Addition and Subtraction Workshop

Courtney Baker  
George Mason University  
cbaker@gmu.edu

Jennifer Ward  
University of South Florida  
jenniferward@usf.edu
Bellwork

• Use two post it notes.
  – On one: Create a word problem for addition.
  – On the other: Create a word problem for subtraction.

• Place your word problems off to the side- we will be using them in an activity later in the training!
MORE or fewer than 100?
What is the exact number? How can it be represented?

107 squares

$28 + 26 + 27 + 26$

$100 + 7$
Workshop Goals

Participants will ---

– Represent and solve problems requiring addition and subtraction (Activity 1).

– Discuss and reflect on pedagogy necessary to teach BIG idea ONE:
  • Discuss and identify representation models for addition and subtraction (Activity 1 & 2)
  • Match and design a variety of addition and subtraction problems (Activity 3)
Workshop Goals

Participants will ---

– Play a computation game and analyze possible fluency outcomes (Activity 3)

– Discuss and reflect on pedagogy necessary to teach BIG idea TWO:
  • Analyze true/false equations illustrating understanding of place value and properties (Activity 4)
  • Explain a variety of students’ addition and subtraction computation methods (Activity 5)
Essential Understanding ONE:

• Addition and subtraction are used to **represent** and **solve** many different kinds of problems.
  – Sequential counting
  – Inverse relationship of addition and subtraction
  – Part, part, whole relationship
  – Number sentence representation
  – Context for problems and different representations
Activity ONE: Representation Models for Addition and Subtraction

• What models can be used to represent and solve addition and subtraction problems?
  – We will pose a story problem with a progression of number choices. For each number choice, consider one or more of the following as a way to represent your solution to the problem:
    • counters
    • 5/10 frames
    • Rekenrek
    • base 10 blocks
    • 100 board
    • open number line
Story Problem 1

Carl has ____ blocks. Rita gives him ______ more blocks. How many blocks does Carl have now?

(2, 3)  (7, 4)  (6, 7)  (12, 10)  (20, 23)  (44, 38)
Story Problem 2

______ penguins were standing on the iceberg.
______ jumped into the water to swim. How many penguins are left on the iceberg?

(5, 4)  (15, 7)  (30, 20)  (50, 18)  (86, 26)  (83, 27)
Story Problem 3

• Tarah has ____ melon balls. Kyrie has ____. How many more melon balls does Tarah have?
  (5, 4)  (15, 7)  (30, 20)  (50, 18)  (86, 26)  (83, 27)
Story Problem 4

- Alicia counted ___ squirrels in the park on Tuesday. On Wednesday, she counted ___ squirrels. How many squirrels did she count on Tuesday and Wednesday?

(2, 3) (7, 4) (6, 7) (12, 10) (20, 23) (44, 38)
So... What Standards Could Be Addressed?

• **Content Standards** —
  - PreK  Decomposing 5 into two parts
  - K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions or equations.
  - 1.NBT.C.4. Add within 100, including...adding a two-digit number and a multiple of 10, using concrete models or drawings ...
  - 2.NBT.B.8 Mentally add 10... to a given number.

• **Practice Standards** —
  - Use appropriate tools strategically.
  - Model with mathematics.
  - Reason quantitatively.
Essential Understanding ONE:

- Addition and subtraction are used to represent and solve many different kinds of problems.
  - Sequential counting
  - Inverse relationship of addition and subtraction
  - Part, part, whole relationship
  - Number sentence representation
  - Context for problems and different representations
Activity TWO: Different Types of Problems

Take a look at the two problems below. Discuss with those at your table the similarities and differences between them.

1. Carl has 5 blocks. Rita gives him 8 more blocks. How many blocks does Carl have now?

2. Alicia counted 5 squirrels in the park on Tuesday. On Wednesday, she counted 8 squirrels. How many squirrels did she count on Tuesday and Wednesday?

What did you discuss?
Activity TWO: Different Types of Problems

Compare and contrast these three problems. What makes them the same or different?

A. Connie has 5 marbles. How many marbles does she need to have 13 marbles now?

B. Connie has 5 marbles. Juan gives her 8 more marbles. How many marbles does she have now?

C. Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie start with?
Activity TWO: Different Types of Problems

A. Connie has 5 marbles. How many marbles does she need to have 13 marbles now?
   – \(5 + \square = 13\) (Join, Change Unknown: JCU)

B. Connie has 5 marbles. Juan gave her 8 more marbles. How many marbles does she have now?
   – \(5 + 8 = \square\) (Join, Result Unknown: JRU)

C. Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie start with?
   – \(\square + 5 = 13\) (Join, Start Unknown: JSU)
What problem did you write?
Four corners
• Join Result Unknown

• Join Change Unknown

• Join Start Unknown

• Other (Part-Part-Whole)
Activity TWO: Different Types of Problems

Take a look at these two problems we solved earlier. Discuss with those at your table the similarities and differences between them.

1. 13 penguins were standing on the iceberg. 5 jumped into the water to swim. How many penguins are left on the iceberg?

2. Tarah has 13 melon balls. Kyrie has 5. How many more melon balls does Tarah have?

What did you discuss?
Activity TWO: Different Types of Problems

• Compare and contrast these three problems. What makes them the same or different?

A. Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to begin with?

B. Connie had 13 marbles. She gave 5 to Juan. How many marbles does she have left?

C. Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did she give to Juan?
Activity TWO: Different Types of Problems

A. Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to begin with?
   \[ \square - 5 = 8 \] (Separate, Start Unknown (SSU))

B. Connie had 13 marbles. She gave 5 to Juan. How many marbles does she have left?
   \[ 13 - 5 = \square \] (Separate, Result Unknown: SRU)

C. Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did she give to Juan?
   \[ 13 - \square = 5 \] (Separate, Change Unknown: SCU)
What problem did you write?
Four corners
Four Corners

• Separate Result Unknown

• Separate Change Unknown

• Separate Start Unknown

• Other (Compare)
Activity TWO: Different Types of Problems

- Joining Stories OR Add To
- Separating Stories OR Take From
- Part, Part Whole Stories OR Put Together, Take Apart
- Comparing Stories

- Result-unknown
- Change-unknown
- Start-unknown

- Number choices
Activity TWO: Different Types of Problems

• How would you adapt the problems you wrote knowing what you know now? Feel free to use multiple number choices to address the range of learners and strategies in your classroom.
So... What Standards Could Be Addressed?

• Content Standards –
  – PreK Identify the missing part of five whole.
  – K.OA.A.2 Solve addition and subtraction word problems...by using object or drawings to represent the problem.
  – 1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.
  – 2.OA.Use addition and subtraction within 100 to solve one-and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.

• Practice Standards -
  – Make sense of problems and perseveres in solving them.
  – Reason abstractly and quantitatively.
Links to PreK, K, 1st, and 2nd

• Use different types of contextual problems as appropriate for the grade level.
• Introduce a variety of strategies to help children represent and solve problems.
Essential Understanding ONE:

• Addition and subtraction are used to **represent** and **solve** many different kinds of problems.
  – Sequential counting
  – Inverse relationship of addition and subtraction
  – Part, part, whole relationship
  – Number sentence representation
  – **Context for problems and different representations**
Activity THREE: “Total the Tiles” Game

- **Materials Needed:**
  - 3 dice, number cards/strips/tiles (2 sets each of 1, 2, 3, 4, 5, 6, 7, 8, 9), recording sheet for each player

- **Object:**
  - To get the LOWEST score after 3 rounds

- **Play:**
  - The number cards/strips/tiles are placed face up. Players take turns.
  - The player rolls all three dice, then removes any tile(s) that match(es) the total roll of the dice. The player continues rolling until tiles cannot be removed.
  - The score for that round is the total of the unused tiles.
Example of a Play -

<table>
<thead>
<tr>
<th>Dice Toss</th>
<th>Tiles Left</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The dice total is 9. You could use a 6, a 2, and a 1 to make 9 OR one 9 tile OR a 5, 3, and 1 OR a 7 and a 2.
Activity THREE: “Total the Tiles” Game

• Materials Needed:
  – 3 dice, number cards/strips/tiles (2 sets each of 1, 2, 3, 4, 5, 6, 7, 8, 9), recording sheet for each player

• Object:
  – To get the LOWEST score after 3 rounds

• Play:
  – The number cards/strips/tiles are placed face up. Players take turns.
  – The player rolls all three dice, then removes any tile(s) that match(es) the total roll of the dice. The player continues rolling until tiles cannot be removed.
  – The score for that round is the total of the unused tiles.
Essential Understanding TWO:

- The mathematical foundation for understanding computational procedures for addition and subtraction of whole numbers are the properties of addition and place value.
  - Commutative / Associative Properties
  - Subtraction is NOT Commutative
  - Place value allows for decomposition and composition
  - Properties of addition are essential to justifying the correctness of computational algorithms.
What is Fluency?

- **Basic Number Computation Fluency** is the ability to recall the combinations of numbers efficiently, accurately, and effortlessly.

- **Computational Fluency** refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. A student cannot be fluent without conceptual understanding and flexible thinking.
Development of Fluency

Understanding Addition and Subtraction

Using Addition and Subtraction Strategies

Fluency in Addition and Subtraction

“A student cannot be fluent without conceptual understanding and flexible thinking.”
• There are fluency requirements for each grade level.
• To develop fluency, emphasize 1) understanding and 2) the use of flexible strategies.
• Encourage games and other activities that help children practice their basic fact combinations.
So... What Standards Could Be Addressed?

• **Content Standards —**
  - PreK  Fluently count from 1 to 10.
  - K.OA.A.5  Fluently add and subtract within 5.
  - 1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.
  - 2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.

• **Practice Standards —**
  - Look for and make use of structure, i.e., associative and commutative properties.
  - Construct viable arguments and critique the reasoning of others.
Activity FOUR: True or False?

8 = 8
4 + 5 = 10
7 + 1 = 8 + 2
9 = 4 + 5
11 − 6 = 10 − 5
Equal
“each side has the SAME value”

2 + 4 = 3 + 3
True or False?
Why do you think so?

• $54 + 10 = 11 + 53$
• $120 + 30 = 148 + 10$
• $23 + 76 + 45 = 144$
• $56 = 50 + 4 + 2$
• $89 + 1 + 40 = 30 + 10 + 80$
Essential Understanding TWO:

- The mathematical foundation for understanding computational procedures for addition and subtraction of whole numbers are the properties of addition and place value.
  - Commutative /Associative Properties
  - Subtraction is NOT Commutative
  - Place value allows for decomposition and composition
  - Properties of addition are essential to computational properties
Links to PreK, 1st, and 2nd

• Number equations are excellent models of addition and subtraction.
• The meaning of the equal sign is essential to an understanding of addition and subtraction.
• The use of a variable and the ability to determine an unknown within an equation is also important.
So... What Standards Can Be Addressed?

• **Content Standards** –
  
  – PreK – Identify different ways to show the parts of five
  
  – K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$.)
  
  – 1.OA.D7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.
  
  – 2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems ... by using drawing and equations with a symbol for the unknown number to represent the problem.

• **Practice Standards** –
  
  – Model with mathematics (e.g., writing an addition equation to describe a situation.)
Activity FIVE: Computational Methods

• When students begin to solve problems, they intuitively direct model the problem.

• Sometimes teachers and students think of problems as a story, as they have a beginning, middle, and ending. Problems with action in them are easier for children to direct model. In classrooms, students should be provided with manipulatives to help them direct model problems.
Activity FIVE: Computational Methods

• Teaching students to identify “key words” is another common instructional approach for teachers. We should be careful not to use this as our only strategy. Sometimes key words can trip students up. For example:

• I have 4 bags of marbles, there are six marbles in each bag. How many marbles in all?
Activity FIVE: Computational Methods

• I have 4 bags of marbles, there are six marbles in each bag. How many marbles in all?
  – The key words, “in all” suggest addition is the correct operation for this problem, but this story is a multiplication problem. The fact that multiplication can be thought of as repeated addition is why we use “in all”.
  – If we allow and encourage students to solve the problem via modeling (to start), we can avoid some of these types of misconceptions.
Activity FIVE: Computational Methods

• After students have had enough experiences with direct modeling, they begin to use counting strategies and then derived facts.
When students start solving problems with multi-digit numbers, they use invented algorithms instead of derived facts. Within those categories of strategies (direct modeling, counting, derived facts, and invented algorithms) are varying levels of sophistication. For instance, counting by tens from 10 to 60 is more efficient than counting by ones. When choosing solution strategies for a sharing session, some teachers will have students share in order of sophistication.
Activity FIVE: Computational Methods

• When students start solving problems with multi-digit numbers, they may invent algorithms using their knowledge of single-digit operations and base ten. Many teachers focus on tens and ones to support students’ algorithm invention.

• For grades K-5, the Common Core has a substantial focus on base-ten understanding. For instance, in 3rd grade, the authors of the Common Core recommend that students, “fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction” (CCSSO & NGA, 2010, p. 24).
Activity FIVE: Computational Methods

Review the student’s computation method. Answer the questions below...

• Explain the method used.
• Is it correct? Why or why not?
• What properties or understanding (or misunderstanding) of place value would justify (or NOT justify) the method used?
Essential Understanding TWO:

• The mathematical foundation for understanding computational procedures for addition and subtraction of whole numbers are the properties of addition and place value.
  – Commutative /Associative Properties
  – Subtraction is NOT Commutative
  – Place value allows for decomposition and composition
  – Properties of addition are essential to computational properties
Computation Method ONE

\[ 431 + 6 \rightarrow 437 + 20 \rightarrow 457 \]
\[ -174 + 6 \rightarrow -180 + 20 \rightarrow -200 \]
\[ 257 \]
Computation Method TWO

606
- 359
  353
Computation Method THREE

438 - 172 = ?

438
- 100
338
- 70
268
- 2
266
Tell how you would find the number of hours in four days.

20 + 20 + 20 - 20 + 4 + 4 = 96. I did it by using 20's and 4's.
Computation Method FIVE

• $400 - 165 = ?$

\[
\begin{array}{c|c|c|c}
165 & 5 & 30 & 200 \\
\hline
\end{array}
\]

$235$
Computation Method SIX

128
+ 258
300
70
+ 16
386
Open number line – count up strategy subtraction

628 – 219 = ?
Open number line – count back strategy subtraction

\[ 628 - 219 = ? \]
Adjust the numbers - addition

99 + 1 \rightarrow 100

- 5 - 5

95 - 1 \rightarrow 94
Decomposing - subtraction

$438 - 172 = ?$

$400 - 100 = 300$
$300 - 70 = 230$
$230 - 2 = 228$
$228 + 30 = 258$
$258 + 8 = 266$
Important Note

• The traditional algorithms for addition and subtraction are not taught until 4th grade – until then, students are developing flexible strategies that use the properties of addition and an understanding of place value.

CCSS.Math.Content.4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.
There are many computation methods for addition and subtraction.

Students need to be given many opportunities to explain or justify their methods.

Making 10 is an important computation strategy.
So... What Standards Were Addressed?

• Content Standards –
  – PreK  Add quantities 1, 2, and 3 by counting all or counting on.
  – K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
  – 1.OA.B.3 Apply properties of operations as strategies to add and subtract.
  – 2.NBTB.6. Add up to four two-digit numbers using strategies based on place value and properties of operations,

• Practice Standards -
  – Construct viable arguments and critique the reasoning of others.
  – Attend to precision.
Where did you see the Mathematics Teaching Practices in these ideas about addition and subtraction?
Plan of Action

Something that squares with your beliefs

Something going ‘round in your head

Three things you will try and when:

1.
2.
3.
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.