Solving Systems of Equations Using Multiple Strategies

NCTM Interactive Institute, 2016

Name
Title/Position
Affiliation
Email Address
Common Core Standards

This session will address the following:

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
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<tbody>
<tr>
<td>8.EE.8a</td>
<td>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
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<tr>
<td>8.EE.8b</td>
<td>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</td>
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<tr>
<td>8.EE.8c</td>
<td>Solve real-world and mathematical problems leading to two linear equations in two variables.</td>
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Warm Up

Find two different equations that are satisfied by the point (5, -2). Describe the relationship of the graphs of your equations.
Warm Up Discussion

• What strategies did you use to find two equations that satisfy the point (5, -2)?
• What are some other strategies we could have used?
Going Bananas!

Consider... 

A banana peel weighs one-eighth the total weight of a banana. If an unpeeled banana balances a peeled banana of the same weight plus seven-eighths of an ounce, how much does a banana weigh with peel?
Going Bananas

What did you have to understand in order to solve this problem?
Consider this student diagram...

Chicken Problem

Fig. 1. The Chicken Problem

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Chicken Problem Solutions

What did you notice about students’ solutions?

Fig. 2. A solution to the Chicken Problem that does not use variables

Fig. 3. A solution to the Chicken Problem that uses variables
What strategies can be used to solve systems of equations?
Strategies

• Elimination
• Substitution
• Graphing
• Calculator
• Manipulatives
Consider

When is one strategy “better” than another?
Sequencing Instruction

Think about the instructional sequence you use in teaching how to solve systems of equations.

What do students do first in the lessons?

How do the lessons change over time?
In groups, you will use the meter stick and rope to complete the Knotty Problem.
Knotty Problem

• What observations and patterns have you noticed?

• What does the y-intercept represent?

• What does the slope represent?
Knotty Problem - extension

• Pair up with another group, and exchange “rules”.
• Make a prediction:

  Assuming each group started with equal lengths of rope, how many knots could you make in your rope so that it’s as close as possible to the same length as the rope from your partnering group with three knots? Use, pictures, words, and symbols to describe your solution.
How much does a fish weigh if its tail weighs 4 kilograms, its head weighs as much as its tail and half its body, and its body weighs as much as its head and tail together?
Fish Problem

• What were your initial thoughts about a strategy to solve the Fish Problem?

• Was this strategy useful?

• What other strategies could you have used?
Reflection

Build procedural fluency from conceptual understanding
Teacher and student actions

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<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
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<tr>
<td>Providing students with opportunities to use their own reasoning strategies and methods for solving problems.</td>
<td>Making sure that they understand and can explain the mathematical basis for the procedures that they are using.</td>
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<tr>
<td>Asking students to discuss and explain why the procedures that they are using work to solve particular problems.</td>
<td>Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.</td>
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<tr>
<td>Connecting student-generated strategies and methods to more efficient procedures as appropriate.</td>
<td>Determining whether specific approaches generalize to a broad class of problems.</td>
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*(Principles to Actions: Ensuring Mathematical Success for All [NCTM 2014], p. 47)*
### Build procedural fluency from conceptual understanding

Teacher and student actions, continued

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<td>Using visual models to support students’ understanding of general methods.</td>
<td>Striving to use procedures appropriately and efficiently.</td>
</tr>
<tr>
<td>Providing students with opportunities for distributed practice of procedures.</td>
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(*Principles to Actions: Ensuring Mathematical Success for All [NCTM 2014], p. 48*)
How were you engaged in the Mathematical Practices?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- Banana Problem
- Chicken Problem
- Knotty Problem
- Fish Problem
Exit Ticket: Reflection

• What new idea(s) do you want to implement into your classroom?

• What challenges did you encounter during this session?
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