The Power of Teacher Collaboration to Support Effective Teaching and Learning

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Your Feelings Looking Ahead?
Algebra Readiness

Content

• Ratios and Proportional Relationships
• Expressions and Equations, Variable
• Linear and Non-linear Functions

Students’ Understanding

• Common misconceptions
• “Rules that expire”
• Connecting representations
Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.
Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism
Guiding Principles for School Mathematics

**Professionalism**

In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and **collective professional growth** toward effective teaching and learning of mathematics.
In too many schools, professional isolation severely undermines attempts to significantly increase professional collaboration ... some teachers actually embrace the norms of isolation and autonomy. A danger in isolation is that it can lead to teachers developing inconsistencies in their practice that in turn can create inequities in student learning.

*Principles to Actions*, p. 100
Incremental Change

• The social organization for improvement is a profession learning community organized around a specific instructional system.

  A. S. Bryk (2009)

• The unit of change is the teacher team.
Collaborative Team Work

- An examination and prioritization of the mathematics content and mathematics practices students are to learn.
- The development and use of common assessments to determine if students have learned the agreed-on content and related mathematical practices.
- The use of data to drive continuous reflection and instructional decisions.
- The setting of both long-term and short-term instructional goals.
- Development of action plans to implement when students demonstrate they have or have not attained the standards.
- Discussion, selection, and implementation of common research-informed instructional strategies and plans.

*Principles to Actions*, pp. 103-104
PLC Collaborative Team Lesson Planning

• Read the collaborative team illustration.

• Discuss with people at your table:
  – How does this team’s work support/undermine the ideas/strategies discussed over the past two days?
  – How is this team’s work similar to the work of teachers in your school?
  – How is this team’s work different from the work of teachers in your school?
  – Implications for your work?
Key Features of the Team’s Work

• Collaborative professional learning
• Collaborative lesson planning—implementing effective teaching practices.
• Implement and refine lesson (mini-lesson study)
• Teaching practice is public
• Repository of collaborative lessons
Key Features of the Team’s Work

• Collaborative professional learning
• **Collaborative lesson planning**—implementing effective teaching practices.
• Implement and refine lesson (mini-lesson study)
• Teaching practice is public
• Repository of collaborative lessons
Five Practices for Orchestrating Productive Mathematics Discussions

- **Anticipating** likely student responses
- **Monitoring** students’ actual responses
- **Selecting** particular students to present their work during the whole class discussion
- **Sequencing** the students’ presentations
- **Connecting** different students’ strategies and ideas in a way that helps students understand the mathematics or science in the lesson.

Smith & Stein, 2011
Planning with the Student in Mind

- Anticipate solutions, thoughts, and responses that students might develop as they struggle with the problem/task.

- Generate questions that could be asked to promote student thinking during the lesson, and consider the kinds of guidance that could be given to students who showed one or another types of misconception in their thinking.

- Determine how to end the lesson so as to advance students’ understanding.

Stigler & Hiebert, 1997
Effective Questions should:

• Reveal students’ current understandings;

• Encourage students to explain, elaborate, or clarify their thinking; and

• Make the mathematics/science more visible and accessible for student examination and discussion.
Pose Purposeful Questions

• Assessing/Advancing
• Reversibility
• Flexibility
• Generalization
• Reveal common misconceptions
# Thinking Through a Lesson Protocol (TTLP) Planning Template

<table>
<thead>
<tr>
<th>Learning Goals (Residue)</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>What understandings will students take away from this lesson?</td>
<td>What will students say, do, produce, etc. that will provide evidence of their understandings?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Task</th>
<th>Instructional Support—Tools, Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the main activity that students will be working on in this lesson?</td>
<td>What tools or resources will students have to use in their work that will give them entry to, and help them reason through, the activity?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Task Launch</th>
<th>Instructional Support—Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>How will you introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Task Enactment</th>
<th>Instructional Support—Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the various ways that students might complete the activity?</td>
<td>What questions might you ask students that will support their exploration of the activity and bridge between what they did and what you want them to learn?</td>
</tr>
</tbody>
</table>

To be clear on what students actually did, begin by asking a set of assessing questions such as: What did you do? How did you get that? What does this mean? Once you have a clearer sense of what the student understands, move on to an appropriate set of questions below.

<table>
<thead>
<tr>
<th>Selecting and Sequencing</th>
<th>Connecting Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which solutions do you want to have shared during the lesson? In what order? Why?</td>
<td>What specific questions will you ask so that students make sense of the mathematical ideas that you want them to learn.</td>
</tr>
<tr>
<td></td>
<td>- make connections among the different strategies/solutions that are presented</td>
</tr>
</tbody>
</table>

Adapted from Smith, Bill, and Hughes, 2008
## Planning with the Student in Mind

<table>
<thead>
<tr>
<th>Strategy/Response</th>
<th>Questions</th>
<th>Students/Group</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Rate: Picture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Rate: Table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scale Factor:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scaling Up: Table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scaling Up: Picture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additive</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Pose Purposeful Questions

• How did you get that?
• How do you know that?
• Can you explain your idea?
• Why?
• Can you convince us?
• Did anyone get something else?

• Can someone tell me or share with me another way?
• Do you think that means the same things?
• Is there another opinion about this?
• Why did you say that, Justin?

# Levels of Classroom Discourse

**Hufford-Ackles, Fuson & Sherin, 2014**

<table>
<thead>
<tr>
<th>Level</th>
<th>Teacher role</th>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Mathematical representations</th>
<th>Building student responsibility within the community</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>Teacher is at the front of the room and dominates conversation.</td>
<td>Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.</td>
<td>Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.</td>
<td>Representations are missing, or teacher shows them to students.</td>
<td>Culture supports students keeping ideas to themselves or just providing answers when asked.</td>
</tr>
<tr>
<td>Level 1</td>
<td>Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.</td>
<td>Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.</td>
<td>Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.</td>
<td>Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.</td>
</tr>
<tr>
<td>Level 2</td>
<td>Teacher facilitates conversation between students, and encourages students to ask questions of one another.</td>
<td>Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.</td>
<td>Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.</td>
<td>Students label their math drawings so that others are able to follow their mathematical thinking.</td>
<td>Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.</td>
</tr>
<tr>
<td>Level 3</td>
<td>Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.</td>
<td>Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse.</td>
<td>Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.</td>
<td>Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
<td>Students believe that they are math leaders and can help shape the thinking of others. They help shape others’ math thinking in supportive, collegial ways and accept the same support from others.</td>
</tr>
</tbody>
</table>
• How would you describe your current classroom discourse?
• How would your colleagues describe their current classroom discourse?
• How might you use this rubric in your setting to improve classroom discourse and increase students’ learning?
Use Pattern Tasks to Support Algebraic Reasoning

### Hexagon Trains

- **Train 1**
- **Train 2**
- **Train 3**
- **Train 4**

Write a description that could be used to compute the perimeter of any train in the pattern.

### Counting Cubes

- **Building 1**
- **Building 2**
- **Building 3**

Write a description that could be used to define any figure in the pattern.

### Zanny Zs

Write a description that could be used to define any figure in the pattern.
...all students can do something mathematical when presented with a geometric pattern. One teacher noted that regardless of your background, you can fly into the task anywhere. You can have the brightest kid in your class and the one who is struggling feel success from the first two weeks. ‘So it makes everybody feel kind they’re on kind of an even playing ground’...

Starting the Year with Pattern Tasks

Establishing Classroom Culture:

- Pattern tasks accessible to all
- Context for discussing multiple solution strategies
- Developing classroom norms and practices
  - Working in partners/groups
  - Presenting work—clarity
  - Being a good audience member—accountable for understanding work of others
  - Respect
- Basis for teacher discussion/collaboration

Smith, Hillen, Catania, MTMS, 2007
Using Pattern Tasks to Develop Mathematical Understandings and Set Classroom Norms

Margaret S. Smith, Amy F. Hillen, and Christy L. Catania
Collaborative Team Work

- An examination and prioritization of the mathematics content and mathematics practices students are to learn.
- **The development and use of common assessments to determine if students have learned the agreed-on content and related mathematical practices.**
- The use of data to drive continuous reflection and instructional decisions.
- The setting of both long-term and short-term instructional goals.
- Development of action plans to implement when students demonstrate they have or have not attained the standards.
- **Discussion, selection, and implementation of common research-informed instructional strategies and plans.**

_Principles to Actions_, pp. 103-104
Assessment

An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions and program improvement.
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Evidence should:

- Provide a window into students’ thinking;
- Help the teacher determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.
Collaborative Team Actions: Assessment Instruments and Tools

1. The team designs and implements agreed-on common assessment instruments based on high quality exam designs. The collaborative team designs all unit exams, unit quizzes, final exams, writing assignments, and projects for the course.

2. The team designs and implements agreed-on common assessment instrument scoring rubrics for each assessment in advance of the exam.

3. The team designs and implements agreed-on common scoring and grading feedback (level of specificity to the feedback) of the assessment instruments to students.
Why Common Assessments?

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

What assessment tasks would you use to assess students’ proficiency with this standard?
Why Common Assessments?

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

- Compute discount price? Total price with tax?
- Find the cost, given amount including tax and tax rate?
- Compute item cost, given both discount rate and tax rate?
A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

A 32-inch television is regularly priced at $295.00. What is the total price of the television, including sales tax, if it was purchased on sale? Fill in the blank to complete the sentence. Round your answer to the nearest cent.

The total cost of the television is $\underline{\hspace{2cm}}$. 
Write your answers to the following problem in your answer booklet.

A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

Adam and Brandi are customers discussing how the discount and tax will be calculated.

Here is Adam's process for finding the total cost for any item in the store.

- Take 10% off the original price.
- Then, add the sales tax to the discounted price.

Adam represents his process as:

\[ T = 0.9p + 0.05(0.9p) \]

\[ \text{sale price} + \text{sales tax} \]

Here is Brandi's process for finding the total cost for any item in the store.

- Determine the original price of the item, including sales tax.
- Then, take 10% off.

Brandi represents her process as:

\[ T = 1.05p - 0.10(1.05p) \]

\[ \text{T.V. price} - 10\% \text{ off plus tax} - \text{discount} \]

In both equations, \( T \) represents the total cost of the television and \( p \) represents the regular price.

Are they both correct? Use the properties of operations to justify your answer.
Adam’s Process

\[ T = 0.9p + 0.05(0.9p) \]
\[ = (0.9p)(1+ 0.05) \]
\[ = (0.9p)(1.05) \]
\[ = (0.9)(1.05)p \]
\[ = 0.945p \]

Brandi’s Process

\[ T = 1.05p - 0.10(1.05p) \]
\[ = (1.05p)(1− 0.10) \]
\[ = (1.05p)(0.9) \]
\[ = (1.05)(0.9)p \]
\[ = 0.945p \]
Amy says, “A 10% discount with 5% sales tax is the same as a 5% discount because 10% – 5% = 5%.

Is Amy correct? Use properties of operations to justify your answer.
Tasks Clarify Expectations

- Range of content
- Depth of knowledge
- Type of reasoning and evidence of it
- Types of applications
Tasks Clarify Expectations

PARCC
“[Sample tasks and tests] are designed to shine a light on important elements of the CCSS . . . “

SBAC
“The sample items and tasks illustrate the knowledge and skills students are expected to demonstrate on the Smarter Balanced assessments, giving educators clear benchmarks to inform their instruction.”
Analyze PARCC & SBAC Released Tasks/Tests

PARCC:
http://www.parcconline.org

SBAC:
http://smarterbalanced.org
The Mathematics Assessment Project (MAP) is part of the Math Design Collaborative initiated by the Bill & Melinda Gates Foundation. MAP set out to design and develop well-engineered tools for formative and summative assessment that expose students’ mathematical knowledge and reasoning, helping teachers guide them towards improvement and monitor progress. The tools are relevant to any curriculum that seeks to deepen students’ understanding of mathematical concepts and develop their ability to apply that knowledge to non-routine problems.

Formative Assessment Lessons: Classroom Challenges
100 lessons for formative assessment, some focused on developing math concepts, others on solving non-routine problems. A Brief Guide for teachers and administrators (PDF) is recommended reading before using these lessons for the first time.

Summative Assessment Tasks
A set of 94 exemplary summative assessment tasks to illustrate the range of performance goals required by CCSSM. The tasks come with scoring rubrics and examples of scored student work.

Prototype Tests
Complete summative test forms and rubrics designed to help teachers and students monitor their progress using a range of task types similar to the ‘Tasks’ section.

Professional Development Modules
5 Prototype modules that encourage groups of teachers to explore the practical and pedagogical concepts behind the MAP materials, such as formative assessment, collaborative learning and the use of unstructured problems.

The TRU Math Tools Suite
performance assessment tasks

These tasks are grade-level formative performance assessment tasks with accompanying scoring rubrics and discussion of student work samples. They are aligned to the Common Core State Standards for Mathematics. You may download and use these tasks for professional development purposes without modifying the tasks.

The tasks for grades 3 through High School were developed by the Mathematics Assessment Resource Service (MARS) of the Shell Centre for Mathematical Education, University of Nottingham, England. The tasks for grade 2 were developed by the Silicon Valley Mathematics Initiative’s Mathematics Assessment Collaborative (MAC).

The tasks below are arranged by grade level. To search for tasks by mathematical strand or to see information about alignment to specific standards, use the "shortcuts" field at the top of the page, or visit the Common Core mathematical content standards page.

2nd grade
Collaboratively analyze assessment tasks to develop common understanding of CCSSM proficiency expectations.
If You Have Common Assessments

Are they really really good?
How Can You Evaluate the Quality of Your Assessments?

On what “basis” do you determine the characteristics of a high quality unit/chapter test?
Analyzing Assessment Quality

<table>
<thead>
<tr>
<th>Assessment Indicators</th>
<th>Description of Level 1</th>
<th>Requirements of the Indicator Are Not Present</th>
<th>Limited Requirements of This Indicator Are Present</th>
<th>Substantially Meets the Requirements of the Indicator</th>
<th>Fully Achieves the Requirements of the Indicator</th>
<th>Description of Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning targets are given appropriate emphasis.</td>
<td>Too much attention on one or two targets or on less important targets; number of points does not reflect importance.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>The most important learning targets receive the most emphasis.</td>
</tr>
<tr>
<td>Balance of procedural fluency and demonstration of understanding</td>
<td>Test is not “rigor” balanced. Emphasis is on procedural knowledge and minimal cognitive demand for demonstrating understanding.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Test is balanced with product- and process-level questions. Higher cognitive demand and understanding tasks are present.</td>
</tr>
<tr>
<td>Question phrasing (precision)</td>
<td>Wording is vague or misleading. Vocabulary and precision of language is a struggle for student understanding.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Vocabulary is direct, fair and clearly understood. Students are expected to attend to precision in responses.</td>
</tr>
<tr>
<td>Format and design of assessment tasks: support valid inferences about students’ knowledge</td>
<td>Assessment contains items that may give misleading information about students’ knowledge. Calculator usage not clear.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Assessment tasks support valid inferences and may include a variety of question types and formats to do so.</td>
</tr>
<tr>
<td>Clarity of directions</td>
<td>Directions are missing or unclear.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Directions are appropriate and clear.</td>
</tr>
<tr>
<td>Visual presentation</td>
<td>Assessment instrument is sloppy, disorganized, difficult to read, and offers no room for work.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Assessment instrument is neat, organized, easy to read, and well-spaced, with room for student work and teacher feedback.</td>
</tr>
<tr>
<td>Time allotment</td>
<td>Few students can complete the assessment in the time allowed</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Test can be successfully completed in time allowed.</td>
</tr>
<tr>
<td>Format and design promotes students’ taking responsibility for their own learning.</td>
<td>Learning targets are unclear; students not expected to analyze their performance.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Learning targets are clear and connected to the assessment questions, either on the test or another sheet.</td>
</tr>
</tbody>
</table>

Analyzing Assessment Quality

• Learning targets are given appropriate emphasis.
• Balance of procedural fluency and demonstration of understanding.
• Question phrasing (precision)
• Format and design of assessment tasks support valid inferences about students’ knowledge
• Clarity of directions
• Visual presentation
• Time allotment
• Format supports students’ taking responsibility for their own learning
Analyzing An Assessment

• Work with a partner. Use the evaluation rubric to rate the quality of the Grade 7 Integer assessment on each dimension.

• What are the strengths/weaknesses of this assessment?

• What would you do to improve the assessment?

• Discuss with your table-mates
Compare Integers Test to “Properties of Integer Addition and Subtraction”

Same  Different
SMP 3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. . . . Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. . . . Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Ms. Lora is discussing properties of arithmetic with integers with students, asking them to say whether a statement is true or false and provide some reasoning to justify their conclusion.

1. For the statement "The sum of a negative integer and a positive integer is always positive." Keisha says "This is false. The sum can be positive, like 10 + -3 = 7. But, it can also be negative. For example, -9 + 3 is -6."

Did Keisha provide a correct argument to explain why the statement is false? Explain why you think so.
2. For the statement "The sum of two negative integers is always negative." Mike says, "This is true. I tried lots of examples, like -3 + -2, -10 + -27, and even ones with big numbers, like -2,000 + -5,000. All the sums were negative. So this must be true."

Has Mike provided a viable argument that the statement is true? Explain why you think so.
3. For the same statement "The sum of two negative integers is always negative." Dev says, “I agree with Mike that the statement is true, but I don’t think giving examples is good enough to prove that it is always true. I wonder if I could use the number line to show that when you add two negative numbers together, the sum is always negative?”

Is Dev’s critique of Mike’s argument correct? Explain why you think so.

How could Dev use a number line to prove that the sum of two negative integers is always negative?
4. For the statement “The difference between two negative integers is always positive.” Joey says "This is true. Just like Keisha gave an example, I see that \(-3 - -8 = -3 + 8 = 5\), so it is true."

Is Joey's argument correct? Explain why you think so.
SMP 3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Understanding a Concept

• Explain it to someone else
• Represent it in multiple ways
• Apply it to solve simple and complex problems
• Reverse givens and unknowns
• Compare and contrast it to other concepts
• Use it as the foundation for learning other concepts
Learning target:
Understanding the definition of a triangle.

Performance task:
Draw a triangle.
Circle All the Triangles
Grade 2: Equal Partitions
Grade 2: Equal Partitions
Analyzing Assessment Tasks

To what extent does the assessment:

• Provide valid information about students’ knowledge?

• Provide information about students’ conceptual understanding?

• Provide information about students’ proficiency in the standards for mathematical practice?
Common Assessment Planning Process

- Plan
- Develop
- Critique
- Administer and Analyze Students’ Performance
- Critique
- Revise
Collaborative Team Work

- An examination and prioritization of the mathematics content and mathematics practices students are to learn.
- The development and use of common assessments to determine if students have learned the agreed-on content and related mathematical practices.
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- The setting of both long-term and short-term instructional goals.
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- Discussion, selection, and implementation of common research-informed instructional strategies and plans.

Principles to Actions, pp. 103-104
Start Small, Build Momentum, Persevere

The process of creating a new cultural norm characterized by professional collaboration, openness of practice, and continual learning and improvement can begin with a single team of grade level or subject-based mathematics teachers making the commitment to collaborate on a single lesson plan.

*Principles to Actions*, p. 207
Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
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5. Assessment
6. Professionalism

Essential Elements of Effective Math Programs
Your Actions?
Specific, research-based teaching practices that are essential for a high-quality mathematics education for all students are combined with core principles to build a successful mathematics program at all levels.

*Principles to Actions* offers guidance to teachers, mathematics coaches, administrators, parents, and policymakers.

http://www.nctm.org/PtA/
Principles to Actions Resources

- *Principles to Actions* Executive Summary (in English and Spanish)
- *Principles to Actions* overview presentation
- *Principles to Actions* professional development guide (Reflection Guide)
- Mathematics Teaching Practices presentations
  - Elementary case, multiplication (Mr. Harris)
  - Middle school case, proportional reasoning (Mr. Donnelly) (in English and Spanish)
  - High school case, exponential functions (Ms. Culver)
- *Principles to Actions* Spanish translation
These grade-band specific professional learning modules are focused on the Effective Teaching Practices and Guiding Principles from *Principles to Actions: Ensuring Mathematical Success for All*.

Presentation, presenter notes, and required materials are provided in each module to support professional learning with teachers through analyzing artifacts of teaching (e.g., mathematical tasks, narrative and video cases, student work samples, vignettes) and abstracting from the specific examples general ideas about how to effectively support student learning.

The Teaching and Learning Modules were developed in collaboration with the Institute for Learning (IFL) at the University of Pittsburgh.

Learning modules are available exclusively to NCTM members. Limited open examples are provided for each grade level (denoted with *).
Effective Teaching Practices
To access module materials, use links on left column of table below.

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<tr>
<th>Establish math goals to focus learning</th>
<th>Implement tasks that promote reasoning and problem solving</th>
<th>Use and connect math representations</th>
<th>Facilitate meaningful math discourse</th>
<th>Pose purposeful Questions</th>
<th>Build procedural fluency from conceptual understanding</th>
<th>Support productive struggle in learning math</th>
<th>Elicit and use evidence of student thinking</th>
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Collaborative Team Tools

Available at nctm.org
e-Book Series

Discovering Lessons for the Common Core State Standards in K–Grade 5

Sarah B. Bush
Karen Karp

Discovering Lessons for the Common Core State Standards in Grades 9–12

Sarah B. Bush
Karen Karp
Frederick Dillon

Discovering Lessons for the Common Core State Standards in Grades 6–8

Sarah B. Bush
Karen Karp

mathematics

Teaching in the Middle School
# GRADE 6

## Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

**Now and Then: Fiber Meets Fibonacci**  
Art Johnson  
January 1999, vol. 4, no. 4, pp. 256–262  

<table>
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This article involves the examination of the golden ratio (1.6:1). Through a context of basket weaving, the background of the golden ratio is discussed. Students will work to identify the golden ratio proportion through measurement, find its unit rate on a table, apply measurements on their own bodies, and write proportional equations. This lesson connects to social studies.

**Proportional Reasoning**  
Jane Lincoln Miller and James Fey  
January 2000, vol. 5, no. 5, pp. 310–313  

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Although this article describes a study, the tasks used focus on meaningful classroom activities. The questions emphasize solving proportional equations to find an unknown and testing the equivalence of ratios. Several error patterns are discussed for the specific tasks, which would help in anticipating obstacles for students.

**Moving to Proportional Reasoning**  
Hannah Stohl  
September 2000, vol. 6, no. 1, pp. 58–60  

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The author focuses on students’ conceptual understanding of the multiplicative relationships involved in proportions. Using visual models of geometric figures, three interesting problems are posed. Since the problems purposely do not include numbers, students have an opportunity to observe the characteristics of proportional figures. These activities prepare students for the skills needed in transformational geometry.
New NCTM Tools
Activities with Rigor and Coherence (ARCs)

Sequence of 2–4 lessons that

• Support *Principles to Actions*
• Address a specific math topic
• Scaffold effective teaching
• Support the 8 SMPs
• Demonstrate the 5 Practices for Orchestrating Productive Mathematics Discussions
• Integrate the wide array of NCTM resources
Discovering Area Relationships of Polygons

Discovering the Area Formula for Triangles

6-8
In this lesson, students develop the area formula for a triangle. Students find the area of rectangles and squares, and compare them to the areas of triangles derived from the original shape.

Finding the Area of Trapezoids

6-8
Students discover the area formula for trapezoids, as well as explore alternative methods for calculating the area of a trapezoid.

Finding the Area of Parallelograms

6-8
Students will use their knowledge of rectangles to discover the area formula for parallelograms.

Finding the Area of Irregular Figures

6-8
Students will estimate the areas of highly irregular shapes and will use a process of decomposition to calculate the areas of irregular polygons.
Activities with Rigor and Coherence - ARCs

What are ARCs?

ARCs are Activities with Rigor and Coherence. Each ARC is a series of lessons that addresses a mathematical topic and demonstrates the vision of *Principles to Actions: Ensuring Mathematical Success for All*. ARCs scaffold effective teaching and support enactment of the eight Mathematics Teaching Practices articulated in *Principles to Actions* as well as the instructional guidance set forth in 5 *Practices for Orchestrating Productive Mathematics Discussions*. ARCs integrate a wide array of NCTM resources to optimize opportunities for learning, including Illuminations and Student Explorations in Mathematics. ARCs also include community features that offer opportunities for social interaction. Engage in online discussions with other math educators, post a comment, and give feedback with ratings and reviews.

Contribute now! The ARCs are still works in progress. We need your voice.
NCTM-Hunt Institute Video Series: Teaching and Learning Mathematics with the Common Core

- Enhance public understanding of what students need to know for college and career
- Why conceptual understanding requires a different approach
- Teachers, educators, leaders, and parents with classroom video
- Primarily for the public; useful to educator outreach
NCTM-Hunt Institute Video Series: Teaching and Learning Mathematics with the Common Core

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- Developing Mathematical Skills in Upper Elementary Grades
- Building Conceptual Understanding in Mathematics
- Mathematical Foundations for Success in Algebra
- Preparation for Higher Level Mathematics
- Parents: Supporting Mathematics Learning
- Standards for Mathematical Practice
NCTM and The Hunt Institute have produced a series of videos to enhance understanding of the mathematics that students need to succeed in college, life, and careers. Beginning in the primary grades, the videos address the importance of developing a solid foundation for algebra, as well as laying the groundwork for calculus and other postsecondary mathematics coursework. The series also covers the Standards for Mathematical Practice elaborated in the Common Core State Standards for Mathematics and examines why developing conceptual understanding requires a different approach to teaching and learning.

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Diane J Briars
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