Taking Action to Ensure Mathematics Works for Each and Every Student!

Matt Larson
NCTM President
@mlarson_math

Standards do not describe or prescribe the essential conditions required to make sure mathematics works for all students.

High Quality Standards are Necessary for Effective Teaching and Learning, But Insufficient

Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism

Essential Elements of Effective Math Programs

Principles to Actions: Ensuring Mathematical Success for All

The overarching message is that effective teaching is the non-negotiable core necessary to ensure that all students learn mathematics.

We Must Focus on Instruction

Teaching has 6 to 10 times as much impact on achievement as all other factors combined ... Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points.

Teaching and Learning Principle

**Eight Research-Informed Instructional Practices**

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning and problem solving.
- Use and connect mathematical **representations**.
- Facilitate meaningful mathematical **discourse**.


---

**Discussion Question**

With a shoulder partner: What cultural beliefs about the teaching and learning of mathematics do you believe stand as obstacles to implementation of effective teaching and learning in mathematics classrooms?


---

**Unproductive Belief**

Students need only learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Build procedural fluency from conceptual understanding.
Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding …

What are Teachers Doing?
Connecting student-generated strategies and methods to more efficient procedures.
Using visual models to support students’ understanding of general methods.

Our Founding Fathers Did NOT Establish the “Standard Algorithms”
Standard algorithms were developed in India in the first centuries of the modern era, and further honed by traders and engineers in the Iraq-Persia region.

The Pedagogical Value of Standard Algorithms
Standard algorithms sacrifice ease of understanding in favor of computational efficiency, and that made sense once. In today’s world we have readily accessible machines to do calculations, so we can turn the educational focus on understanding the place-value system that lies beneath those algorithms.

The Area Model Builds Understanding of the Standard Algorithm

\[
\begin{align*}
43 \times 17 &= 2 \times 17 + 3 \times 17 \\
&= 301 + 430 \\
&= 731
\end{align*}
\]

\[
\begin{align*}
40 + 3 &= 400 + 30 + 1 \\
&= 430 \\
720 + 21 &= 731
\end{align*}
\]
Don’t confuse an “instructional strategy” with a “mathematical standard”

Moving Forward: Consider How you Communicate with Parents

We should emphasize visual representations or models to build understanding -- not “alternate” “different” or “new” algorithms

Which Makes More Sense?
Which is More Grounded in Place Value?

$$\begin{array}{c}
\text{2} \\
43 \\
\times \ 17 \\
\hline
301 \\
+430 \\
\hline
731 \\
\end{array}$$

$$\begin{array}{c}
43 \\
\times \ 17 \\
\hline
21 \\
280 \\
30 \\
\hline
731 \\
\end{array}$$

If the past, or “traditional instruction,” were so wonderful, why do so many adults proudly proclaim that they “cannot do math”?

We Need to Be Clear: There is No “New Math” & There is No Such Thing as Common Core Math

At the K-8 level there is no “new math,” but there are “new” research-informed instructional strategies!

Unproductive Belief

An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

**Eight Research-Informed Instructional Practices**

**Support productive struggle in learning mathematics.**

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.


**Struggle vs. Frustration**

Struggle does not mean needless frustration or extreme levels of challenge. It means students expend some effort to make sense of mathematics.


**Successful Productive Struggle**

- Engages students with a worthwhile task—one that captures the central idea of a lesson.
- Stretches students’ thinking and performance just beyond the level they can do on their own.
- Teachers provide timely assistance.


**Perseverance: Learning from Our Mistakes**

With parents we should talk about perseverance and learning from mistakes, not “productive struggle.”


**If your students are going home at the end of the day less tired than you are, the division of labor in your classroom requires some attention.**

Caution About Productive Struggle

It is critical to recognize that a focus on ‘grit’ or ‘growth mindset’ is highly cognitive, places the burden of change on the individual, and fails to interrogate institutional structures/practices that disadvantage students who have been marginalized.


Facilitate Meaningful Mathematical Discourse

Students who learn to articulate and justify their own mathematical ideas, reason through their own and others’ mathematical explanations, and provide a rationale for their answers develop a deep understanding …


Eight Research-Informed Instructional Practices

Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.


Eight Research-Informed Instructional Practices

Facilitate meaningful mathematical discourse.

Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.


Facilitate Meaningful Mathematical Discourse

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</td>
<td>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</td>
</tr>
<tr>
<td>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</td>
<td>Engaging in writing and critiquing the writing of peers, using examples to support or counterexamples to refute arguments.</td>
</tr>
<tr>
<td>Facilitating discourse among students by highlighting the work of peers, using examples to support or counterexamples to refute arguments.</td>
<td>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.</td>
</tr>
<tr>
<td>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning</td>
<td>Identifying how different approaches to solving a task are the same and how they are different.</td>
</tr>
</tbody>
</table>


Anticipating ... Monitoring ... Selecting ...

Sequencing ... Connecting


Five Practices to Promote Productive Math Discussions

Effectivte teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.
Effective Teachers are Effective Questioners

“Effective mathematics teachers … pose more questions with higher cognitive demand and ask more follow-up questions”


Classroom Mantras


Five Essential Elements of Effective Mathematics Programs

Effective teaching and learning, while the non-negotiable core of successful mathematics programs, are part of a system of essential elements of excellent mathematics programs.


Five Essential Elements of Effective Mathematics Programs

Access and Equity
Curriculum
Tools and Technology
Assessment
Professionalism


Guiding Principles for School Mathematics: Access and Equity

Access and Equity. An excellent mathematics program requires that all [each and every] students have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.


NCTM Broadens its Access and Equity Work

• NCTM has re-framed its work to focus on Access, Equity, and Empowerment, to capture the critical constructs of *identity, agency, and social justice.*
Breaking Barriers: Actionable Approaches to Reach Each and Every Learner in Mathematics

Access and Equity Obstacles

A range of obstacles exists ... one of these involves the quality of instruction available to different groups of students ... another involves differential opportunities to learn high-quality grade level mathematics content and to be held to high expectations for mathematics achievement.


Beliefs about access and equity in mathematics, continued

<table>
<thead>
<tr>
<th>Unproductive beliefs</th>
<th>Productive beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics learning is independent of students' culture, conditions, and language, and teachers do not need to consider any of these factors to be effective.</td>
<td>Effective mathematics instruction leverages students' culture, conditions, and language to support and enhance mathematics learning.</td>
</tr>
<tr>
<td>Tracking promotes students' achievement by allowing students to be placed in “homogeneous” classes and groups in which they can make the greatest learning gains.</td>
<td>The practice of isolating low-achieving students in low-level or slower-paced mathematics groups should be eliminated.</td>
</tr>
<tr>
<td>Only high-achieving or gifted students can reason about, make sense of, and persevere in solving challenging mathematics problems.</td>
<td>All students are capable of making sense of and persevering in solving challenging mathematics problems and should be expected to do so. Many more students, regardless of gender, ethnicity, and socioeconomic status, need to be given the support, confidence, and opportunities to reach much higher levels of mathematical success and interest.</td>
</tr>
</tbody>
</table>

Who Teaches Whom What?

The power and status of school mathematics often manifest themselves in decisions about what content gets taught, to which students, and by which teachers ... what gets taught in the mathematics classroom shapes the mathematics identities of both students and teachers.


Access Remains a Critical Issue

Across OECD countries, more than 70% of students attend schools whose principal reported that students are grouped by ability for math ... reducing ability-grouping can reduce the influence of socio-economic status on students’ opportunities to learn.

High-Rigor Course Access is Not a Reality in the United States

- Nationwide 48% of high schools offer calculus.
- Nationwide 78% of high schools offer Algebra II.


High-Rigor Course Access is Not a Reality in the United States

- 33% of high schools with high black and Latina/o student enrollment (greater than 75%) offer calculus, compared to 56% of high schools with low black and Latina/o student enrollment (less than 25%).


Type and Quality of Instruction Matters

While education systems have generally done well in providing equitable access to the *quantity* of mathematics education – in the sense that marginalized students spend about the same time in mathematics classes in school as their non-marginalized peers – the data show large differences in the *quality* of learning experiences.


Type and Quality of Instruction Matters

While marginalized students tend to learn simple facts and figures and are exposed to simple applied problems, their privileged counterparts experience mathematics instruction that help them think like a mathematician, develop deep conceptual understanding and advanced mathematical reasoning skills.


Quality of Instruction Matters Beyond School

Learning environments where students are actively engaged in mathematics, i.e. involved in problem solving, the discussion of ideas, and the application of methods, not only enhance individual understanding but may also be related to positive outcomes later in life including the adaptive expertise and the propensity to engage successfully with and use mathematics in their lives as adults.

Tracking Persists in New Forms

Although many schools have done away with traditional three-track sorting, hidden forms of tracking persist ... For example, an algebra course might sort students into fast and slow speeds of learning, so that by the end of the year students in the same class have not had the same opportunity to learn.


All Too Often the Teachers are Tracked

Teachers themselves are tracked, with those judged to be the most competent, experienced, or high status assigned to the top tracks and those with the least experience and training assigned to the lower tracks.


Who is Teaching Whom?

In a study of 29 districts in 16 states, marginalized students in grades 4 through 8 had access to less effective instruction than non-marginalized students, and that lack of access persisted over time.


We expect that the very best doctors will treat the most grievously ill patients.

It should be no different in education. Great teachers have the skills to help the students who struggle the most.


Guiding Principles for School Mathematics: Curriculum

Curriculum. An excellent mathematics program includes curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.


A Curriculum Obstacle

Grade level mathematics curriculum standards are often treated as a checklist of topics. When conceptualized as such, mathematics content becomes nothing more than a set of isolated skills ...
A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades (p. 14).


Assessment Obstacle

Traditionally assessment tends to emphasize the evaluation of student achievement (e.g., assigning grades), and more recently, the rating of schools and the performance of teachers – the cultural perception that links assessment to grading and rating ...

Valid and Reliable Classroom-Based Assessment is Not the Enemy

Assessment diagnoses student learning needs and facilitates effective instruction just as medical tests diagnose illness and direct effective treatment protocols.


Guiding Principles for School Mathematics: Professionalism

Professionalism: In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.


Professionalism Obstacle

In too many schools, professional isolation severely undermines attempts to significantly increase professional collaboration ... A danger in isolation is that it can lead to teachers developing inconsistencies in their practice.


Overcoming the Obstacle: Professional Learning Communities

Teachers have a professional responsibility to participate in group decision making to improve the art and practice of teaching. One of the most powerful forums for teacher improvement is involvement in a professional learning community.


But What Happens in Your PLCs?

A Chinese teacher sees a lesson as a performance and puts in many hours of preparation to cover the standard forty-five minute period ...


Lesson Planning is Cultural

The tendency to spend relatively little time developing lessons and to produce lesson outlines appears to be a cultural style specific to the U.S.

You Should Collaboratively Plan One Lesson in Each Unit

The lack of time to devote this careful planning and reflection to all lessons cannot be used as an excuse to never collaboratively learn, plan, and reflect on the effectiveness of key lessons.


Why Focus on Lesson Planning?

…the co-planning of lessons is the task that has one of the highest likelihoods of making a marked positive difference on student learning.


It Can All Seem Overwhelming and Change Often Feels Sisyphean!

Change is Hard!

The most likely reason for the stability of teaching practices over time is that teaching is a cultural activity and cultural activities, by their very nature, are highly resistant to change.

Some Practices are a Cultural Trap

Cultural routines evolve over time to enable adaptation to the environment. However, sometimes the environment changes, and yet, the cultural routine persists, even if it is now highly maladaptive.


Why is it so hard to reform mathematics education?

The traditional math “teaching script” is VERY embedded in our culture ... The inertia of the past is incredibly strong.

Moving Forward: Support Research-Informed Instructional Practices

We expect physicians to use research-informed treatments. We must do the same.


Moving Forward: Support and Implement Research-Informed Instructional Practices

The six guiding principles constitute the foundation of high-quality mathematics education.

Standards-Based Reform Has Improved Mathematics Learning

Math achievement in this country is up over the long-term ... Since we’ve been doing Standards-based reform!
**Standards-Based Reform Has Improved Mathematics Learning**

<table>
<thead>
<tr>
<th>Subscale</th>
<th>Grade 4</th>
<th>2015</th>
<th>'15-'13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite</td>
<td>240</td>
<td>-1</td>
<td>*</td>
</tr>
<tr>
<td>Number properties and operations</td>
<td>243</td>
<td>+1</td>
<td>*</td>
</tr>
<tr>
<td>Measurement</td>
<td>238</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td>236</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>Data analysis, statistics, and probability</td>
<td>238</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>Algebra</td>
<td>243</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

*Statistically significant (p < .05).*

Don’t panic (yet) over the slight drop in 2015.

**Standards-Based Reform Has Improved Mathematics Learning**

Based on the NAEP long-term trend assessment, initiated in 1973, today’s fourth and eighth graders are performing at a significantly higher level than their parents and grandparents did in mathematics.

**We Know The Elements of Effective Instruction: We Just Need to Do It!**

“It is critical that schools learn the lesson that ‘best practice’ in effective organizations is rarely new practice.

“. . . [In The Knowing-Doing Gap, Pfeffer and Sutton emphasize], most effective actions are ‘well-known practices, with the extra dimension that they are reinforced and carried out reliably (2000, p. 14).’”

—Schmoker, Focus: Elevating the Essentials to Radically Improve student learning (2011), p. 17

**Eight Research-Informed Instructional Practices**

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse. Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

**Change Takes Perseverance**

“When teachers try to change more than two or three things about their teaching at the same time, the typical result is that their teaching deteriorates and they go back to doing what they were doing before.”


**The Document Is Entitled Principles to Actions**

With a shoulder partner:

- What research informed instructional strategy will you make the focus of your second semester this year?
- Who will you enlist to support you?
<table>
<thead>
<tr>
<th><strong>You Can Make It Happen!</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>“[Effective] teachers/leaders believe that success and failure in student learning is about what they, as teachers or leaders, did or did not do . . . We are change agents!”</td>
</tr>
</tbody>
</table>