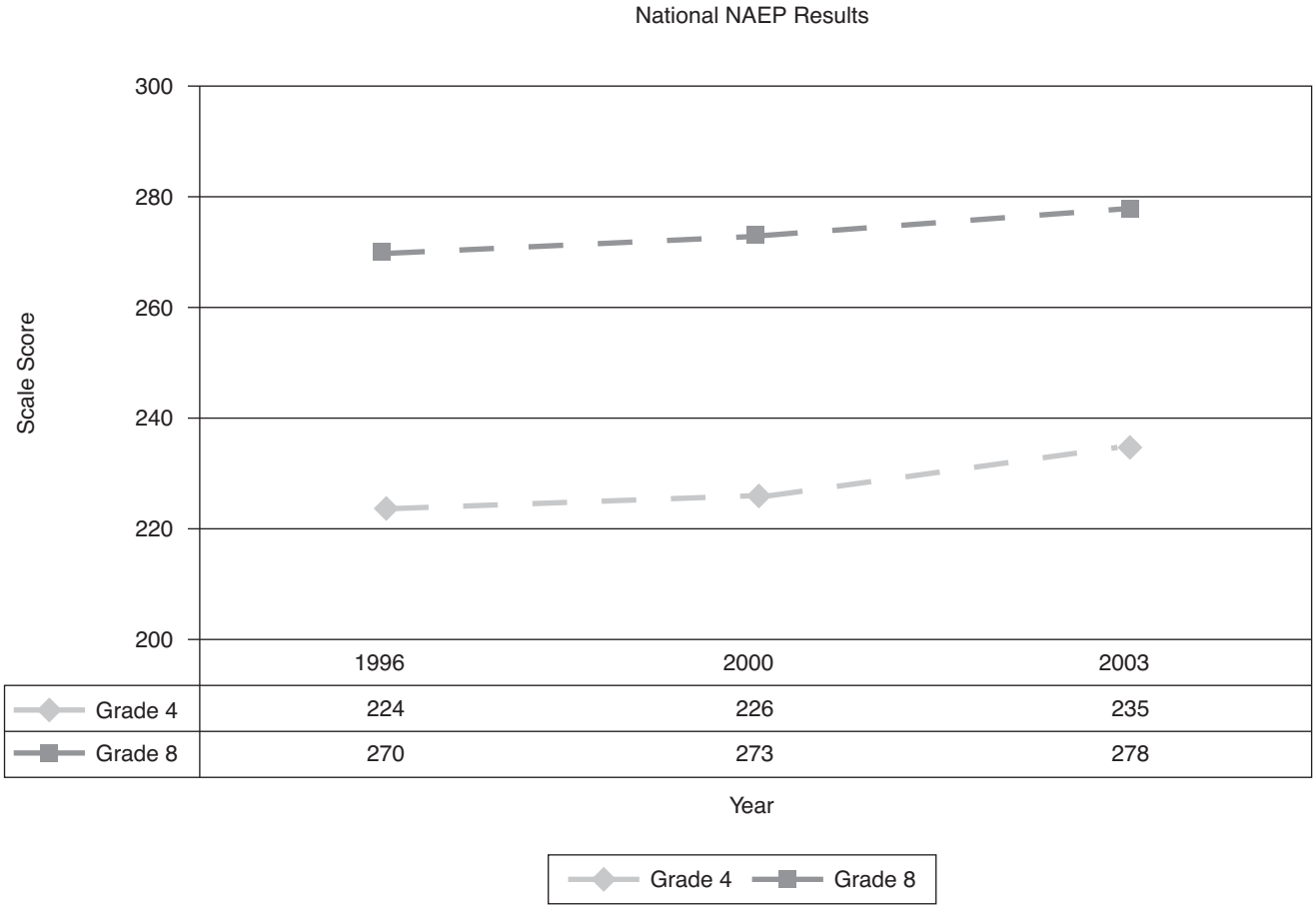


Fig. 2. National NAEP overall scale results for students with and without accommodations (NCES 2003c)

students to use calculators on examinations. Seventy percent of 4th-grade teachers (NCES 2003b) reported instructing their students in the use of calculators, as contrasted with the 77% reporting doing the same in 1996. Eighty percent of 8th-grade teachers reported giving such instruction in 2000, down slightly from the 83% reporting so in 1996. Possibly fewer students are using calculators, or perhaps teachers think that their students need no such instruction.

The video study that was part of the overall Third International Study of Mathematics and Science (TIMSS) involved the videotaping of 8th-grade classrooms in Germany, Japan, and the United States. The analysis of the lessons from the video study presented on these tapes reflected major differences in teacher-student interactions and in the conduct of instruction in mathematics classrooms across the three countries. U.S. researchers concluded that students in U.S. classrooms experienced little or no reasoning as part of their mathematics study (Peak 1996; Manaster 1998; Stigler and Hiebert 1999). Portions of the video study were repeated in 1999, and the list of countries was expanded to include Australia, the Czech Republic, Hong Kong SAR, Japan (reanalysis of original tapes), the Netherlands, Switzerland, and the United States. This more thorough study's findings indicated that U.S. classes at the 8th-grade level tend to introduce less new content per lesson than classes in Hong Kong and Switzerland, and tend to review more content than classes in Japan and Hong Kong. U.S. classrooms tended to have less complex problems—both in terms of the number of steps and the number of concepts involved. The problems in U.S. classrooms also tended to be less mathematically related to one another than those presented in many other countries. However, those analyzing the data could not identify any one country's instructional approach as an international model to emulate (Hiebert, et al. 2003).

The most recent NAEP data on 12th-grade calculator use is from 2000, when 69% of students reported using a calculator every day and another 14% reported using a calculator once or twice a week. More than 90% of students have access to a calculator for their mathematics school work, a level that has remained constant since 1992. At this grade level, the data do not indicate what percent of these calculators were school owned as opposed to student owned.

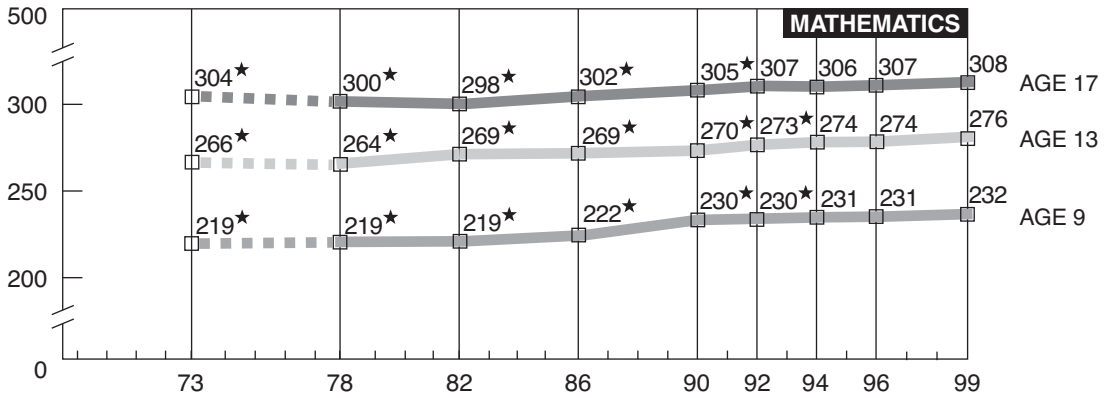
Table 10 reports the use of various instructional methods as gleaned from CBMS studies. Some questions were not asked both of four-year and of two-year colleges. Dramatic increases have occurred in all kinds of institutions in the use of graphing calculators, required computer assignments, group projects, and writing assignments since 1990. The *italics*, plain, and **bold** type indicate the data, when available, from the CBMS studies in *1990*, 1995, and **2000**, respectively.

In the 1995 CBMS assessment, 29% of mainstream and 10% of nonmainstream calculus sections in four-year colleges used a "reform" textbook. In 2000, the greater use of computer assignments, the prevalence of graphing calculators, and the more frequent appearance of a writing component suggest that aspects of reform calculus tended to blend into other sections of both types of calculus courses.

TABLE 10

Percent of First-Semester Calculus Sections Using Various Instructional Methods (Loftsgaarden, Rung, and Watkins 1997; Lutzer, Maxwell, and Rodi 2002)

Method	**Two-Year College**		**Four-Year College or University**	
	Mainstream	Nonmainstream	Mainstream	Nonmainstream
Graphing calculators	65 **78**	44 **42**	*3* 37 **51**	26 **45**
Computer assignments	23 **35**	8 **15**	*9* 18 **31**	6 **13**
Meet in computer lab once/week	15	5		
Use computer-aided instruction	3	<1		
Assign group projects	22 **27**	20 **20**	*10* 22 **19**	7 **9**
Have writing component	20 **31**	17 **20**	*9* 22 **27**	7 **14**
Taught via lectures	82	88		
Taught by television	<1	<1		
Use "reform" text			29	10



AGE 17	304	300	299	302	302	307	306	307	308
AGE 13	266	264	269	269	269	273	274	274	276
AGE 9	219	219	219	222	222	230	231	231	232

Fig. 3. Average mathematics scale scores for the nation, 1973 to 1999 (Campbell, Hombo, and Mazzeo 2000)

A facet of university instruction that has shown a marked increase in the past decade has been the use of distance education in instructing those who travel to campus or need more flexible learning environments. In 1997–98 about one-third of the nation's postsecondary institutions were offering distance-education courses, the majority of them through two-way interactive video formats. In mathematics these courses were generally remedial or first-year entry-level survey courses, accounting for approximately 3% of the enrollments in distance education nationally. About 8% of the institutions offered college-level degree or certificate programs to be completed totally through distance education. By the 2000–01 year, 56% of two- and four-year institutions had distance-education programs, and the number of degree or certificate programs that were to be completed by distance education alone were found on 19% of campuses (Lewis, et al. 1999; Waits and Lewis 2003). The 2000 CBMS study included a special look at distance education in the undergraduate years. The findings indicated a small increase in the number of sections but reported rather small enrollments in many instances. The courses most often taught by distance education at two-year campuses were at the precalculus level or below, for example, elementary statistics or mathematics for liberal arts. At the four-year level, the only courses with more than 3% of the sections taught by distance education were trigonometry by mathematics departments, and a variety of computer science courses and statistical literacy courses by statistics departments (Lutzer, Maxwell, and Rodi 2002).

The National Assessment of Educational Progress (NAEP)

The U.S. government, through its Department of Education's National Center for Education Statistics, conducts a large-scale assessment known as the National Assessment of Educational Progress (NAEP). This program, which periodically assesses the knowledge of and opportunity to learn mathematics and other subjects with random samples of youth in the United States, is one of the best barometers of the direction of U.S. mathematics education from assessment period to assessment period.

The most recent NAEP assessment in mathematics was conducted in 2003 and collected data from a national sample of 4th and 8th graders. With the 2003 assessment, the NAEP program shifted its data trend line for national data from a sample that did not allow the use of testing accommodations (such as more time if needed) to one that includes data from students who complete the examination with any accommodations as provided by law. Similar data, using samples working with accommodations, were collected in the 1996 and 2000 administrations of NAEP, and together they provide a basis for some trend analysis (NCES, 2003c), as shown in figure 2.

Student performance on the NAEP mathematics assessment is reported on a 0–500 scale through the use of common items from grades 4 and 8 and from grades 8 and 12 to enable comparisons of different grade levels. Given the approximate 50-point gaps between grade 4 and grade 8 performances, one might consider a difference of 12 to 15 points as being equivalent to one grade level of performance. If this informal metric is used, the increases observed at grade 4 and grade 8 between 1996 and 2003 are both over one-half a grade-placement level. Statistically, the 2003 performances at both grade levels were significantly higher than those observed in either 1996 or 2000. This finding agrees with data from students without accommodations at each grade level on the national NAEP scale in 1996, which showed about a one-grade-level increase from both 1990 and 1992. Student performance at grade 12 will be added to that for grades 4 and 8 with the next NAEP mathematics assessment in 2005.

The framework for the 2003 assessment was the same as that for the 1996 and 2000 assessments. That framework includes small but significant changes from previous assessment frameworks. Most notable are changes in the emphasis given to the role of algebra and functions at grades 4 and 8, whereas the changes at grade 12 reflect the increased emphasis on data and chance in the secondary school curriculum. More attention was focused on students' proficiency in context-based problem solving, in constructing their own responses, and in knowing when and how to use technology in solving problems on assessments. In addition, the previous uses of conceptual, procedural, and problem-solving labels to assign a level of cognitive demands of an item was discontinued and replaced by a factor that focuses on the demand, in terms of complexity, that an item places on students: low, medium, and high (NAGB 2002). A new framework, reflecting changes in school mathematics that have occurred since the mid-1990s, is being used in the development of the next NAEP assessment in 2005. Among the changes is the appearance of questions at grade 12 involving content of high school geometry and second-year algebra courses, reflecting the increases in enrollment in those courses.

The NAEP long-term-trend study

Although national NAEP assessments and their frameworks are designed to change as the curriculum and school programs change, the NAEP also administers a long-term-trend assessment program with nationally representative samples of students using the instruments developed and used in the first NAEP assessment in 1973. These instruments have remained unchanged since that time and have been administered under exactly the same conditions across the intervening 30 years. Thus, data from the NAEP long-term assessment provide a basis for interpreting changes in mathematics curriculum and teaching against a constant baseline consisting of the 1973 NAEP data.

The NAEP trend assessment is far more focused on students' knowing definitions, basic facts, and algorithms than it is on students' knowing and being able to apply problem-solving strategies, show their reasoning, or communicate about mathematics. On this assessment, students are not allowed to use calculators. As such, the NAEP trend assessment provides valuable information on whether student performance on items considered important in 1973 (such as paper-and-pencil computation skills, direct application of measurement formulas in geometric settings, and the use of mathematics in daily-living skills involving time and money) has changed over time.

The line graphs in figure 3 depict the overall trend in student performance on the NAEP scale for each of the

three age groups sampled. The 1999 level of performance for the 9- and 13-year-old groups is statistically higher than that of the same age groups at every testing period from 1992 or earlier. The 1999 performance level for 17-year-olds was significantly higher than that observed for every t[illegible] year from 1990 and before (Campbell, Hombo, and Mazzeo 2000). These findings indicate that, on average, students today have a better command of items considered important in 1973 than their historical peers across the past 30 years.

Longitudinal trends in performance at NAEP benchmark levels

To help in understanding trends in student knowledge and skills, levels of performance were established by anchoring five points on the mathematics scale: 150, 200, 250, 300, and 350. These five levels are accompanied by descriptions that outline the concepts, procedures, and processes associated with performance at each level. They are briefly described as shown in the left column of table 11, which gives the results from the 1978 to 1999 assessments with respect to these levels (Campbell, Hombo, and Mazzeo 2000).

TABLE 11
Trends in Percent of Students at or above Five Mathematics Performance Levels, 1978 to 1999

Performance Levels	Age	1978	1986	1990	1996	1999
Level 350	9	0	0	0	0	0
Multistep problem solving	13	1	0	0	1	1
and algebra	17	7	7	7	7	7
Level 300	9	1	1	1	2	2
Moderately complex	13	18	16	17	21	23
procedures and reasoning	17	52	52	56	60	79
Level 250	9	20	21	28	30	31
Numerical operations and	13	65	73	75	79	79
beginning problem solving	17	92	96	96	97	97
Level 200	9	70	74	82	82	83
Beginning skills and	13	95	99	99	99	99
understandings	17	100	100	100	100	100
Level 150	9	97	98	99	99	99
Simple arithmetic facts	13	100	100	100	100	100
	17	100	100	100	100	100

An analysis of the data in table 11 shows that the percent of students reaching various benchmark levels from 150 through 300 has increased significantly across the period from 1978 through 1999. These analysis indicate that much of the increase in students' long-term-trend results comes from growth in such mathematical topics as basic number facts and operations and in reading and interpreting graphs, tables, and charts. However, few students at any age achieved the 350 benchmark, a level that indicates substantial ability in elementary algebra and geometry and in multistep problem solving.

An analysis was made of the performance of students in the bottom quartile, middle half, and upper quartile of each of the age groups over the same periods of time. The resulting growth patterns for each of the three quartile groups paralleled the increases shown in table 10. This outcome indicates that the increases were not just an artifact of the performance of the most able students, but rather, an increase indicative of change in the students in each of the three quartile-defined groups (Campbell, Hombo, and Mazzeo 2000).

NAEP results for various subgroups

Considerable research has been done in recent years on the differences in the performance of students of different racial, ethnic and gender groups, a matter of great concern (Willingham and Cole 1997; Oakes 1990; Oakes and Wells 1998; Pellagrino et al. 1999). Differences between white and black student performance on the long-term-trend NAEP have varied over the assessment's history, but the differences were significantly smaller in 1999 than

in 1973 at all three age levels. The linear trend over the same time, across all age levels, also indicated a significant decrease in differential performance. The same pattern was true for the difference between white and Hispanic student performance at ages 9 and 13. However, for 17-year-olds, no significant change was observed in the overall difference or trend in the difference for the period 1973 to 1999 (Campbell, Hombo, and Mazzeo 2000).

Some studies have suggested that these differences are the result of opportunities afforded the students in school and in their homes and communities; other studies point to the atmosphere of encouragement about education and its roles in students' lives (Mullis, Jenkins, and Johnson 1994; Oakes 1990; Eakin and Backler 1993). Narrowing these gaps remains a central challenge for mathematics education in the United States.

On the 2003 national NAEP, male students outscored female students at grades 4 and 8 (NCES 2003b). The linear trend for the difference of male minus female scores across the 26-year history of the NAEP long-term-trend assessment shows a significant narrowing of the performance gap between genders at age 17 and a shift from female leads to male leads at the 9- and 13-year-old levels. On the long-term-trend NAEP, the gap between male and female performance was not statistically significant in 1999 at any age level (Campbell, Hombo, and Mazzeo 2000).

NAEP results for various states

In the late 1980s, the United States Congress passed legislation directing NAEP to allow the possibility of state-by-state analysis of performance. The first state assessment took place in 1990 in grade 8 in 32 jurisdictions. The NAEP 2003 mathematics assessment was conducted state-by-state in grades 4 and 8 in all 50 states, the District of Columbia, the Department of Defense Domestic Dependent Elementary and Secondary Schools (DDESS), and the Department of Defense Dependent Schools (DoDDS) located on U.S. bases in foreign countries. In light of the mandates of the No Child Left Behind (NCLB) legislation, the state-by-state NAEP assessments will take on additional importance, because the degree to which NAEP data correlates with the states' own assessments will serve as a criterion in judging the degree to which the individual states are meeting the federal mandates for educational reform.

Results, shown in table 12, demonstrate major differences among the states. Taking a difference of 10 points as a rough indication of a year (the difference between the national means for grades 4 and 8 being 42 points using the state-by-state NAEP samples), some jurisdictions differ by two or more years at grade 4 and three or more years at grade 8.

TABLE 12
Selected Statistics for Public Schools on the 2003 NAEP

	Grade 4		Grade 8	
Jurisdiction	**Mean Scale Score**	**% Proficient or Above**	**Mean Scale Score**	**% Proficient or Above**
Alabama	223	19	262	16
Alaska	233	30	279	30
Arizona	229	25	271	21
Arkansas	229	26	266	19
California	227	25	267	22
Colorado	235	34	283	34
Connecticut	241	41	284	35
Delaware	236	31	277	26
District of Columbia	187	7	243	6
DDESS	237	30	282	27

TABLE 12 *(continued)*

	Grade 4		Grade 8	
Jurisdiction	**Mean Scale Score**	**% Proficient or Above**	**Mean Scale Score**	**% Proficient or Above**

DoDDS	237	31	286	35
Florida	234	31	271	23
Georgia	230	27	270	22
Hawaii	227	23	266	17
Idaho	235	31	280	28
Illinois	233	32	277	29
Indiana	238	35	281	31
Iowa	238	36	284	33
Kansas	242	41	284	34
Kentucky	229	22	274	24
Louisiana	226	21	266	17
Maine	238	34	282	29
Maryland	233	31	278	30
Massachusetts	242	41	287	38
Michigan	236	34	276	28
Minnesota	242	42	291	44
Mississippi	223	17	261	12
Missouri	235	30	279	28
Montana	236	31	286	35
Nebraska	236	34	282	32
Nevada	228	23	268	20
New Hampshire	243	43	286	35
New Jersey	239	39	281	33
New Mexico	223	17	263	15
New York	236	33	280	32
North Carolina	242	41	281	32
North Dakota	238	34	287	36
Ohio	238	36	282	30
Oklahoma	229	23	272	20
Oregon	236	33	281	32
Pennsylvania	236	36	279	30
Rhode Island	230	28	272	24
South Carolina	236	32	277	26
South Dakota	237	34	285	35
Tennessee	228	24	268	21
Texas	237	33	277	25
Utah	235	31	281	31
Vermont	242	42	286	35
Virginia	239	36	282	31
Washington	238	36	281	32
West Virginia	231	24	271	20
Wisconsin	237	35	284	35
Wyoming	241	39	284	32
Nation	**234**	**31**	**276**	**27**

College Entrance Examinations

Typically, a student in the United States applies for college in the 12th grade (the last year of high school). The selectivity of colleges in the United States ranges from community colleges and others that require no more than a high school diploma or its equivalent for entrance, to select colleges at which 10% or fewer of applicants are accepted. Occasionally the selectivity of an institution varies internally among the majors it offers.

Because college entrance examination scores provide the only easily comparable measure for students coming from different high schools and different areas of the country, they are often given great importance by colleges. As a result, most college-intending students in the United States take a college entrance examination during their junior or senior year.

Two independent test batteries exist. The SAT tests, administered by the Educational Testing Service under the auspices of The College Board, are more commonly taken in the east, south, and west. The ACT tests, administered by the American College Testing Service, are most commonly taken in the midwestern states. The percent of students taking the SAT-I (the test taken by all students who take the SAT) remained almost constant through the 1990s but increased from 43% to 48% of high school graduates between 1999 and 2003. The percent of high school graduates taking the ACT increased from 38% in 1999 to 40% in 2003. This increase was mainly in the number of non-college-bound students taking the test; that number increased 15% between 1999 and 2003 (on top of an earlier increase of 20% between 1990 and 1999). Some students take both tests.

The SAT-I test yields a verbal and a mathematics normed score on a 200–800 scale. In 1995, this score was renormed so that 500 would be the mean, and scores for 1989–95 were recomputed after individual scores were converted from an old scale to the new scale. The ACT test has English, mathematics, reading, and science sections, each scored on a 1–36 scale, and a summary score also on that scale. On both tests the mean mathematics score in the past decade has shown substantial improvement (see table 13).

TABLE 13
Mean Scores of High School Seniors on SAT-I and ACT Mathematics and English Tests in the 1990s (The World Almanac and Book of Facts 2004, p. 295)

	Test			
Year	**SAT-I Math**	**SAT-I Verbal**	**ACT-Math**	**ACT-English**
1990	501	500	19.9	20.5
1991	500	499	20.0	20.3
1992	501	500	20.0	20.2
1993	503	500	20.1	20.3
1994	504	499	20.2	20.3
1995	506	504	20.2	20.2
1996	508	505	20.2	20.3
1997	511	505	20.6	20.3
1998	512	505	20.8	20.4
1999	511	505	20.7	20.5
2000	514	505	20.7	20.5
2001	514	506	20.7	20.5
2002	516	504	20.6	20.2
2003	519	507	20.6	20.3

The general public has come to view these mean scores as a barometer of how well the education system is performing as a whole, despite their not being designed for that purpose and despite their obvious shortcomings when used for this purpose.

Advanced Placement Programs in Calculus

A high school student who completes the standard mathematics content of grades 9–12 before entering 12th grade, and who desires to continue the study of mathematics in 12th grade, has three potential paths to follow. If the school is very small and no college is located nearby, the student may be able to take an individualized course. If a college or university is located nearby, the student may be able to take a college course there, earning credit both for high school graduation and college credit if the student passes the course. If enough students in the school are in this situation, then the school may wish to offer Advanced Placement courses.

Under the auspices of the College Board, in 1955 the Educational Testing Service (ETS) created the Advanced Placement Program to enable students to take college-level work before they graduate from high school (ETS Web site). High schools participating in this program offer courses whose syllabi are designed to be in agreement with introductory college courses. At present, 34 courses are offered in 21 different subject areas, and more than 14 000 high schools worldwide participate. In 2003, 1 017 396 students took 1 737 231 examinations. Both numbers were the highest on record and approximately three times the numbers of students participating and examinations taken in 1991 (AP Central Web site).

Most advanced placement (AP) courses are a year in length. However, some high schools offer "block schedules" with longer class periods each day, and in these schedules AP courses are compressed into one-semester configurations. In May of each year, ETS administers nationwide examinations for each of the courses. Colleges have the option of offering college credit, placement into more advanced classes (with or without credit), or ignoring the scores students receive. Most colleges take scores on Advanced Placement tests into account when placing students into courses.

When AP courses are taken in the 11th grade or earlier, they can be considered along with a student's application to a college and may factor into admissions decisions. Although scores on AP tests in 12th grade are not available to colleges before admissions decisions are made, enrollment in AP courses itself tends to signify a more serious student and a more scholastically oriented high school, and can increase a student's chances of admission to some colleges.

Scores on AP tests range from 1 to 5. The American Council on Education recommends that colleges give credit to students who score 3 or higher, but some colleges have higher cutoffs, and some give credit for part of the year-long course, depending on the score (ETS Web site).

Two AP examinations are offered in calculus: Calculus AB (since 1956) and Calculus BC (since 1969). Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Each test consists of two multiple-choice parts for 105 minutes and a free-response section for 90 minutes. One of the multiple-choice parts does not allow the use of a calculator; the other part and the free-response section require a graphing calculator.

The 166 821 students who took the AB test and the 45 973 students who took the BC test in 2003 together equaled about 7% of graduating high school seniors. These numbers are the highest on record for these examinations. In 2003, 66% of students taking the Calculus AB test scored 3 or higher, and 81% of students taking the Calculus BC test scored 3 or higher (Jones 2004, AP Central Web site).

The syllabi for Calculus AB and BC are developed by a national committee of the College Board and modified periodically, and they might be said to represent a consensus regarding what a good calculus course should cover. Both are primarily concerned with developing student understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed geometrically, numerically, analytically, and verbally. The connections among these representations also are important (College Board Web site). All Calculus AB topics are covered in Calculus BC; those covered only in BC are identified here with an asterisk (*).

I. Functions, graphs, and limits
   - Analysis of graphs
   - Limits of functions (including one-sided limits)

Asymptotic and unbounded behavior
Continuity as a property of functions
*Parametric, polar, and vector functions

II. Derivatives
Concept of the derivative
Derivative at a point
Derivative as a function
Second derivatives
Applications of derivatives
Computation of derivatives

III. Integrals
Interpretations and properties of definite integrals
*Applications of integrals
Fundamental theorem of calculus
Techniques of antidifferentiation
Applications of antidifferentiation
Numerical approximations to definite integrals

*IV. Polynomial approximations and series
*Concept of series
*Series of constants
*Taylor series

Advanced Placement Program in Statistics

A single AP examination is offered in statistics. It is meant to be equivalent to a one-semester, introductory, non-calculus-based college course in statistics. Graphing calculators with statistical capabilities are required on the examination, but the College Board emphasizes that they are not equivalent to computers in the teaching of statistics. In 2003, 58 203 students took this examination, over twice the number who took it in 1999; 62% of these students scored 3 or higher (AP Central Web site).

Here is an outline of the areas covered by the AP Statistics examination.

I. Exploring data: observing patterns and departures from patterns
Interpreting graphical displays of distributions of univariate data (dot plot, stem plot, histogram, cumulative frequency plot)
Summarizing distributions of univariate data
Comparing distributions of univariate data (dot plots, back-to-back stem plots, parallel box plots)
Exploring bivariate data
Exploring categorical data: frequency tables

II. Planning a study: deciding what and how to measure
Overview of methods of data collection
Planning and conducting surveys
Planning and conducting experiments
Generalizability of results from observational studies, experimental studies, and surveys

III. Anticipating patterns: producing models using probability theory and simulation
   - Probability as relative frequency
   - Combining independent random variables
   - The normal distribution
   - Sampling distributions

IV. Statistical inference: confirming models
   - Confidence intervals
   - Tests of significance
   - Special case of normally distributed data

Advanced Placement Programs in Computer Science

Two AP exams are offered in computer science: Computer Science A and Computer Science AB. Computer Science A is meant to be the equivalent of a first-semester college course in computer science. Its content emphasizes object-oriented programming methodology with a concentration on problem solving and algorithm development. Computer Science AB includes all the topics of Computer Science A as well as a more formal and in-depth study of algorithms, data structures, and abstraction, and is meant to cover a second-semester course in computer science. Students need access to a computer system that represents relatively recent technology. Beginning in 2004, the examination will use the JAVA programming language. In 2003, 14 674 students took the A examination and 7071 took the AB examination. Of those, 61% scored 3 or higher on the A examination, and 76% scored 3 or higher on the AB examination (AP Central Web site).

Here is an outline of the areas to be covered in the computer science examinations starting in 2004. All Computer Science A topics are covered in Computer Science AB; those covered only in AB are identified here with an asterisk (*).

I. Object-oriented program design
   - Program design
   - Class design

II. Program implementation
   - Implementation techniques
   - Programming constructs
   - Java library classes

III. Program analysis
   - Testing
   - Debugging
   - Understand and modify existing code
   - Extend existing code using inheritance
   - Understand error handling
   - Reason about programs
   - Analysis of algorithms
   - Numerical representations and limits

IV. Standard data structures
   - Simple data types (int, Boolean, double)
   - Classes
   - One-dimensional arrays
   - *Two-dimensional arrays
   - *Linked lists (singly, doubly, circular)

*Stacks
*Queues
*Trees
*Heaps
*Priority queues
*Sets
*Maps

V. Standard algorithms
Operations on A-level data
Searching
Sorting
Computing in context
Major hardware components
Systems software
Types of systems
Responsible use of computer systems

Special Schools and Programs

The National Consortium for Specialized Secondary Schools for Mathematics, Science, and Technology includes more than 80 [1999; 60] institutional members with 35 000 students. The goal of the Consortium is to foster, support, and advance the efforts of those specialized schools to attract and academically prepare students for leadership in these areas. Some members are boarding schools requiring state residence and highly competitive examinations for entrance. A few are local, specialized high schools. Others are regional centers that students may attend for a half day or full day for a single year. (NCSSSMST Web site, www.ncsssmst.org)

Summer programs in mathematics for very capable students are of two types. Some programs follow a model initiated by Julian Stanley at Johns Hopkins University in the 1970s, identifying talent in the upper elementary or middle school grades and offering accelerated courses (usually in the summer but sometimes through the school year) to enable those students to study more advanced mathematics at a younger age. Other programs follow a model initiated by Arnold Ross at Notre Dame University at about the same time, in which students are taught mathematics different from that which they would normally see in school and in which they are expected to solve problems and deduce propositions in somewhat the same manner as a professional mathematician. These programs recruit students either regionally or nationally, and the entire nation is covered.

The largest organization of mathematics clubs in the United States is Mu Alpha Theta, founded in 1957. Mu Alpha Theta has more than 1300 [1999; 1500] high school and community college chapters and more than 55 000 [1999; 50 000] student members across the United States. Its purpose is to stimulate interest in mathematics by recognizing superior mathematical scholarship in students. In addition to holding regional meetings and an annual national meeting, Mu Alpha Theta also publishes a newsletter, *The Mathematical Log*, and several other resources for its student members. (Mu Alpha Theta Web site, www.mualphatheta.org)

Within the United States, 472 schools (of 1305 worldwide in 115 countries) are authorized to offer the programme of the International Baccalaureate Organization. Of these, about 400 offer the Diploma Programme, a demanding two-year precollege course that leads to examinations and is designed for students ages 16–19. The remainder offer either the Middle Years Programme or the Primary Years Programme designed for younger students. (IBO Web site, www.ibo.org)

Programs for College and Graduate Students

The National Science Foundation funds a large number of research opportunities for undergraduate students through

its Research Experiences for Undergraduates (REU) Sites program. An REU Site consists of a group of 10 or so undergraduates who work in the research programs of the host college or university. Each student is associated with a specific research project, in which he or she works closely with the faculty and other researchers. Students are granted stipends and, in many instances, assistance with housing and travel. Undergraduate students supported with NSF funds must be citizens or permanent residents of the United States or its possessions. In March 2004, 34 REU Sites with research opportunities in mathematics were in existence. A list of the REU Sites can be found at www.nsf.gov.

To increase the number of U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences, the NSF VIGRE (Vertical Integration of Research and Education in the Mathematical Sciences) program supports postdoctoral positions, graduate research traineeships, and research experiences for undergraduates in departments of mathematics and statistics. From 1999 through 2003, VIGRE awards were granted to 39 departments at 37 different universities. A list of the VIGRE sites can also be found at www.nsf.gov.

Mathematics Competitions

Mathematics competitions in the United States are voluntary both for individuals and schools. Some middle schools and high schools have mathematics teams, often competing in events run by local professional organizations. Described here are the larger competitions of national scope.

MATHCOUNTS was founded by the National Society of Professional Engineers, CNA (an insurance company), and the National Council of Teachers of Mathematics in 1982 to increase interest and involvement in mathematics and to assist in developing a technologically literate population. It is now operated by the MATHCOUNTS foundation. Its sponsors include the Dow Chemical Company Foundation, the General Motors Foundation, Lockheed Martin, NEC Foundation of America, Texas Instruments Inc., the 3M Foundation, and the National Aeronautics and Space Administration. Participation is restricted to students in grades 7 and 8. In 2003, more than 500000 students were exposed to MATHCOUNTS materials, and more than 28000 participated in the national competition at some level (Bauer and Fagan 1999; MATHCOUNTS Web site, mathcounts.org).

The American Mathematics Competitions, now centered at the University of Nebraska—Lincoln, in 2003 involved more than 700000 participants and about 20% of the high schools in the country. These competitions began in 1950 as the American High School Mathematics Examination (AHSME) for students in grades 9–12, cosponsored by the Mathematical Association of America and the Society of Actuaries. This examination is now called the AMC 12. Gradually other organizations became involved. In 1985, an examination for students below grade 9, the American Junior High School Mathematics Examination (AJHSME), now called the AMC 8, was initiated. In 2000, the AMC 10, an examination for students below grade 11, was begun. The AMC 12 is a qualifying examination for the American Invitational Mathematics Examination, whose highest scorers become eligible to participate in the U.S. Mathematical Olympiad, a six-question, six-hour examination that is used to determine the U.S. Olympiad team. The AMC also operates a summer program for qualifying students. (AMC Web site, www.unl.edu/amc/)

The Math League, founded in 1977, specializes in mathematics contests, books, and computer software designed to stimulate interest and confidence in mathematics for students from the 4th grade through high school. In recent years, more than 1 million students have participated in Math League contests each year. Contest problems are designed to cover a range of mathematical knowledge for each grade level and require no additional knowledge of mathematics beyond the grade level they test. (Math League Web site, www.mathleague.com)

The American Regions Mathematics League (ARML), begun in 1976 as the Atlantic Region Mathematics League, is a competition of teams of high school students who represent their school, local area, state, or country (outside the U.S.). A power contest runs through the school year. A national competition, which takes place toward

the end of the school year, occurs at three sites. In May 2003, 99 schools participated in the national competition. (ARML Web site, www.arml.com)

The Consortium for Mathematics and Its Applications (COMAP) operates a Mathematical Talent Search (USAMTS) for very high performing high school students in its newsletter *Consortium*. It also sponsors—in conjunction with the MAA, the NCTM, the Information Science and Operations Research Society, and the Society for Industrial and Applied Mathematics—the High School Mathematical Contest in Modeling (HiMCM). The results from this contest are published in COMAP's *Consortium*. It also organizes the Mathematical Contest in Modeling for teams of college students (the winning entries of which are published in the *UMAP Journal*) and a similar contest restricted to high school students. (COMAP Web site, www.comap.com)

The Putnam Exam (officially the William Lowell Putnam Mathematical Competition) for undergraduates, administered by the Mathematical Association of America, celebrated its 64th anniversary in December 2003. In 2002, the most recent year for which results have been published (Klosinski, Alexanderson, and Larson 2003), 3349 [1999; 2581] individuals and 376 [1999; 319] three-person teams competed from 476 [1999; 419] colleges and universities in the United States and Canada. Problems and solutions for the 2003 examination have been published in *Mathematics Magazine,* volume 77 (February 2004), pages 78–82.

Overview

A bachelor's degree and a state teaching certificate are needed to teach in most public schools in the United States at any level, K–12. The teaching certificate is generally obtained through a combination of course-taking at the college level and in-school experience (observations and work in schools, including supervised practice teaching) at or around the grade levels at which the teaching is to take place. Some states require also that the teacher pass a test, usually consisting of specific subject-matter knowledge and general knowledge about teaching and the education system.

Most teachers earn certification before having had a full-time teaching position, and gain tenure after two to four years of full-time teaching. With tenure comes job security; a tenured teacher cannot be removed from a teaching position without evidence of incompetence, breach of contract, or other wrongdoing.

A teaching certificate is not required to teach in private or parochial schools but is often desired because the agencies that accredit schools want schools to have certified teachers. (Accreditation is necessary so that a school's graduates and students who transfer from the school will be recognized automatically by other schools.) Also, in situations of teacher shortage or the movement of a teacher from one state to another, provisional certification is possible until all the requirements for full certification have been met.

Mathematics courses taken by preservice and in-service teachers in their training are typically offered by mathematics departments, and in some institutions mathematics courses for teachers may be offered in education departments. In 2003, in response to a report of a commission headed by former senator John Glenn (National Commission 2000), the Mathematical Association of America initiated a program called Preparing Mathematicians to Educate Teachers (PMET) to help college and university mathematicians take a larger role in training and supporting classroom teachers (Katz and Tucker 2003). The PMET project has three major components: summer workshops and minicourses for faculty training; articles, Web sites, and other materials, and panels at meetings to support faculty instruction; and minigrants and regional networks to nurture and support grassroots innovation in teacher education on individual campuses.

Data on the quantity and quality of the teacher workforce comes from education agencies in individual states, a schools and staffing survey conducted by NCES, a national survey of science and mathematics education conducted by Horizon Research, Inc. (Weiss et al. 2001), and NAEP. The state data are not always complete, and some of the data raise questions about accuracy and completeness (CCSSO 2002). Data from 38 states and Puerto Rico indicate that the number of high school mathematics teachers in these jurisdictions increased during the 1990s, from about 112 000 in 1990 to 134 000 in 2000. Many of these teachers teach other subjects, as evidenced by the finding that in this time period high school teachers with a main assignment in mathematics increased from 61 000 to 71 000. The percentage of these teachers who are female increased from 45% in 1990 to 56% in 1998 and then decreased to 55% in 2000.

For high school teachers, certification is commonly granted in a particular subject or subjects. In 2000, 86% of the high school teachers of mathematics in 28 reporting states were certified in mathematics, a decrease of 2% since 1998 and of 4% since 1990. Nearly half the states reported that at least 95% of their high school mathematics teachers were certified. Ten states had fewer than 90% certified high school mathematics teachers.

Judging from data from 35 states and Puerto Rico, the number of mathematics teachers in grades 7 and 8 also increased, from about 81 000 in 1994 to 125 000 in 2000. In 29 states and Puerto Rico, the number of teachers with a main assignment in mathematics increased from 34 000 in 1992 to 47 000 in 2000. For teachers in grades 7 and 8, a major effort has focused on certifying teachers in subjects rather than as generalists (which is the common certification for teachers in grades K–6). In 1997–98, 66% of mathematics teachers in grades 7 and 8 in reporting states were certified in mathematics, an increase of 12% since 1994. Of the rest, 15% were certified in elementary school teaching as generalists, and 19% were not certified. NAEP data from the 2000 assessment indicate that 44% of grade 8 mathematics teachers had a major in mathematics and 27% had a major in mathematics education, down from 47% and 23% in 1996.

In 1994, on the basis of an analysis of the NCES Schools and Staffing Survey data by Ingersoll (1999), 69% of

mathematics teachers in grades 7–12 had a major in mathematics or mathematics education at the undergraduate level. More recent comparable data are not available. We do provide below some teacher degree data collected in the NAEP national studies of 1996 and 2000. No such data were collected in the 2003 national NAEP study.

Elementary School Mathematics Teachers and Teacher Education

The last 50 years of the 20th century saw vast changes in the preparation of teachers for the nation's elementary schools. In 1952, nearly half of the nation's 600 000 public elementary school teachers did not hold college degrees (Lucas 1997). By the early 1990s, all states required an undergraduate degree in order for an individual to receive a teaching certificate. However, even at present, the amount of mathematics included in the collegiate program for someone preparing to teach grades K–6 is minimal. Individual university programs vary in their nature, as do state requirements for certification. However, most programs for students preparing to teach in grades K–6 consist of a major in education with only a modicum of coursework beyond the institution's general education requirements (Hawkins et al. 1998).

The 2000 national NAEP assessment asked teachers to indicate their college majors, but they could provide multiple responses that are difficult to interpret. We show estimates from the 2000 assessment here with the 1996 national NAEP mathematics assessment immediately following in parentheses. From these surveys, of 4th-grade teachers, about 3% (9%) had an undergraduate or graduate degree in mathematics, 4% (4%) had teachers with a degree in mathematics education, 87% (83%) had a degree in education, and the remaining 6% (4%) had a degree in some other major. However, Weiss et al. (2001) report that virtually no teachers in grades K–4 in their national sample had either a mathematics or mathematics education major. Data from the NAEP school reports in 1996 (not asked in 2000) suggest that 53% of the nation's 4th graders were being taught all subjects by a single teacher in a self-contained classroom and that another 41% were being taught all but one or two subjects by a single teacher. Additional data reflect that about 29% (22%) of students had teachers with five years or less of experience, 18% (24%) had teachers with 6 to 10 years of experience, 33% (35%) had teachers with 11–24 years of experience, and 20% (19%) had teachers with 25 or more years of experience. Data from TIMSS reveal that the average U.S. 4th-grade class spent about 156 hours in mathematics instruction during the school year in 1995–96 (Martin et al. 1999, NCES 2003b).

Elementary school teachers have been found not to have great depth of understanding of mathematics. Liping Ma (1999) compared the mathematical knowledge of elementary school mathematics teachers in the U.S. with that of their counterparts in Shanghai, China, most of whom who teach only mathematics. She found that the U.S. teachers had far less depth of knowledge than their Chinese counterparts. The Ma book argues for deep conceptual knowledge on the part of teachers and for its role in teachers' planning and guidance of lessons in their classrooms. This work had great influence on the CBMS (2001) document *The Mathematical Education of Teachers* discussed below.

Of the K–4 teachers surveyed in the Weiss et al. (2001) national studies, 96% reported having completed a course in mathematics for elementary school teachers; 42%, a course in college algebra, trigonometry, or elementary functions; 33%, a course in probability or statistics; 21%, a course in applications of mathematics or problem solving; 21%, a course in geometry for teachers; and 12%, a course in calculus. These figures are not always consistent with the 2000 (1996) NAEP, in which 83% (84%) of grade 4 students had teachers who had taken a course in the teaching of mathematics; 39% (43%), in number systems and numeration; 31% (37%), in measurement; 30% (34%), in geometry; 46% (45%), in college algebra; and 39% (36%), in probability and statistics. In general, these data suggest that the overall mathematics preparation of elementary school teachers is short of the goals outlined for the mathematics education of elementary teachers in the CBMS recommendations for the mathematical education of teachers (CBMS 2001). This document calls for preservice elementary teachers to have at least nine semester hours (equivalent to three courses) of coursework in mathematics that should provide experiences in number and operations, in geometry and measurement, in algebra and functions, and in data analysis, statistics, and probability. Further, this training should be taught with the goal of developing teachers' in-depth understanding of the mathematics they teach.

Such goals will take significant effort to accomplish, because many prospective elementary teachers take much of their general content coursework at community colleges and then transfer to a four-year college or university to

complete their undergraduate degree program. In some states, these students must complete a five-year program before gaining certification. These points of discontinuity in their preparation make the development of a carefully articulated sequence of courses and experiences in mathematics and mathematics education a formidable task.

Middle School Mathematics Teachers and Teacher Education

In 1996, 33 states and the District of Columbia (containing Washington, D.C., the nation's capital) offered certification for middle grades different from either elementary or secondary certification (Blank and Langesen 1999, p. 58). All but 3 of those states had special mathematics requirements for that certification, ranging from passing a test to taking 27 credit hours (equivalent to 6 to 9 semester courses) in mathematics. The CBMS (2001) recommendations call for the teaching of mathematics in grades 5–8 to be conducted by mathematics specialists, teachers specifically educated to teach mathematics to the students of the grade levels they instruct. These teachers should have at least 21 semester courses in mathematics, including at least 12 semester hours on fundamental ideas of mathematics appropriate for middle school teachers.

From their 2000 survey, Weiss et al. (2001) report that 9% of mathematics teachers in grades 5–8 in their sample had degrees in mathematics and 6% had degrees in mathematics education. The 2000 (1996) national NAEP reports (NCES 2003b, Hawkins et al. 1998) lead to estimates that 41% (49%) of the nation's 8th graders had teachers with an undergraduate degree in mathematics, 25% (13%) had teachers with an undergraduate degree in mathematics education; 30% (32%) had a degree in education, and 4% (7%) were taught be a teacher with a degree in some

other discipline. The fact that about two-thirds of the nation's 8th-grade students were taught mathematics by teachers without substantial mathematics training is a matter of major concern. Data on the NAEP examination show a significant difference between the performance of students taught by teachers with degrees in mathematics or mathematics education and that of students taught by teachers with degrees in education or some other discipline (Hawkins et al. 1998; NCES 2003b). The school structures reported on the NAEP school questionnaire indicate that 87% of the nation's 8th-grade students in 1996 were in schools organized along departmental lines. Data from TIMSS show that schools offering only one course of study at the 8th-grade level tended to spend about 146 hours a year on mathematics instruction, but schools with different tracks averaged about 135 hours a year of instruction in mathematics in 1995–96 (Martin et al. 1999).

In the 2000 (1996) NAEP study, 53% (44%) of the grade 8 students had teachers who had completed a methods course in the teaching of mathematics. These percents are significantly lower than in 1992 (58%) and may again reflect shortages of mathematics teachers for middle schools. However, the mathematics coursework taken by middle school teachers is increasing. In 2000 (1996), the percent of students having a teacher with coursework in number systems and numeration was 54% (50%); in measurement was 43% (37%); in geometry was 65% (64%); in college algebra was 78% (74%); in probability and statistics showed a particularly large increase to 69% (36%); and in calculus was 68% (68%).

In the 2000 (1996) NAEP, 78% (81%) of the grade 8 students had teachers with certification in mathematics and 33% (21%) had certification in some other area. The percents total more than 100% because some teachers have more than one area of certification. Additional data reflect that about 30% [1996: 20%] of students had teachers with 5 years or less of experience, 17% (19%) had teachers with 6 to 10 years of experience, 32% (38%) had teachers with 11–24 years of experience, and 21% (24%) had teachers with 25 or more years of experience.

Secondary School Mathematics Teachers and Teacher Education

For secondary school mathematics teacher certification, states require from 18 (in South Dakota) to 45 (in California) semester hours of mathematics, equivalent to 6 to 15 semester courses, or they require a major in the subject. When a number of credit hours of mathematics is specified for the certificate, almost half require 30 credit hours. When specific courses are mentioned, they include three courses in calculus (two single variable and one multivariable), linear algebra, geometry, and abstract algebra, plus a host of various electives.

The 2000 study of Weiss et al. (2001) reports that 58% of mathematics teachers in grades 9–12 in their sample had an undergraduate major in mathematics, 21% had a degree in mathematics education, 10% had a degree in some other education field, and 10% had a degree in a field other than education or mathematics. In this sample, 96% of teachers had completed a course in calculus; 86%, in probability and statistics; 82% in geometry; 81%, in linear algebra; 70%, in advanced calculus; 68%, in computer programming or other computer science; 65%, in differential equations; and 64%, in abstract algebra.

The CBMS report (2001) recommends that secondary school teachers of mathematics have a major in mathematics that includes a six-hour capstone course connecting their college mathematics courses with high school mathematics. This recommendation stems from the view that teachers need to know the subjects they will teach, they need to understand the broad range of the mathematical sciences their students will encounter in their careers (i.e., core subjects plus dynamical systems, graph theory, combinatorics, operations research, computer science, etc.), and they need to develop the habits of mind and dispositions toward doing mathematics that characterize effective workers in the field.

In addition to specific courses, the report notes that teachers of the secondary grades (9–12) need to develop understanding and skills associated with the use of technology in representing and exploring mathematical concepts

and relationships. This foundation includes experience writing computer programs in a high-level language, such as C++, and experience with a computer algebra system, geometry drawing software, and a statistical software package. These experiences should also be designed to enable teachers to become thoughtful and effective in using educational technology and to keep abreast of changes in the field.

In-service Programs

In 1998, 44 states reported policies defining requirements for continuing professional development of teachers in order for them to keep their state certification. The policy in half the states is 6 semester credits (equivalent to two college courses) every five years; three states require more. Seven states require from 120 to 180 clock hours of professional development every five years. Four states require continuing education units, which are given after various kinds of professional development experience. Eight other states allow a combination of credits, units, and contact hours (CCSSO 1998).

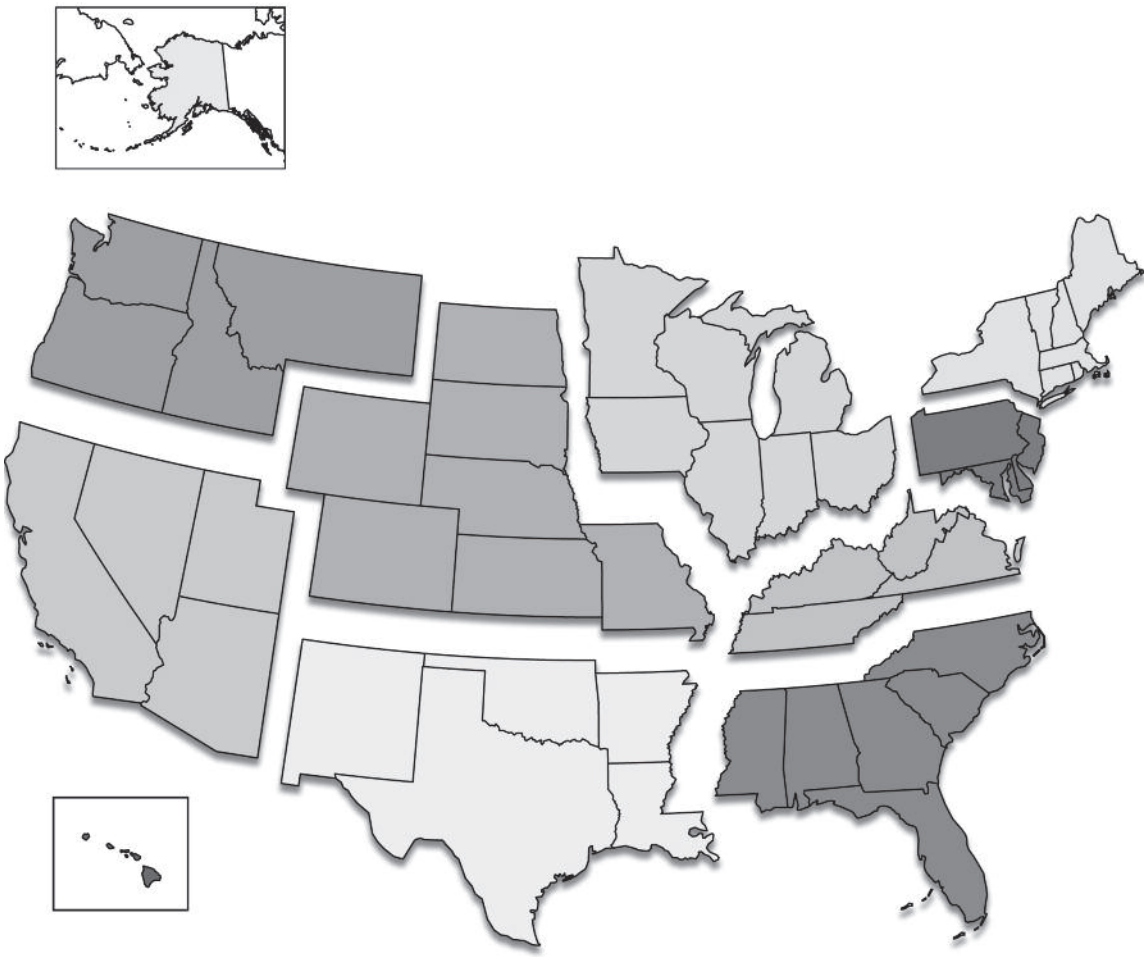


Fig. 4. Regions serviced by the Eisenhower Regional Consortia

In-service opportunities are widely offered within school districts, by professional organizations, by local colleges and universities, by regional education centers, and by commercial enterprises. Within state policies, local school districts and often individual teachers have the freedom to choose the kinds of in-service activities they desire. A teacher is rarely required to participate in a particular in-service program unless that program is given within the teacher's own school district.

NAEP teacher questionnaires in 1992, 1996, and 2000 asked teachers in grades 4 and 8 to report on their professional development in their teaching field during the previous year. Table 14 displays the percents of teachers who

reported having 16 or more hours of professional development. As might be expected, teachers of mathematics in grade 8 had considerably more professional development in mathematics than teachers of mathematics in grade 4.

TABLE 14
Hours of Professional Development in Mathematics Education, 1992–2000 (CCSSO 2002, p. 76)*

	1992	1996	2000
Grade 4			
16–35 hours	10%	15%	12%
36+ hours	11%	13%	7%
Grade 8			
16–35 hours	25%	21%	29%
36+ hours	22%	27%	23%

*Hours = total time in professional development workshops or seminars in mathematics or mathematics education during the previous year.

NCTM and its more than 250 affiliated national, state, and local organizations in mathematics education provide a number of professional development opportunities for teachers of mathematics. In addition to journals and publications, these organizations hold a number of conferences with special sessions for teachers of all grade levels from kindergarten through undergraduate teacher preparation.

The annual meeting of NCTM, held in the spring of each year, has been attended by 17 000 to 19 000 mathematics teachers and other mathematics educators in recent years. In addition, NCTM sponsors three or more regional meetings geographically scattered across the United States and Canada throughout the fall of the year to serve teachers on a local and regional basis, serving as many as 15 000 teachers. All these meetings feature nationally known speakers; workshops; grade-level curriculum and teaching sessions; displays of the most recent textbook materials, manipulatives, and other supplementary materials; and technology for teaching mathematics.

In recent years, new methods of assessment have been a popular subject for in-service offerings. Sometimes these in-service sessions revolve around new tests that school systems and states have developed, often as a result of NCLB. Other in-service programs emphasize assessment using open-ended questions, real-world tasks, and portfolios, as recommended in reports of the Mathematical Sciences Education Board (MSEB 1991, 1993a, 1993b; Stenmark 1991).

Another popular subject for in-service offerings is technology. Of the meetings devoted to technology, the largest are those organized as Teachers Teaching for Technology (T^3).

Doctoral Programs

Mathematics education is a small area within education. Of 19 301 doctorates awarded in the United States in education in the three-year period 1998–2001, only 273 were in mathematics education (National Research Council 1999, 2000, 2001). Still, in this period more doctorates were earned in mathematics education than in any other teaching field except physical education. Of these 273 doctorates, 56% were awarded to women. In comparison, 3139 doctorates were awarded in mathematics, and 811 of these (26%) were earned by women.

The number of doctorates per year in mathematics education during each of the years 1999–2001 was 101, 91, and 81, respectively, relatively close to the 1970–2002 national average of 86 doctorates per year. The 15 U.S. institutions awarding the greatest number of doctoral degrees in mathematics since 1993 are listed here in order of number of doctorates awarded: Teachers College—Columbia University, University of Georgia, University of Texas at Austin, Ohio State University, Georgia State University, Florida State University, North Carolina State University at Raleigh, American University, Rutgers University at New Brunswick, University of Oklahoma, Illinois State University, University of Wisconsin—Madison, University of Maryland, Indiana University, and University of Northern Colorado (Reys et al. 2003). All but Teachers College are public institutions.

During the same period, 1025 dissertations related to mathematics education were documented. This finding indicates that research in mathematics education is often done by individuals who are not identified with mathematics education as their field when finishing their degree. For instance, some of the doctorates awarded in curriculum and instruction may be awarded to people who consider themselves to be mathematics educators. Also, some dissertations that are categorized as being in, or relevant to, mathematics education are done by individuals who do not consider themselves to be mathematics educators (e.g., psychologists, sociologists, or school administrators or supervisors). Coincidentally, 1025 dissertations related to mathematics education are also listed for the two-year period 2002–2003, indicating a growing number of researchers entering the field.

In the most recent survey of doctoral programs in mathematics education (Reys et al. 2000), most doctoral programs reported requiring for entrance a minimum Graduate Record Exam score (75% of programs have this requirement), a minimum grade-point average in previous work (71%), and teaching experience in grades K–12 (56%). Almost half (48%) required a prior bachelor's degree in mathematics, and more than 60% reported that by completion of the program, students must have attained a master's level in mathematics. To enter the program or be admitted to candidacy, 71% of the largest institutions and 54% of all reporting institutions required the passing of qualifying examinations. The number of semester hours needed to complete a doctoral program ranged between 45 and 125, the wide disparity perhaps due to assumptions about prior completion of a master's degree.

Most schools (85%) required residency (a period of time, usually one year, when the doctoral student is a full-time student), and almost all (96%) required the completion of comprehensive examinations. The doctoral programs tended to emphasize research in mathematics education, the teaching and learning of mathematics, research methods, mathematics curriculum, and mathematics content more than technology or general foundations, although all these areas received emphasis in most programs. All institutions required a doctoral dissertation, and all allowed those dissertations to involve quantitative or qualitative research.

Of doctoral students at the time of this survey (1999), 15% were from outside the United States and 62% were female. Minorities (blacks, Asians, and Native Americans) were represented close to their percent in the population as a whole, except for Hispanics (underrepresented at 3%). More than three-fourths of respondents (78%) believed that the demand for individuals with doctorates in mathematics education was currently greater than the supply, and 85 percent gave the same response when asked to predict the demand 5 or 10 years into the future.

Government-Supported Research and Training Centers

The U.S. Department of Education supports 10 regional laboratories. Within each of these regional laboratories is a regional consortium created to provide technical assistance and professional development opportunities on topics in mathematics and science education important to their regions and the nation. Among their many activities, the consortia collaborate with states in developing plans for systemic reform, train teachers, identify and disseminate

exemplary practices, develop strategies for addressing needs of underserved and underrepresented groups, and provide information and facilitate communication among their constituents. Clockwise, from the upper-left corner of figure 4, the regions are named Northwest (NWREL), Mid-continent (McREL), North Central (NCREL), The Education Alliance (LAB, formerly Northeast and Islands), AEL (formerly Mid-Atlantic), Appalachia, Southeast (SERVE), Southwest (SEDL), Far West (WestEd), and Pacific (PREL).

NSF-Funded Centers

Since 2000, the National Science Foundation has funded 17 centers for learning and teaching. The Centers for Learning and Teaching (CLT) program addresses the need to enrich and diversify the national infrastructure for standards-based science, technology, engineering, and mathematics education. The goal is to increase the number of K–12 educators prepared in content, pedagogy, and assessment methodologies. Each center has a specific concentration, but all offer an environment that merges education research, high-quality professional development, and the teaching of innovative instructional practices. Each CLT consists of at least one doctoral degree–awarding university and one or more school districts, plus partnering organizations.

Of the 17 centers, 6 are devoted to the learning and teaching of mathematics. Below is a URL for, and a list of the partners in, each of these centers.

THE MID-ATLANTIC CENTER FOR MATHEMATICS TEACHING AND LEARNING (INITIATED IN 2000)
www.education.umd.edu/mac-mtl/
University of Maryland
Pennsylvania State University
University of Delaware
Prince Georges County, Maryland

THE APPALACHIAN COLLABORATIVE CENTER FOR LEARNING, ASSESSMENT AND INSTRUCTION IN MATHEMATICS (ACCLAIM) (2001)
www.acclaim-math.org
University of Kentucky
Ohio University
University of Tennessee
Marshall University
Appalachian Rural Systemic Initiative

DIVERSITY IN MATHEMATICS EDUCATION (DIME) (2002)
www.wcer.wisc.edu/dime/default.htm
University of Wisconsin—Madison
University of California, Berkeley
University of California, Los Angeles
Madison Metropolitan School District
Berkeley Unified School District
California Subject Matter Project

CENTER FOR PROFICIENCY IN TEACHING MATHEMATICS (CPTM) (2003)
www.cptm.us/
University of Georgia

University of Michigan (Ann Arbor)
University of Michigan—Dearborn
Henry Ford Community College
Gwinnett County, Morgan County, and Social Circle City Schools, Georgia
Washtenaw Intermediate School District, Michigan

MATHEMATICS IN AMERICA'S CITIES (METRO-MATH) (2004)
(no Web site at the time of this writing)
Rutgers University
City University of New York
University of Pennsylvania
New York City School District, New York
Newark and Plainfield School Districts, New Jersey
Philadelphia School District, Pennsylvania

CENTER FOR THE STUDY OF MATHEMATICS CURRICULUM (CSMC) (2004)
mathcurriculumcenter.org/
Michigan State University
University of Missouri
Western Michigan University
Battle Creek, Kalamazoo, and Novi Public Schools, Michigan
Columbia Public Schools, Missouri
University of Chicago School Mathematics Project
Horizon Research, Inc.

Since 2002, NSF has given 5-year grants to 12 comprehensive K–12 math-science partnerships (MSPs). Their names and the lead institution for each are named here.

NORTH CAROLINA PARTNERSHIP FOR IMPROVING MATHEMATICS AND SCIENCE (NC-PIMS)
University of North Carolina General Offices

NEW JERSEY MATH SCIENCE PARTNERSHIP
Rutgers University—New Brunswick

SYSTEM-WIDE CHANGE FOR ALL LEARNERS AND EDUCATORS
University of Wisconsin—Madison

APPALACHIAN MATHEMATICS AND SCIENCE PARTNERSHIP
University of Kentucky Research Foundation

EL PASO MATH AND SCIENCE PARTNERSHIP
University of Texas—El Paso

FACULTY OUTREACH COLLABORATIONS UNITING SCIENTISTS, STUDENTS AND SCHOOLS (FOCUS)
University of California—Irvine

SUPER STEM EDUCATION
Baltimore County Public Schools

Puerto Rico Math and Science Partnership
University of Puerto Rico—Rio Piedras

Promoting Rigorous Outcomes in Mathematics/Science Education (PROM/SE)
Michigan State University

Milwaukee Mathematics Partnership: Sharing in Leadership for Student Success
University of Wisconsin—Milwaukee

Math and Science Partnership of Southwest Pennsylvania
Allegheny Intermediate Unit

Partnership for Reform in Science and Mathematics
University System of Georgia

NSF has also awarded 23 grants for targeted math-science partnerships between schools and universities. A list of these awardees and further information about the program can be found at www.ehr.nsf.gov/msp.

PART IX: BIBLIOGRAPHY

Professional Organizations in Mathematics Education

Closed-membership organizations

CONFERENCE BOARD OF THE MATHEMATICAL SCIENCES (CBMS) (FOUNDED 1960)
e-mail: ronrosier@math.georgetown.edu; Web site: www.cbmsweb.org

The CBMS is an umbrella organization consisting of the major professional societies in the mathematical sciences in the United States and composed of the presidents and executive directors of the member societies. Its purpose is to promote understanding and cooperation among the national professional organizations in mathematics so that they can work together, supporting one another in research, the improvement of education, and the expansion of the mathematical sciences. The following societies belong: American Mathematical Association of Two-Year Colleges (AMATYC), American Mathematical Society (AMS), Association of Mathematics Teacher Educators (AMTE), American Statistical Association (ASA), Association for Symbolic Logic (ASL), Association for Women in Mathematics (AWM), Association of State Supervisors of Mathematics (ASSM), Benjamin Banneker Association (BBA), Institute for Operations Research and the Management Sciences (INFORMS), Institute of Mathematical Statistics (IMS), Mathematical Association of America (MAA), National Association of Mathematicians (NAM), National Council of Supervisors of Mathematics (NCSM), National Council of Teachers of Mathematics (NCTM), Society for Industrial and Applied Mathematics (SIAM), and Society of Actuaries (SOA).

MATHEMATICAL SCIENCES EDUCATION BOARD (MSEB) (1986)
e-mail: mseb@nas.edu; Web site: www7.nationalacademies.org/mseb

The MSEB is a standing board of the National Research Council (NRC) with appointed members. The current mission of MSEB is to provide a continuing national leadership and guidance for policies, programs, and practices supporting the improvement of mathematics education of all students at all levels. Its current initiatives focus on the learning, instruction, and assessment of mathematics; equity in mathematics; attracting and retaining students in mathematics majors and in mathematically intensive careers; capacity building in, and professionalization of, mathematics education; evidence of effectiveness in mathematics education; and the public perception of mathematics, mathematics learning, and mathematics teaching.

United States National Commission on Mathematics Instruction (USNCMI)
Web site: www.ncssm.edu/usncmi

The national adhering body to the International Commission on Mathematical Instruction (ICMI) is the U.S. National Academy of Sciences (NAS). The NAS, through the National Research Council (NRC), appoints the U.S. National Commission on Mathematics Instruction (USNCMI) to conduct the work of the ICMI and foster other international collaborations in mathematics education. The NRC Board of Mathematical Sciences, MSEB, CBMS, and NCTM nominate candidates for selection to the USNCMI.

Open-membership organizations—grades K–12

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS (NCSM) (1969)
Web site: mathforum.org/ncsm/

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM) (1920)
e-mail: nctm@nctm.org; Web site: www.nctm.org

Journals: *Teaching Children Mathematics, Mathematics Teaching in the Middle School, Mathematics Teacher, Journal for Research in Mathematics Education, ON-Math* (online journal)

SCHOOL SCIENCE AND MATHEMATICS ASSOCIATION (SSMA) (1902)
Web site: www.ssma.org

Journal: *School Science and Mathematics*

WOMEN AND MATHEMATICS EDUCATION (WME) (1978)
Web site: www.wme-usa.org

Open-membership organizations—college level

AMERICAN MATHEMATICAL ASSOCIATION OF TWO-YEAR COLLEGES (AMATYC) (1974)
e-mail: web@amatyc.org; Web site: www.amatyc.org

Journal: *AMATYC Journal*

AMERICAN MATHEMATIAL SOCIETY (AMS) (1894)
Web site: www.ams.org

Journals: *Notices of the American Mathematical Society* (and several research journals)

AMERICAN STATISTICAL ASSOCIATION (ASA) (1839)
Web site: www.amstat.org

Journals: *Chance, Stats, The American Statistician* (and four others devoted to research in statistics)

MATHEMATICAL ASSOCIATION OF AMERICA (MAA)
e-mail: maahq@maa.org; Web site: www.maa.org

Journals: *The American Mathematical Monthly, College Mathematics Journal, Mathematics Magazine*

ASSOCIATION OF MATHEMATICS TEACHER EDUCATORS (AMTE) (1993)

Web site: www.amte.net

K–12 Textbook Publishers

Textbooks for grades K–12 are not listed in *Books in Print,* and currently available textbooks are likely not to be listed even in online bookstore catalogs. For this reason, we provide this list of publishers, their locations, and a Web address to assist those who might be interested in obtaining more information about K–12 textbooks and other curriculum materials used in the United States. These publishers and their college publishing counterparts often have auxiliary materials available online.

AGS Publishing, Circle Pines, MN 55014; www.agsnet.com
Amsco School Publications, New York, NY 10013: www.amscopub.com
AnsMar Publishers, Inc., Poway, CA 92064; www.excelmath.com
Bates Publishing Company, Sandwich, MA 02563: batespub.com
Carnegie Learning, Pittsburgh, PA 15222; www.carnegielearning.com
CORD Communications, Waco, TX 76702; www.cord.org/home.cfm
CPM Educational Program, Sacramento, CA 95822; www.cpm.org
Curriculum Research and Development Group, Honolulu, HI 96822; www.hawaii.edu/crdg
Glencoe/McGraw-Hill, Blacklick, OH 43004; www.glencoe.com
Globe Fearon, Upper Saddle River, NJ 07458; www.globefearon.com
Harcourt School Publishers, Orlando, FL 32887; www.harcourtschool.com
Holt, Rinehart & Winston, Austin, TX 78746; www.holtinfo@hrw.com
Houghton Mifflin Company, Boston, MA 02116; www.storemanager@hmco.com
It's About Time, Inc., Armonk, NY 10504; www.its-about-time.com
Kendall/Hunt Publishing Company, Dubuque, IA 52002; webmaster@kendallhunt.com
Key Curriculum Press, Emeryville, CA 94608; www.keypress.com
Macmillan/McGraw-Hill, Desoto, TX 75115; www.mhschool.com
McDougal Littel, Geneva, IL 60134; www.mcdougallittell.com
Pearson Learning Company, Upper Saddle River, NJ 07458; www.pearsonlearning.com
Prentice Hall School Division, Upper Saddle River, NJ 07458; www.prenhall.com
Sadlier-Oxford, New York, NY 10005; www.sadlier-oxford.com
Saxon Publishers, Norman, OK 73071; webmaster@saxonpub.com
Scott Foresman, Glenview, IL 60025; www.scottforesman.com
SRA/McGraw-Hill, Desoto, TX 75155; www.sra4kids.com
W. H. Freeman & Co., New York, NY 10003; www.whfreeman.com
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