Teacher Notes for the Core Math Tools Simulation Lesson

Title: Waiting for Blood Donors

Overview
In this lesson, students are presented with a scenario in which the percent of donors having type B blood is known but we want to estimate the number of donors that will have to arrive in order to have a particular number of type B blood donors. Students will investigate this scenario using the Simulation App of the Core Math Tools.

CCSSM Standards
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
7.SP.8c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
S-IC- 2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

CCSSM Mathematical Practices
1. Make sense of problems and persevere in solving them.
4. Model with mathematics
5. Use appropriate tools strategically.

Core Math Tools
This investigation uses the Simulation App.

Instructional Plan
1. Introduce the following scenario: On a certain day, the local blood center needs 4 pints of type B blood. Assuming that donors arrive independently and randomly, how many donors, on average, should the blood center have to see in order to obtain 4 donors with type B blood.
2. Ask the students if they have an estimate of how many donors they think will need to arrive at the blood center in order to obtain 4 donors with type B blood. Ask what would be the fewest number that would need to arrive and ask about the maximum number.
3. Suggest to the students that we will first investigate how many donors will the blood center have to see in order to see just one donor with type B blood. Ask the students to explore the features of the Simulation Tool to find a model that could be used to match the probability to that of a random donor.
having type B blood and other features of the Tool that would help count the number of rolls until a 1 is rolled.

4. One student suggested that the 10-sided die could be used with a 1 representing a donor with type B blood. The student said that we would have to keep track of how many times the die had to be rolled before a 1 was rolled.

5. Show the following screen and ask if the students found a different way to set up the simulation. Ask the students to share their models and justify why their models will work.
**Teacher Notes:**
The following is an example of using the Core Math Tools Simulation App.

Select the green 10-sided die.
Under Build select Count till see. Double-click on the 1.

6. Share with the students that one student conducted one trial and saw the following output. Ask the students what the results represent. Ask them to conduct one trial using their own model. Have them share their results and explain what the results indicate.
Teacher Notes:
This example shows that it took 15 rolls of the die before a 1 was rolled. If you click on the small triangle next to “Count till see” you will see the outcomes of the 15 rolls of the die. In the output, the first column is the outcome number or the roll number, the second column is what number was rolled each time, and the third column is how many rolls were needed get a 1.

7. Share with the students that one student ran a second trial. Show the results. Ask the students in explain what each column in the output represents.
Teacher Notes:
For this second simulation, it took 16 rolls of the die to roll a 1. Notice that outcome number 15 was a 1 so the second simulation is starting with outcome number 16. The next 1 that was rolled did not occur until the 31st outcome. This means that it took another 16 rolls until a 1 was rolled.

8. Share that the student conducted 100 additional trials. Show the results and ask the students how can they use the results from all these trials to answer the question: How many donors, on average, should the blood center have to see in order to obtain 1 donor with type B blood?
Teacher Notes:
Click on the small triangle by the Count till see; by doing this we only see the number of rolls needed to get a 1. We do not see the individual rolls. The outcome number is now the simulation number.

To view a histogram of the results, select Graph and Label Bars under the View menu.

9. One student suggested to use the Info menu and find the mean of the distribution.
Teacher Notes:

Example of sample output.

<table>
<thead>
<tr>
<th>10-sided Die</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1025</td>
</tr>
<tr>
<td>mean = 5.58</td>
</tr>
<tr>
<td>minimum = 1</td>
</tr>
<tr>
<td>q1 = 3</td>
</tr>
<tr>
<td>median = 6</td>
</tr>
<tr>
<td>q3 = 8</td>
</tr>
<tr>
<td>maximum = 10</td>
</tr>
<tr>
<td>sample standard deviation = 2.92</td>
</tr>
<tr>
<td>sample variance = 8.54</td>
</tr>
</tbody>
</table>

Count till see: 1

<table>
<thead>
<tr>
<th>Count till see: 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 102</td>
</tr>
<tr>
<td>mean = 10.05</td>
</tr>
<tr>
<td>minimum = 1</td>
</tr>
<tr>
<td>q1 = 3</td>
</tr>
<tr>
<td>median = 8</td>
</tr>
<tr>
<td>q3 = 15</td>
</tr>
<tr>
<td>maximum = 58</td>
</tr>
<tr>
<td>sample standard deviation = 8.89</td>
</tr>
<tr>
<td>sample variance = 79.06</td>
</tr>
</tbody>
</table>

In this example, the distribution is skewed to the right and has a mean of 10.05 and a standard deviation of 8.89. This would mean that on average about 10 donors are needed to have one person have type B blood. Students should observe that the minimum number is 1 while the maximum number can be quite high—in this example, 58 rolls of the die were needed before a 1 was rolled.

10. Tell your students that we have investigated how many donors are needed to have 1 donor with type B blood, and now we want to find the average number needed to obtain 4 donors with type B blood. Ask the students what they think the average number will be and why.

11. Ask the students to explore the features of the Simulation Tool to find a model that could be used to match the probability to a random donor having type B blood and other features of the Tool that would help count the number of rolls until a 1 is rolled at least 4 times.

12. Share that one student decided to use four 10-sided dice and set up each one like the die was set up when using just one 10-sided die. Share that the student decided then to find the sum of the number of rolls needed for each of the four 10-sided dice to roll a 1. Share that the student used the function option under the Build menu to find the sum.
Teacher Notes:
Highlight the four boxes (click on the first and then shift-click on the last to highlight all four).
Select Function under the Build menu. Enter a+b+c+d.

13. Ask the students if they found another method to set up the simulation. Have them share their methods and ask that they justify why their methods are valid.
14. Ask the students to use their simulation setup and conduct one trial. Have the students share their results and explain what their results represent.
Teacher Notes:
The sample results indicate that the first die needed 1 roll, the second die needed 12 rolls, the third die needed 3 rolls, and the fourth die needed 1 roll to see a 1, for a total of 17. This would mean that 17 donors were necessary before 4 of them had type B blood.

15. Discuss that we will need to conduct a large number of simulations. Share the results from one student who conducted 100 runs. Show the histogram and the mean of the 100 runs.
**Teacher Notes:**
Under the View menu, check Graph, Label Bars, and Relative Frequencies.
For this example, the distribution is skewed right with a mean of 40.93 and a standard deviation of 17.94. These results would indicate that the average number of about 41 donors are needed before 4 are found with type B blood.

$$a+b+c+d$$

a: Count till see: 1  b: Count till see: 1  c: Count till see: 1  d: Count till see: 1

n = 101
mean = 40.93
minimum = 9
q1 = 27
median = 40
q3 = 54
maximum = 90
sample standard deviation = 17.94
sample variance = 321.97

16. Ask the students to summarize their findings and report the answer to the question: What is the average number of donors needed to obtain 4 donors with type B blood?

17. Ask how the results of their simulations are related to the results when they found the number of donors needed to see just 1 with type B blood.

18. Have the students work in pairs to design a simulation that could help answer the following question: How many donors, on average, should the blood center have to see in order to obtain 4 donors with type A blood assuming that approximately 40% of people have type A blood?
Teacher Notes:
Additional question that can be considered:

Using the blood type distribution

<table>
<thead>
<tr>
<th>Blood Type</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>45%</td>
</tr>
<tr>
<td>A</td>
<td>40%</td>
</tr>
<tr>
<td>B</td>
<td>10%</td>
</tr>
<tr>
<td>AB</td>
<td>5%</td>
</tr>
</tbody>
</table>

How many donors on average would you have to see in order to have one of each blood type?

Credit:
Patrick Hopfensperger, University of Wisconsin–Milwaukee

References: