Grade 6:
Expressions & Equations
NCTM Interactive Institute, 2015

Name
Title/Position
Affiliation
Email Address
With your table, decide the similarities and differences about the four phrases below:

- Numerical expression
- Numerical equation
- Algebraic expression
- Algebraic equation
This session will address the following:

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.EE.2</td>
<td>Write, read, and evaluate expression in which letters stand for numbers.</td>
</tr>
<tr>
<td>6.EE.4</td>
<td>Identify when two expressions are equivalent.</td>
</tr>
<tr>
<td>6.EE.9</td>
<td>Write an equation to express one quantity (dependent variable) in terms of the other quantity (independent variable).</td>
</tr>
<tr>
<td>6.EE.7</td>
<td>Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$.</td>
</tr>
</tbody>
</table>
Algebra Magic

• Think of a number.
• Multiply the number by 3.
• Add 8 more than the original number.
• Divide by 4.
• Subtract the original number.

Compare your answer to others at your table.

*Why did this happen? Show in 2 different ways.*
What could be done to the steps in order to get the **number you started with**?

- Think of a number.
- Multiply the number by 3.
- Add 8 more than the original number.
- Divide by 4.
- Subtract the original number.
Writing Expressions

• Enter the first three digits of your phone number.
• Multiply by 80.
• Add 1.
• Multiply by 250.
• Add the last four digits of your phone number.
• Repeat the above step.
• Subtract 250.
• Divide by 2.

Describe the number you have.

How did the problem work?
Which of the following steps can you reverse without changing the result? Why?

1) Think of a number.
2) Subtract 7.
3) Add 3 more than the original number.
4) Add 4.
5) Multiply by 3.
6) Divide by 6.
Algebra Magic

The following trick is missing the last step.

• Think of a number.
• Take its opposite.
• Multiply by 2.
• Subtract 2.
• Divide by 2.

?? ?????? ????

Decide what the last step should be for the given condition so final result is:

a) One more than original number.
b) Opposite of original number.
c) Always 0.
d) Always -1.
Make up a separate algebra magic trick with at least five steps that will meet one of the bullets listed below:

- Final result is one more than the original number.
- Final result is 0.
- Uses all four operations.
- Result is same, whether steps are done backwards or forward.
What errors might occur as students translate the following sentences into algebraic expressions?

- Multiply \( n \) by 5 then add 4.
- Add 4 to \( n \) then multiply your answer by 5.
- Add 4 to \( n \) then divide your answer by 5.
- Multiply \( n \) by \( n \) then multiply your answer by 3.
- Multiply \( n \) by 3 then square your answer.
Matching Expressions, Words, Tables, & Areas

Work collaboratively with your tablemates.

• Match cards to make a complete set with an equivalent expression, description, table, and area cards.

• If there is not a complete set, make a card for the missing type(s) with one of the blank cards.
Matching Expressions, Words, Tables, & Areas

Large group discussion:

• Which, if any, of the groups of expressions are equivalent to each other? How do you know?
• What will students learn as a result of this activity?
• What challenges might students encounter with this activity?
Expressions to Equations

8 + 4 = □ + 7

What responses do students give for box?

Operational vs Relational
“answer” vs “equivalence”
Equality

The notion of equality is surprisingly complex, is often difficult for students to comprehend, and should be developed throughout the curriculum.
Many students at all grade levels have not developed adequate understanding of the meaning of the equal sign.

“Limited conception of what the equal sign means is one of the major stumbling blocks in learning algebra. Virtually all manipulations on equations require understanding that the equal sign represents a relation.”

Equality

Is the number that goes in the box the same number in the following two equations?

\[2 \times \square + 15 = 31 \quad 2 \times \square + 15 - 9 = 31 - 9\]

In the equation \[\square + 18 = 35\], the number that goes in the box is 17. Can you use this fact to figure out what number goes in this box:

\[\square + 18 + 27 = 35 + 27\]
Transitioning to Relational Thinking

• No calculators – No computations
• Use relational thinking to justify answer.

True or False:

471 – 382 = 474 – 385
674 – 389 = 664 – 379
583 – 529 = 83 – 29
37 x 54 = 38 x 53
5 x 84 = 10 x 42
64 ÷ 14 = 32 ÷ 28
42 ÷ 16 = 84 ÷ 32
Transitioning to Relational Thinking

• No calculators – No computations
• Use relational thinking to justify answer.

**What is the value of variable?**

73 + 56 = 71 + d
67 − 49 = c − 46
234 + 578 = 234 + 576 + d
94 + 87 − 38 = 94 + 85 − 39 + f
92 − 57 = 94 − 56 + g
68 + 58 = 57 + 69 − b
56 − 23 = 59 − 25 − s
Relational Thinking

What properties are important to developing relational thinking with students?

\[
\begin{align*}
    a + 0 &= a \\
    a \times 1 &= a \\
    a + b &= b + a \\
    a + b &= (a + n) + (b - n) \\
    a - b &= (a + n) - (b + n) \\
    ab &= (na) \left( \frac{1}{n} b \right) \\
    a - 0 &= a \\
    a \div 1 &= a \\
    a \times b &= b \times a \\
    a + b &= (a - n) + (b + n) \\
    a - b &= (a - n) - (b - n) \\
    \frac{a}{b} &= \frac{na}{nb}
\end{align*}
\]
Equality Sign Caution

3 + 5 = 8 + 2 = 10 + 5 = 15

Equality strings written by students (and teachers!) provide opportunity to discuss meaning of equal sign and its proper use.

3 + 5 = 8
8 + 2 = 10
10 + 5 = 15
Interpreting Equations

Which is greater, x or y? Explain your reasoning.

\[ y = 4x \]

- x is greater because it's multiplied by 4.
- y is greater because it is four times the size of x.
- It depends what x and y are.
Interpreting Equations

Let $e$ represent the number of eggs.
Let $b$ represent the number of egg boxes.
There are 6 eggs in each box.
Find an equation linking $e$ and $b$.

\[ b = 6e \]
\[ e = 6b \]
Interpreting Equations

Let $e$ represent the cost of an egg.
Let $b$ represent the cost of a box of eggs.
The price per egg is the same whether you buy them separately or in a box.

Find an equation linking $e$ and $b$.

$$b = 6e$$
$$e = 6b$$
Interpreting Equations

Working together at your tables:

• Match an equation card with a statement card.
• Explain/challenge reasoning.
• Use blank cards to write equation or statement cards so that each card is grouped with at least one other card.
Solving Equations
Strip Diagram Method

• Helps students conceptualize the characteristics of the problem to solve
  Make sense of variable to represent unknown quantity

• Helps students formulate an algebraic equation to solve the problem
  Analyze relationship(s) between components of problem

• Helps empower students
  Develop competence and confidence in using the algebraic method.
There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let's denote the number of girls as $g$ and the number of boys as $b$. According to the problem, we have the following relationships:

- $b = g + 10$ (since there are 10 more boys than girls)
- $b + g = 50$ (since the total number of children is 50)

We can solve these equations to find the number of girls.

Substituting the first equation into the second, we get:

$g + 10 + g = 50$

$2g = 40$

$g = 20$

Therefore, there are 20 girls in the dance group.
Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let $x$ be the number of girls.

What could be possible algebraic equation(s)?

Boys

Girls

50

10
There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let $x$ be the number of girls.

What could be possible algebraic equation(s)?

$$x + (x + 10) = 50$$
There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let $x$ be the number of girls.

What could be possible algebraic equation(s)?

$$(50 - x) - x = 10$$
Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let $x$ be the number of boys.

What could be possible algebraic equation(s)?

- Boys
- Girls

Boys: $x + 10$

Girls: $x$

Total: $x + 10 + x = 50$

$2x + 10 = 50$

$2x = 40$

$x = 20$

So, there are 20 boys and $20 - 10 = 10$ girls.
There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let $x$ be the number of boys.

What could be possible algebraic equation(s)?

$$x + (x - 10) = 50$$

Boys: $x$

Girls: $x - 10$
There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let $x$ be the number of boys. What could be possible algebraic equation(s)?

$$x - (50 - x) = 10$$
Solving Equations
Strip Diagram Method

Use the Strip Diagram Method to solve the problems on the handout.

Set up the diagram/algebraic equations in as many ways as possible.
Cover up method: $5 + \frac{3x - 1}{4} = 7$
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5 + [ ] = 7  [ ] = 2  $\frac{3x - 1}{4} = 2$
Solving Equations

Cover Up Method

**Cover up method:** \( 5 + \frac{3x - 1}{4} = 7 \)

<table>
<thead>
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<th>( 5 + \boxed{\phantom{0}} = 7 )</th>
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Solving Equations

Cover Up Method

Cover up method: $5 + \frac{3x - 1}{4} = 7$

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Solving Equations
Cover Up Method

Cover up method: \( 5 + \frac{3x - 1}{4} = 7 \)

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<td>(3 \square = 9)</td>
<td>(\square = 3)</td>
<td>(x = 3)</td>
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Solving Equations
Cover Up Method

• Practice solving equations using the Cover Up Method with your tablemates.

• What will students learn as a result of this activity?

• What challenges might student encounter with this activity?
Reflection

• What new idea(s) do you want to implement into your classroom as a result of this session?

• What challenges did you encounter during this session?
Reflection

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<th>What are students doing?</th>
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<tr>
<td>Providing students with opportunities to use their own reasoning strategies and methods for solving problems.</td>
<td>Making sure that they understand and can explain the mathematical basis for the procedures that they are using.</td>
</tr>
<tr>
<td>Asking students to discuss and explain why the procedures that they are using work to solve particular problems.</td>
<td>Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.</td>
</tr>
<tr>
<td>Connecting student-generated strategies and methods to more efficient procedures as appropriate.</td>
<td>Determining whether specific approaches generalize to a broad class of problems.</td>
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(Principles to Actions: Ensuring Mathematical Success for All [NCTM 2014], p. 47)
**Reflection**

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<tr>
<td>Using visual models to support students’ understanding of general methods.</td>
<td>Striving to use procedures appropriately and efficiently.</td>
</tr>
<tr>
<td>Providing students with opportunities for distributed practice of procedures.</td>
<td></td>
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*(Principles to Actions: Ensuring Mathematical Success for All [NCTM 2014], p. 48)*
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