Base Ten and Place Value
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Workshop Goals

Participants will ---

– Represent and solve a problem (Activity 1) requiring an understanding of counting and place value (Essential Understandings 1, 2, and 3).
Goal 1

Participants will –
– Discuss and reflect on pedagogy necessary to teach counting, comparing, and representing numbers (Essential Understandings ONE and TWO):

• Identify counting and sequencing criteria (Activity 2)
• Match counting and comparing standards to activities appropriate in Prek – 2nd grade (Activity 3)
• Use number counting and comparing vocabulary to solve a color tile clue problem (Activity 4)
Goal 2

Participants will –

- Discuss and reflect on pedagogy necessary to teach place value concepts. (Essential Understandings ONE and THREE):
  - Play two different Place value games or activities. Identify the place value concepts that could be addressed (Activity 5)
  - Use a variety of strategies to assess students’ concepts of place value (Activity 6)
An understanding of number involves integrating several key concepts, such as unit, place value, and one-to-one correspondence... Ideas such as unit and decomposition that children encounter in prekindergarten through grade 2 set the stage for their development of fluent use of place value, leading to number flexibility and computational proficiency.

(Dougherty, Flores, Louis, Sophian, 2010, p. 41)
Characteristics of the Numeration System
(Adapted from Reys, Lindquist, Lambdin, Smith, & Suydam, 2004)

• **Place value:** The position of the digit represents its value: for example, the 2 in 23 names “twenty” and has a different mathematical meaning from the 2 in 32 which names “two.”

• **Base of ten:** The term base simply means a collection. Thus in our system, 10 is the value that determines a new collection, and the system has 10 digits, 0 through 9.

• **Use of zero:** A symbol for zero exists and allows us to represent symbolically the absence of something. For example, 309 shows the absence of tens in a number containing hundreds and ones.

• **Additive property:** Numbers can be summed with respect to place value. For example, 123 names the number that is the sum of 100 + 20 + 3.
Place Value Development

• **Counting by Ones**
  – Physical item worth a count of “one more”
  – Each item is a unit of one.
  – Items being counted compose the total.
  – Number word stand for its numerical value to represent the entire quantity.
  – Example: *One, Two, Three, Four...Four blocks*

• **Counting by Groups and Singles**
  – A collection of items can be thought of as a group.
  – Groups can contain different quantity of items.
  – Example: *One, two, three groups of two, and one, two, three, singles*
Place Value Development

- **Counting by Tens and Ones**
  - “Ten” is used as a unit.
  - Count by tens and ones: 10, 20, 30, 40, 41, 42, 43
  - Tens and Ones are dealt with separately.
  - Example: *When I add 15 blocks with 17 blocks, I have two tens and 12 ones* 

- **Equivalent Groupings**
  - “Ten” is used as a unit.
  - Tens and ones are dealt with simultaneously
  - Mental flexibility permits crossing between thinking of tens and ones
  - Example: *Three tens and 13 ones is also equal to 43.*
It is critical that students see the relationship between number names such as “fifty-three” with the grouping of tens concept. They must also see that the way we write numbers (ones on the right, tens on the left of ones and so on) must be coordinated with the idea of groupings.

Activity 1

Positional System and Representations of Place Value
Models for Place Value

• Physical models can help children develop the idea of a “ten” as both a single entity and a set of ten units.
• The models do not “show” the concept to the children.
• Children must construct the concept, and impose it on the model.
Quick Write

• Identify the tools that students in your classroom use during study of place value
Groupable and Pre-grouped

• Groupable models most clearly reflect the relationship of ones and tens
  – The ten can actually be made by grouping the singles. When children stack 10 cubes the tower of ten is literally the same as the ten ones from which it was made.
  – These can be called “put-together/ take-apart” models.

• Pre-grouped models are trading/exchanging models.
  – Pre-grouped models are introduced when children need to represent hundreds.
  – Children cannot actually take them apart or put them together.
  – When 10 single pieces are accumulated they must be exchanged, regrouped or traded, for a ten, ten tens must also be traded for a hundred.

Students should have exposure to groupable models first to see the one to one relationship
Proportional and Non-Proportional

- Proportional models help to demonstrate an exact size ratio between values.
  - The material for 10 is ten times the size of 1; 100 is ten times the size of 10.

- Non-proportional models lack an exact ratio between the quantity presented and size of tool used.
  - Within money, ten pennies are not the same size as one dime.
**Proportional** – The material for 10 is ten times the size of 1; 100 is ten times the size of 10.

*Ex: base ten blocks, bean sticks, bundled sticks*

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**Nonproportional** – The material does not maintain any size relationships.

*Ex: money, abacus, color tiles or chips*
Base 10 Models

• Base 10 models are considered both groupable and proportional

• These models support students in experiencing the relationship between places by
  – Building quantities
  – Regrouping to see the relative magnitude of places
  – Supporting regrouping during addition and subtraction operations
Straws, Bundles and Bags

- Frequently items such as straws or popsicle sticks are used to model place value concepts.
- These models are considered groupable and proportional.
- It is important for students to engage in the task of putting together and taking these models apart to explore the relationship between places.
Can you model...

• With the materials at your table, create representations of the following numbers. Try to use at least two different materials (1 groupable, 1 pre-grouped)
  • 17
  • 45
  • 89
  • 123
Activity 2

Composing, Decomposing, & Renaming Numbers
Representing Numbers

• In order to develop children’s understanding of number, it is important that we ensure they have multiple opportunities to experience and express numbers in a variety of ways.
  – The Place Value Triangle: Experiences in “saying, writing and building” numbers
  – Renaming Numbers: Experiences in representing the same value in multiple ways
Place Value Triangle:
Connecting Concepts with Oral and Written Forms

Base-Ten Concepts
Standard and equivalent groupings meaningfully used to represent quantities

Counting
• By ones
• By groups and singles
• By tens and ones

Oral Names
• Standard: Thirty-two
• Base-Ten: Three tens and two

Written Names
32
1. **Make It!** Draw 2 cards and place them on the sentence strip.

1. **Build It!** Use the Base 10 blocks to represent your number.

1. **Say It!** Read your number.

Extension: Include question cards that ask about:
- Digit
- Value
- Place
1. Play with a partner!
2. Each player will **Make It! Build It! Say It!**
3. Then one student will **Compare It!** This student will place a symbol between the numbers to make a true expression!
• We would like children to develop the idea that there are many equivalent (equal) names for one number. Because arithmetic, for the most part, involves replacing numbers or expressions with equivalent numbers or expressions (we replace $5 \square + 7$ with $12$ or $\square \square \frac{1}{2}$ with $\frac{1}{4} + \frac{1}{4}$), this is an important concept for children to understand.
Representations of 23

- 23 ones
- 1 ten and 13 ones
- 2 tens and 3 ones
Here are two ways to show 42. How many ways can you find?
Decomposing Numbers

Here is one way to decompose 85. How many ways can you find?

85

50  30  2  3
Name-Collection Boxes
(from UCSMP, 2008)

- Name-collection boxes feature a label attached to a box with an open space for writing. The idea is to fill the box with different names for the number on the label. Numbers can be named using one or more operations (addition, subtraction, multiplication, and division), words in any language, tally marks, arrays, Roman numerals, and so on.
Name-Collection Boxes
(from UCSMP, 2008)

This is a name collection box for the number 8.

Inside the box are many different ways to name "eight."

You can use pictures, numbers and words in a name collection box.

Some names are addition expressions, some are subtraction, one is a tally, and one is an array.

This box even has the Spanish name for eight. Can you find it?

There are many ways to show the number 8.
Number of the Day

• Write as many mathematical expressions as you can that equal 48.

• Share some expressions

• What mathematical ideas/relationships can you see within pairs of expressions?
Number of the Day

• Number of the Day (NOTD) is another activity that can be used to provide opportunity for children to represent numbers in a variety of ways.

• NOTD involves asking students to generate mathematical expressions that equal a given number. The advantage of conducting NOTD as a whole-group activity is that it provides opportunities for teachers and students to talk about important mathematical concepts such as equivalent fractions, the commutative property, use of parenthesis, and multiplying with zero.
Activity 3

Skip Counting and 100’s Boards
Skip Counting

• Skip counting concepts support young learners in discerning a pattern within our counting system.

• It can support later operation strategies such as “make a ten” or “count on”.

• Initial skip counting may begin at 0, but as students become proficient, counting should begin at any number.

• Typical skip counting patterns include by 2’s, 5’s and 10’s.
Skip Counting on a 100’s Chart

Using a Number Line to record the counts
100 Dots

• Use a 10x 10 array of dots.
• Cover a certain number of rows.
• Ask children to count how many rows of ten are visible.
• Ask children to give the name for that many rows of ten.
• Continue to slide the cover up and down the array, asking how many tens and the name for that many.
Counting Rows of Ten

How many tens? 2
Two tens is called…
*twenty.*

How many tens? 4
Four tens is called…
*forty*
### Counting Tens and Ones

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The diagram illustrates counting tens and ones, with the tens represented by large dots and the ones by smaller dots.
Patterns on 100’s Board

- Begin with an empty 10x10 grid.
- How many individual squares are on the grid?
- Model moving one more, one less, ten more, ten less, etc.
Pairs of Squares

- Give each child a 10-by-10 grid of one hundred squares.
- Ask children to tell how many squares are on the grid and to explain their thinking.
Pairs of Squares

- Introduce how to count from one square to another, in each different direction.

  one more
Pairs of Squares

- Introduce how to count from one square to another, in each different direction.

one less
Pairs of Squares

• Introduce how to count from one square to another, in each different direction.

ten more
Pairs of Squares

• Introduce how to count from one square to another, in each different direction.

  ten more
• Introduce how to count from one square to another, in each different direction.
More and Less on a 100’s Chart
Beyond 100

• Children may work through similar experiences using alternatives to the 100 board such as the 200, 300 etc. board.

• These boards follow the same layout of the 100 board, but extend place value concepts beyond 100, up to the number you are working in.
Activity 4

Rounding and Estimating in Base 10
Estimation Using Magnitude

Figure 4
Estimation
Using Magnitude
Estimation
The Closest Ten

A great visual way to explore estimation.
• Find the closest tens to an identified number
  – Shade in the ten that is in the tens digit.
  – Shade in ten more.
  – Which ten is closest?

• What ten is closest to 17?
Estimation

The Vertical Number Line

- Great Visual
- Easy Transition
  - Closest 10
- Aligns with Computation Norms
Activity 5

Place Value Games
Games and Books for Place Value

• Select and play a place value game or activity with others at your table.
  • Grouping and Grazing
  • Clear the Board
  • 100s Chart Cover-Up
  • Two Dashes and a Throw Away
  • Books and Place Value

• Answer the questions on the handout.
Activity 6

Assessing Place Value
Activity Six: Assessments of Place Value Understanding

- Review children’s responses to the place value problems –
  - What does the child understand about place value?
  - What does the child still need to know?
  - What activities/games would be helpful to increase his/her understanding?
Problem: Yesterday we wrote the numeral “39” on our “days in school” chart. How would we write today’s number?

Answer: “410” “forty ten”
Assessment TWO

• **Problem:** (Show the numeral 42. Point to the digit 4) Use your counters to show how many this is. (Point to the digit 2) Use your counters to show how many this is.

• **Answer:**
Assessment THREE

• **Problem:** How many ways can you show 125 using place value blocks?

• **Answer:** There are only 5 ways.
Assessment FOUR

• Problem: (Place 36 tiles on the table) How many groups of ten can you make? How many are left over? How many would you have if I gave you ten more tiles? How could you find out?

• Answer: Student made groups correctly. She counted out ten more to answer the question, and correctly said 46.
Assessment FIVE

• Problem: Find 37 on the hundreds chart. How could you count up from 37 to 58?

• Answer: Begin with 37, then say... 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58.
Would your students understand this comic?
Where did you see the Mathematics Teaching Practices in these ideas about addition and subtraction?

<table>
<thead>
<tr>
<th>Mathematics Teaching Practices</th>
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<tbody>
<tr>
<td><strong>Establish mathematics goals to focus learning.</strong> Effective teaching of mathematics</td>
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<tr>
<td>establishes clear goals for the mathematics that students are learning, situates goals within</td>
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<td>learning progressions, and uses the goals to guide instructional decisions.</td>
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<td><strong>Implement tasks that promote reasoning and problem solving.</strong> Effective teaching of</td>
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<tr>
<td>mathematics engages students in solving and discussing tasks that promote mathematical</td>
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<td>reasoning and problem solving and allow multiple entry points and varied solution strategies.</td>
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<td><strong>Use and connect mathematical representations.</strong> Effective teaching of mathematics engages</td>
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<td>students in making connections among mathematical representations to deepen understanding of</td>
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<td>mathematics concepts and procedures and as tools for problem solving.</td>
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<td><strong>Facilitate meaningful mathematical discourse.</strong> Effective teaching of mathematics facilitates</td>
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<td>discourse among students to build shared understanding of mathematical ideas by analyzing and</td>
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<td>comparing student approaches and arguments.</td>
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<td><strong>Pose purposeful questions.</strong> Effective teaching of mathematics uses purposeful questions</td>
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<td>to assess and advance students' reasoning and sense making about important mathematical ideas</td>
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<td>and relationships.</td>
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<td><strong>Build procedural fluency from conceptual understanding.</strong> Effective teaching of mathematics</td>
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<td>builds fluency with procedures on a foundation of conceptual understanding so that students,</td>
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<td>over time, become skillful in using procedures flexibly as they solve contextual and</td>
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<td>mathematical problems.</td>
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<tr>
<td><strong>Support productive struggle in learning mathematics.</strong> Effective teaching of mathematics</td>
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<tr>
<td>consistently provides students, individually and collectively, with opportunities and supports</td>
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<td>to engage in productive struggle as they grapple with mathematical ideas and relationships.</td>
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<tr>
<td><strong>Elicit and use evidence of student thinking.</strong> Effective teaching of mathematics uses</td>
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<tr>
<td>evidence of student thinking to assess progress toward mathematical understanding and to adjust</td>
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<tr>
<td>instruction continually in ways that support and extend learning.</td>
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Plan of Action

Something that squares with your beliefs

Something going ‘round in your head

Three things you will try and when:
1.
2.
3.
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