Counting and Cardinality
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Friendly Facts

• Write three facts about yourself on a white index card. Each fact must contain a number. When you write the fact, leave out the number.
• Write the three numbers for your facts IN ORDER on a blue index card.
• Do NOT write your name on the white or blue cards.
• Place your white and blue cards in the middle of the table.
• Mix them up.
Friendly Facts Match

• Match the blue cards with the correct white cards

• How were the numbers used in each fact?
• Number sense can be described as a good intuition about numbers. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

(Howden, 1989)
Three Big Number Ideas

• Number Meaning
• Number Relationships
• Number Magnitude
**Number Meaning**

<table>
<thead>
<tr>
<th>Cardinal</th>
<th>Ordinal</th>
<th>Nominal</th>
</tr>
</thead>
</table>
| • How much?  
• How many?  
• What part of? | • In what order? | • Label  
• Number used as name |
Number Relationships

- Decomposing and recomposing numbers
- Ideas about less than, greater than, in between
- Subitizing and noticing patterns
Understanding that numbers are neither large nor small unless you have a context in which to understand them

Operation Sense
Understanding the relative effect of operating on numbers
What Number Ideas Did You Hear?

“Papa, you’re almost at the end! Did you start at 1?”
Workshop Goals

Participants will ---

– Represent and solve a problem (Activity 1) requiring an understanding of number (Essential Understandings 1, 2, and 3).
Essential Understanding #1

Number is an extension of more basic ideas about relationships between quantities.

• Quantities can be compared without assigning numerical values to them.
• Physical objects are not in themselves quantities. All quantitative comparisons involve selecting particular attributes of objects or materials to compare.
• The relation between one quantity and another quantity can be an equality or inequality relation.
• Two important properties of equality and order relations are conservation and transitivity.
• The equality relation between two quantities remains unchanged when one or both quantities are decomposed into parts and when one of the quantities is combined with another quantity to form a larger quantity.
Essential Understanding #2

The selection of a unit makes it possible to use numbers in comparing quantities.

• Using numbers to describe relationships between or among quantities depends on identifying a unit.

• The size of a unit determines the number of times that it must be iterated to count or measure a quantity.

• Quantities represented by numbers can be decomposed (or composed) into part-whole relationships.
Essential Understanding #3

*Meaningful counting integrates different aspects of number and sets, such as sequence, order, one-to-one correspondence, ordinality, and cardinality.*

- The number-word sequence, combined with the order inherent in the natural numbers, can be used as a foundation for counting.
- Counting includes one-to-one correspondence, regardless of the kind of objects in the set and the order in which they are counted.
- Counting includes cardinality and ordinality of sets of objects.
- Counting strategies are based on order and hierarchical inclusion of numbers.
Workshop Goals

Participants will –

– Discuss and reflect on pedagogy necessary to teach counting, comparing, and representing numbers (Essential Understandings ONE and TWO):

  • Identify counting and sequencing criteria (Activity 2)
  • Match counting and comparing standards to activities appropriate in Prek – 2nd grade (Activity 3)
  • Use number counting and comparing vocabulary to solve a color tile clue problem (Activity 4)
Goal ONE

Participants will ---

– **Represent and solve** a problem (Activity 1) requiring an understanding of number (Essential Understandings 1, 2, and 3).
Activity ONE: Problem

Find out the current population of Denver. If 50% of Denver’s population lined up one behind another, how far would that line stretch?

What points of interest in the United States would the line reach?

Use the tools available to you to discover your answer(s).

Be ready to share your process with the group.
Discussion Questions

• How did we address –
  – Essential Understanding One?
  – Essential Understanding Two?
  – Essential Understanding Three?

• How did our solution show that we understood the concepts of number, counting, and decomposing and composing number?
Goal TWO

Participants will –

– Discuss and reflect on pedagogy necessary to teach counting, comparing, and representing numbers (Essential Understandings ONE and TWO):
  • Identify pre-number, counting and comparing concepts (Activity 2),
  • Match counting and comparing standards to activities appropriate in Prek – 2nd grade (Activity 3), and
  • Use number counting and comparing vocabulary to solve color tile problems (Activity 4).
Prenumber Concepts

• Classification
  – Done with or without numbers
  – Helps children make some sense of things around them and become flexible thinkers
  – Helps children identify what is to be counted

(Reys, Lindquist, Lambdin, & Smith, 2006)
Prenumber Concepts

• Patterns
  – Mathematics is the study of patterns.
  – Exploring patterns requires active mental involvement and often physical involvement.
  – In early grades, patterns help children develop number sense, ordering, counting, and sequencing.
  – In later grades, it helps children develop thinking strategies for basic facts and algebraic thinking.

(Reys, Lindquist, Lambdin, & Smith, 2006)
• **Repeating Patterns** → → Emphasis is on the cyclical nature of the repetition and the identification of the elements in the cycle.
  (e.g., blue, red, blue, red, blue, red, two elements (blue, red) that repeat)

• **Growing Patterns** → → Show an arithmetic change between pairs of elements in the pattern or a progression from one step to the next.
  (e.g., 2, 4, 6 where each pair differs by 2)
Counting with Patterns
Prenumber Concepts

• Comparisons
  – Leads to one-to-one correspondence
  – Helps children become aware of relationships such as *more than, less (fewer) than and as many as*.  
    (Reys, Lindquist, Lambdin, & Smith, 2006)
  – The concept of more is understood first.
  – The concept of less is more difficult for children to understand because children have more opportunities to use the word more.
  – To help children, pair less with the word more often.

MathWorks/MAEF, 2004
Who has more cups? Bonnie or Sammy?

Bonnie’s cups:

Sammy’s cups:
How about this?
When making comparisons, **students must be able to discriminate between important and irrelevant attributes.** For instance, with who has more cups – Bonnie or Sammy? the cups are very different: their sizes, shapes, colors, contents, etc. Still, the procedure for setting up a correspondence is the same.
• Children need to become familiar with descriptions of relationships such as more than, less (fewer) than, and as many as.
  – Bonnie has fewer cups than Sammy.
  – Sammy has more cups than Bonnie.
  – Bonnie has one less cup than Sammy.
  – Sammy has one more cup than Bonnie.

• Note that the child at the prenumerical stage has not yet developed the number concept to know that 6 is one more than 5, there are 5 ones in one 5 (lack of quantitative reasoning).
Activity TWO: More or Less Game
Three Models for Making Comparisons

1. Counting
2. Physically comparing without counting
3. One-to-one
Prenumber Concepts

• **Subitizing** – Recognizing the Group

The skill to “instantly see how many” in a group is called *subitizing*, from the Latin word meaning “suddenly.” Sight recognition of quantities up to five or six is important for several reasons:

– It saves time.
– It is a forerunner of some powerful number ideas.
– It helps develop more sophisticated counting skills.
– It accelerates the development of addition and subtraction.

(Reys, Lindquist, Lambdin, & Smith, 2006)
Domino Flash
(Spatial Subitizing)
Experiences that help develop understanding of number concepts and relationships

- Instant recognition of patterned sets (dot arrangements, dot cubes, or dominos)
- 5 as a benchmark (5-frames)
- 10 as a benchmark (10 frames)
- 1 or 2 more, 1 or 2 less (dot patterns, flashing dots, domino trains)
- Part-part-total (shake and spill with bi-colored counters, part-part-total mats)

MathWorks/MAEF, 2004
Prenumber Concepts

• Conservation
  – A given number does not vary when rearranged

(Reys, Lindquist, Lambdin, & Smith, 2006)
Counting

• To count successfully, a learner must...
  • Know the verbal sequence
  • Demonstrate one–to–one correspondence
  • Keep track
  • Say the last number to answer “how many” (cardinality)
Examples of Beginning Counting...

“One....two...three...four...
You’re gonna be in BIG trouble!”

“Ten....teen...teen....teen....teen..... twenty!”
“PreKindergarten Counting” Video

https://www.youtube.com/watch?v=bw8gEvh1rYs

• What do you observe that she understands?

• What does she not understand about counting?

• How did the parent resolve this?
For children to orally count to 100, they need to know:

- the single-digit sequence 1 – 9
- that 9 signals a transition
- the decade vocabulary
- each decade term is combined with the single-digit sequence 1 – 9
- there is an exception in the counting term between 10 and 20
# Counting Principles

<table>
<thead>
<tr>
<th>Principle</th>
<th>The child understands that:</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-One Principle</td>
<td>Each item is named with a distinct “tag”.</td>
</tr>
<tr>
<td>Stable Order Principle</td>
<td>The items are tagged in a stable, repeatable order.</td>
</tr>
<tr>
<td>Cardinal Principle</td>
<td>The final tag represents a property of the set as a whole.</td>
</tr>
<tr>
<td>Abstract Principle</td>
<td>These principles apply to any array or collection.</td>
</tr>
<tr>
<td>Order Relevance Principle</td>
<td>The same cardinal number results regardless of the order of enumeration.</td>
</tr>
</tbody>
</table>
Counting Strategies

• **Counting ‘What is there’:** Children count groups of objects and answer the “How many?” question.

• **Counting Out:** Children count out a specified number of objects from a larger set of objects.

• **Counting All:** When asked to make and count two groups of objects, children recount all the objects in both groups.
Counting Strategies

• **Counting On**: the child gives number names as counting proceeds and can start at any number and begin counting.

• **Counting Back**: when counting back, children give correct number names as they count backward from a particular point.

• **Skip Counting**: In skip counting, the child gives correct names, but instead of counting by ones, counts by twos, fives, tens, or other values. The starting point and direction are optional.
Relationships with Numbers

• Anchoring Numbers to 5 and 10
• Part-part-Whole Relationships
• Numbers 10 through 20
Counting with Models

ten-frame
Counting on a Ten Frame
Counting on the Hundreds Board

Start at 34. Count by ones to 78. How many numbers did you say?
Counting on the Hundreds Board

Start at 6. Count by tens to 86. How many numbers did you say?
Find That Number!

Pick a number and count 10 more. Where did you stop?

Pick a number and count 20 more. Where did you stop?

Pick a number and count 50 more. Predict where you would stop.

Tell how you can use the hundred chart to mentally add 30 to a number.

Here is only a part of a hundred chart:

Use what you know about a hundred chart to find the values:
A _____  B _____  C _____  D _____

Tell how you found C.
Could you find C in more than one way? Explain your answer.
Activity THREE: Counting and Comparing Centers

- Okta’s Rescue from *Illuminations*
- Numeral Comparing Game
- Counting and Data
- Days in School
- Bean Out
- Dominoes
- Before and After
Center Assignments

• Table 1: Numeral Comparing Game
• Table 2: Days in School
• Table 3: Bean Out
• Table 4: Domino Activities
• Table 5: Before & After

Do your assigned table activity and at least one other activity. Using poster paper, answer the three questions on the next slide.
Center activity questions

• What standard does the activity address?

• What does a child need to know mathematically to do the activity?

• What evidence will you note that shows a child is learning the mathematics in the activity?
Days in School

Connecting the representation of 100 to literacy and the word wall
Activity FOUR: Color Tile Riddles

- I have 9 color tiles and they are connected to make one filled square.
- I did NOT use any yellow tiles.
- The top row contains 1 green tile and 2 red tiles.
- The bottom row includes twice as many green tiles as the top row and half of the red tiles.
- The middle row has tiles that are all the same color. These tiles are NOT colored like the bottom and top row.
- In the whole square there are the same number of blue, red, and green tiles.
- Except for the blue tiles, like-colored tiles are NOT touching.
Plan of Action

Something that squares with your beliefs

Something going ‘round in your head

Three things you will try and when:

1.
2.
3.
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