Reasoning Algebraically about Functions

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What does it mean to reason algebraically?
Core Algebraic Thinking Practices

- Generalizing relationships
  (between co-varying quantities)

- Representing these relationships
  (in words, variables, tables, and graphs)

- Justifying relationships
  (from the problem context)

- Reasoning with these relationships
  (to predict function behavior)

(Blanton, Levi, Dougherty, & Crites, 2011; Kaput, 2008)
In the context of “algebra readiness”:

What do you wish elementary students knew as they entered middle grades to be prepared to reason algebraically with functions?
What do your students find most difficult about functions?
Growing Train

There was a train that ran the same route everyday. As it went along, it picked up two train cars at each stop.

1. If we don’t count the engine, how many train cars would the train have at stop 1? How many would it have at stop 2? How many would it have at stop 3?

2. Organize your information in a function table. Write an equation that shows the relationship between the values in your table for each set of values.

3. Find a relationship between the number of stops and the total number of cars on the train. Represent your rule in words and letters (variable notation).

4. If we count the engine, how would this affect your function table? How would this affect your rule?
How would students in your class solve this task?

What knowledge do you wish they would bring to this task...
  - about functional relationships?
  - about representing relationships with variables?
  - about reasoning with relationships?
VIDEO: One Student’s Thinking about Growing Train

- Research project designed to explore cognitive foundations of students’ functional thinking
- 8 week classroom teaching experiment
- Two 30-45 minute lessons each week (16 lessons total)
- Growing Train was the task for the post-interview
VIDEO: One Student’s Thinking about Growing Train

Think about

- the kind of relationship she noticed and what supported her noticing this;
- use of variable and how she represented the relationship;
- how she reasoned with the relationship in a new situation (counting the engine).
- (Students’ written work)
the kind of relationship she noticed and what supported her noticing:

- Correspondence rule (not recursive pattern)
- Equations showing a relationship between specific values allowed her to think about structure across the set of equations and generalize the rule (not “guess” a rule)

use of variable and how she represented the relationship:

- Variable as representing quantity, not object; choice of letter arbitrary
- Relationship was represented by an equation, not expression

how she reasoned with the relationship in a new situation (counting the engine):

- Reasoned with her function rule as object to create new rule (without constructing a new table)
How is this student’s thinking similar to or different from what you see with middle grades students?
Is this an anomaly?
Brady’s Birthday Problem

Brady is celebrating his birthday at school. He wants to make sure he has a seat for everyone. He has square desks. He can seat 2 people at one square table in the following way:

If he joins another square table to the first one, he can seat 4 people:

If he joins another desk to the second one, he can seat 6 people:

a) Fill in the table below to show how many people Brady can seat at different numbers of desks.

b) Do you see any patterns in the table? If so, describe them.

c) Think about the relationship between the number of desks and the number of people. Use words to write the rule that describes this relationship.

d) Use variables (letters) to write the rule that describes this relationship.

e) If Brady has 100 desks, how many people can he seat? Show how you got your answer.
How successful are 3rd-grade students in comparison to 6th and 7th grade students in generalizing the relationship and representing it in words or variables?
Some context...

From grade 3:
- 105 3rd-grade students, six experimental classrooms in a school in the North East (US)
- Early algebra intervention: 18 one-hour early algebra lessons throughout the year
- One-hour pre/post written assessment containing the Brady Problem

From grades 6-7
- 200 6th- and 7th-grade students in a middle school located in the same district
- One-hour written assessment containing the Brady Problem (and other items common to the grade 3 assessment)
Writing function rule in words: Correctness

- Third-grade Pre: 4%
- Third-grade Post: 46%
- Sixth-grade: 48%
- Seventh-grade: 26%
Writing function rule in variables: Correctness

- Third-grade Pre: 0%
- Third-grade Post: 38%
- Sixth-grade: 35%
- Seventh-grade: 48%
This suggests that

- elementary-grades children can **generalize and represent functional relationships** as well as middle school students!

- our six-year-old student is not an anomaly.
What else do we know about whether elementary grades children are capable of reasoning algebraically about functions?

Results for the Brady Problem from our Grades 3-4, two-year early algebra intervention
Brady Problem: Using variables or words to represent a functional relationship

- Intervention
- Comparison

Gr 3 Pre
Gr 3 Post
Gr 4

Words
Variables
In Year 2 of our 3-year intervention, 4<sup>th</sup>-grade students* were significantly better able to write a function rule in variables characterizing a relationship in data displayed in a table

(60% correct for experimental, 0% for control)

*students in the lowest two groupings ("Needs Improvement" and "Warning") as measured by state standardized assessments.
So what?

- Reasoning algebraically (with functions) is NOT just for “higher performing” students.
- Third-grade students can perform as well as middle grade students in traditional settings.
- Children as young as ages 5-6 can make sense of functional relationships: generalizing these relationships, representing them with variable notation and in their own words, and reasoning with them to explore novel situations.
Imagine how prepared students might be to reason algebraically about functions in middle grades if they had the experiences described here using a sustained, longitudinal approach across elementary grades.
Questions?

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