NAME ______________________

“Math Talk”

Table 1

Levels of Discourse in a Mathematics Classroom

Levels	Characteristics of Discourse
0	The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher’s questions.
1	The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
2	The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another’s work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
3	The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

Adapted from Hufferd-Ackles, Fuson, and Sherin (2004)

Stein, C. (2007). Let’s talk: Promoting mathematical discourse in the classroom. *The Mathematics Teacher.* 101(4), p. 285-289.

NAME ______________________

Using Teacher Discourse Moves

- Waiting (e.g., Can you put your hands down and give everyone a minute to think?)
- Inviting Student Participation (e.g., Let's hear what kinds of conjectures people wrote.)
- Revoicing (e.g., So what I think I hear you saying is that if there was only one point of intersection, it would have to be at the vertex. Have I got that right?)
- Asking Students to Revoice (e.g., Okay, can someone else say in their own words what they think Emma just said about the sum of two odd numbers?)
- Probing a Students' Thinking (e.g., Can you say more about how you decided that?)
- Creating Opportunities to Engage with Another's Reasoning (e.g., So what I'd like you to do now is use Nina's strategy to solve this other problem with a twelve-by-twelve grid.)

NAME ______________________

What Are Some Strategies for Facilitating Productive Classroom Discussions? Brief

Cirillo, M. (2013). S. DeLeeuw, (Series Ed.) National Council of Teachers of Mathematics. http://www.nctm.org/news/content.aspx?id=35386

Use Teacher Discourse Moves In Classroom Discussions, Chapin, O'Connor, and Anderson (2003, 2009) introduced five "productive talk moves," which they described as suggested actions that were found to be effective in "making progress toward achieving [their] instructional goal of supporting mathematical thinking and learning" (p. 11). This claim was based on data from their work in Project Challenge, an intervention project initially aimed to provide disadvantaged elementary and middle school students with a reform-based mathematics curriculum that focused on mathematical understanding, with a heavy emphasis on talk and communication about mathematics. A goal of using the talk moves was to increase the amount of high-quality, mathematically productive talk in classrooms. Building on Chapin et al. (2003), Herbel-Eisenmann, Cirillo, and Steele expanded this earlier work through a five-year project aimed at supporting teachers' facilitation of classroom discourse through the design of a professional development curriculum program. The curriculum supports secondary mathematics teachers in becoming more purposeful about engaging students in mathematical explanations, argumentation, and justification. A modified set of talk moves serves as a centerpiece of the curriculum. This set of Teacher Discourse Moves (TDMs) is a tool that can help facilitate productive and powerful classroom discourse. As part of the curriculum's overarching goals, productive focuses on how discourse practices support students' access to mathematical content. Powerful refers to how classroom discourse supports students' developing identities as knowers and doers of mathematics. There are six TDMs (cf. the five talk moves), which are defined in such a way that highlights what is special about thinking and reasoning in mathematics class as opposed to any other subject area (Herbel-Eisenmann, Steele, & Cirillo, in press). These six moves are:

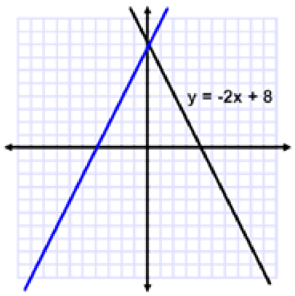
- Waiting (e.g., Can you put your hands down and give everyone a minute to think?)
- Inviting Student Participation (e.g., Let's hear what kinds of conjectures people wrote.)
- Revoicing (e.g., So what I think I hear you saying is that if there was only one point of intersection, it would have to be at the vertex. Have I got that right?)
- Asking Students to Revoice (e.g., Okay, can someone else say in their own words what they think Emma just said about the sum of two odd numbers?)
- Probing a Students' Thinking (e.g., Can you say more about how you decided that?)
- Creating Opportunities to Engage with Another's Reasoning (e.g., So what I'd like you to do now is use Nina's strategy to solve this other problem with a twelve-by-twelve grid.)

The six TDMs can be particularly productive and powerful when they are purposefully used in combination with each other (e.g., Asking Students to Revoice after Probing a Students' Thinking). These moves can be used in conjunction with the Five Practices introduced above.

NAME ______________________

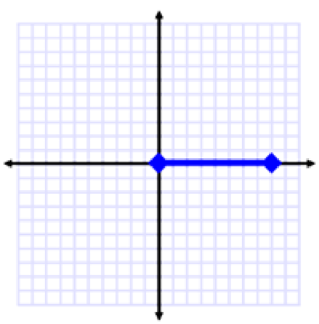
Build an Isoceles Triangle, Then Make a Rhombus

Part A: Build An Isosceles Triangle That Has The X-axis As Its Base And Its Vertex On The Y-axis



- What is the equation of the line (blue)?

Part B: Make An Isosceles Triangle Congruent To The Last One That Has The Segment Connecting (0,0) to (8,0) as its base.



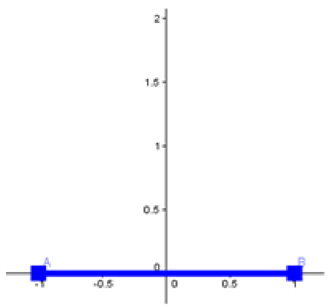
- What are the equations of the lines?
- How do they compare to the lines in the previous problem?
- What is going on with the slopes and y-intercepts?

NAME ______________________

NAME ______________________

Use Algebra To Make An Equilateral Triangle

Given The Base AB Of An Equilateral Triangle With A(-1,0) And B(0,1)



- What is the height of the triangle?
- What are the equations of the lines that make an equilateral triangle?
- What is the vertex?
- What is going on with the slopes and y-intercepts?

NAME ______________________

Using Algebra To Make Polygons

Partner A: Graph	Partner B: Graph
$y = x - 5$	$y = x - 10$
$y = -x - 5$	$y = -x - 10$
$y = x + 5$	$y = x + 10$
$y = -x + 5$	$y = -x + 10$

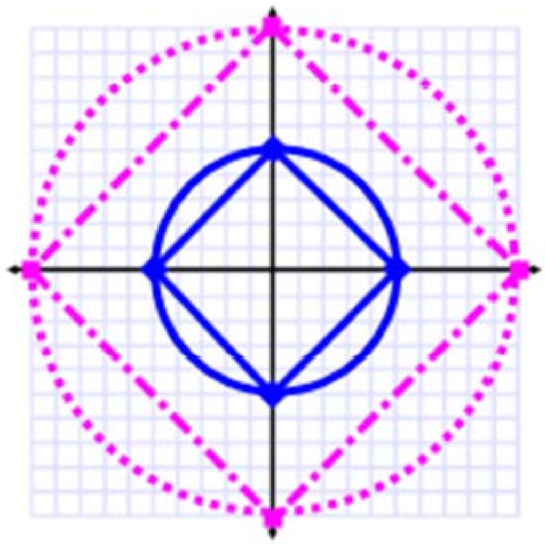
Questions for Both Partners

- **What shape is it? How do you know?**
- **What is its perimeter? area?**
- **How do the two graphs compare?**
- **What about $y = 4$, $y = -4$, $x = 4$ and $x = -4$**

NAME ______________________

Labelling Circles Around Squares

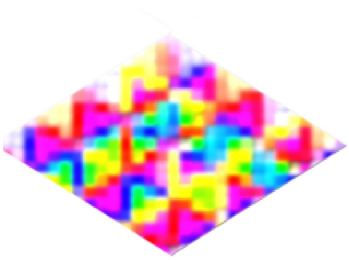
Extend: What are the equations of those circles?



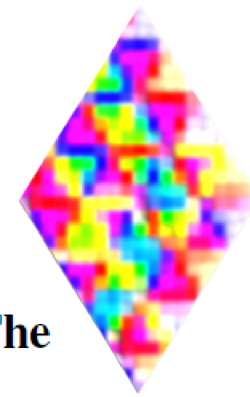
NAME ______________________

Using Algebra To Build A Regular Hexagon

- **What Equations Make an Equilateral Triangle With Base AB at A(0,0) and B(1,0)? Draw The Triangle.**
- **What is The Vertex of That Triangle?**
- **On The Same Graph, Sketch The Lines Below to Form a Rhombus:**



$$y = x\sqrt{3} + \sqrt{3}$$
$$y = -x\sqrt{3} + \sqrt{3}$$
$$y = x\sqrt{3} - \sqrt{3}$$
$$y = -x\sqrt{3} - \sqrt{3}$$



- **Label The Midpoints of The Sides of The Rhombus.**
- **Do You See The Hexagon?**

NAME ______________________

Another Way To Ask…

- What Equations Make An Equilateral Triangle With Base AB at A(0,0) and B(1,0)? Draw The Triangle.
- Reflect This Triangle Across Both The X And Y Axes.
- Reflect One of Those Triangles Into Quadrant 3.
- Do You See A Hexagon?
- Which Equations Form The Regular Hexagon?
- What Are Its Vertices?

NAME ______________________

Modeling Problems

1. Build All The Trains of Length 1, 2, 3, and 4. Record how many of each are there? Organize your findings. Describe in words or pictures any patterns you see. Do you have any conjectures? Describe something on your paper with someone at your table or near you.

Some questions to think about:
How do you know you have all of the trains of a given length?
How many trains of length 5 will there be?
How many trains of length n are there? How do you know for sure?

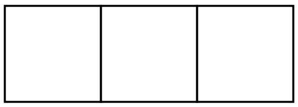
NAME ______________________

2. Build all the trains of Length 1,2,3,4, and 5 using only cars of length 1 and 2. Record how many of each are there. Organize your findings. Describe in words or pictures any patterns you see. Compare your results with someone at your table . Do you have any conjectures?

Some questions to think about:
How do you know you have all of the trains of a given length?
How many trains of length 6 will there be?
How many trains of length n are there? How do you know for sure?

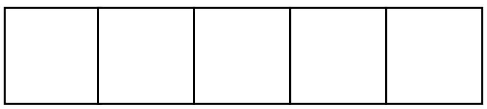
NAME ______________________

3.* Build the next two trains. How do you get from one train to the next? Describe the pattern in words on your paper or to someone next to or near you. Use your pattern to write/describe a rule that relates the train number to the number of unit squares. What does **TRAIN 0** look like? Does your **TRAIN 0** match someone near you?



TRAIN 1

TRAIN 2

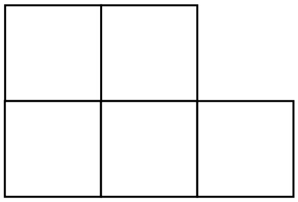


TRAIN 3

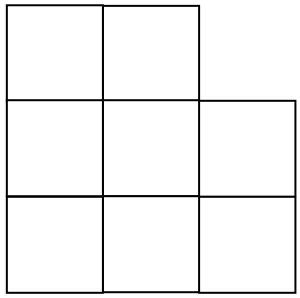
*Problems #3-8 were inspired by a workshop at an NCTM affiliate conference (SCCTM) given by Dr. Megan Che (Clemson University) and Angela Watt (Lakeside Middle School)

NAME ______________________

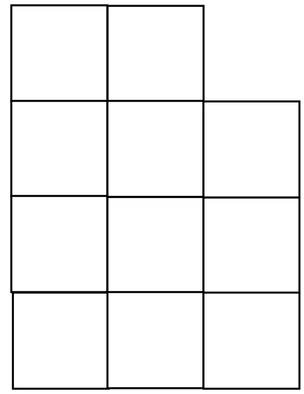
4. Build the next two trains. How many unit squares are in each train? Make a table, a graph and an equation that relate the train number to the number of unit squares.



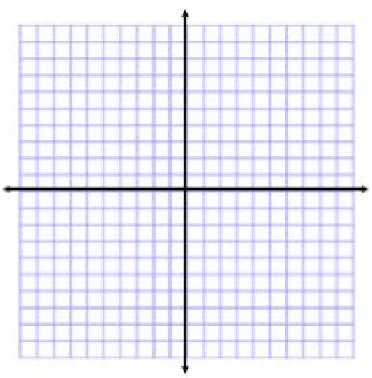
TRAIN 1



TRAIN 2



TRAIN 3



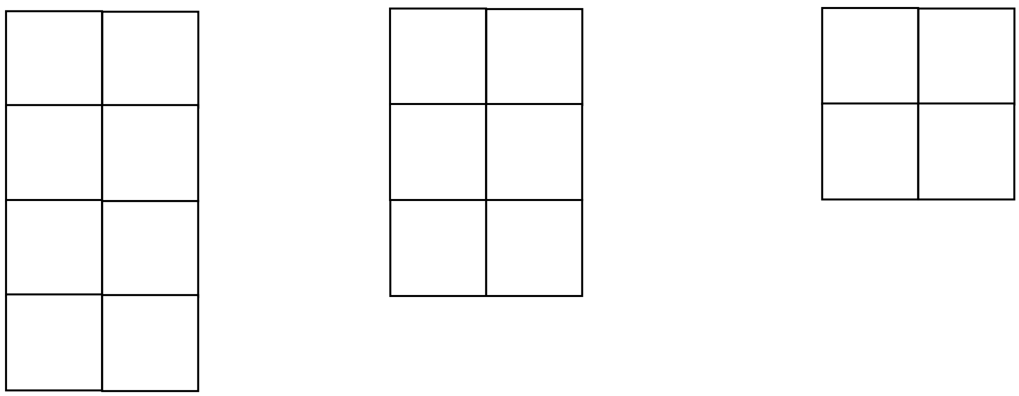
Some questions to think about:

- How is this pattern related to the pattern in number 3?
- What is the domain of your rule?
- Is there a **TRAIN -1**?

NAME ______________________

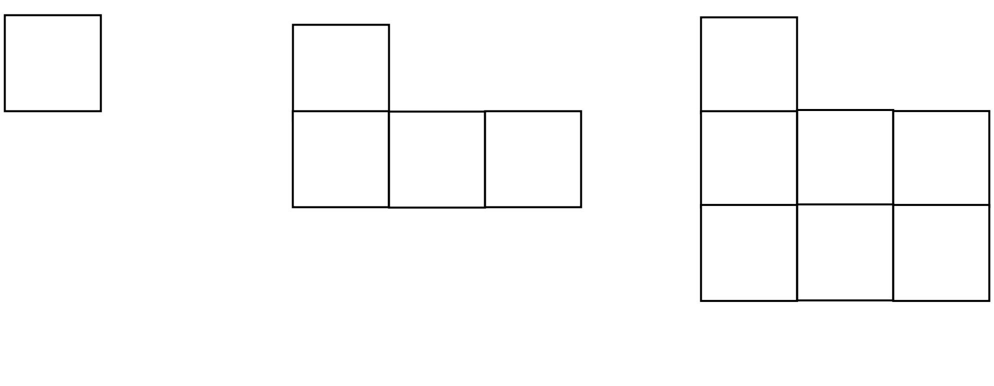
5. (IF YOU HAVE EXTRA TIME) Build and record trains 1, 2, and 3 so that they fit the rule "three times the train number minus 2." Graph it. Make a table. Show or describe how the table, graph, rule and trains are related.

6. Build the next two trains. What will the next one look like? Can you build it? Write a rule that relates the train number to the number of unit squares. Can you build **TRAIN 5**?



TRAIN 1 TRAIN 2 TRAIN 3

7. Build **TRAIN 0**. How can you record **TRAIN 0** on paper? Share your representation with someone near you. Build and record **TRAIN -1** and **TRAIN -2** Write a rule that relates the train number to the number of unit squares.



NAME ______________________

TRAIN 1 TRAIN 2 TRAIN 3

8. What do **TRAIN -1** and **TRAIN -2** look like for problems 3, 4, and 7? How many unit squares are in **TRAIN -1.5**? Can you build **TRAIN 6** for problem number 6?

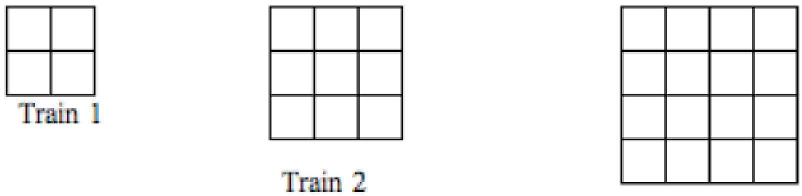
9. (IF YOU HAVE TIME) Build the trains that match the table:

Train #	1	2	3	4
# of unit squares	-5	-2	1	4

Write a rule.

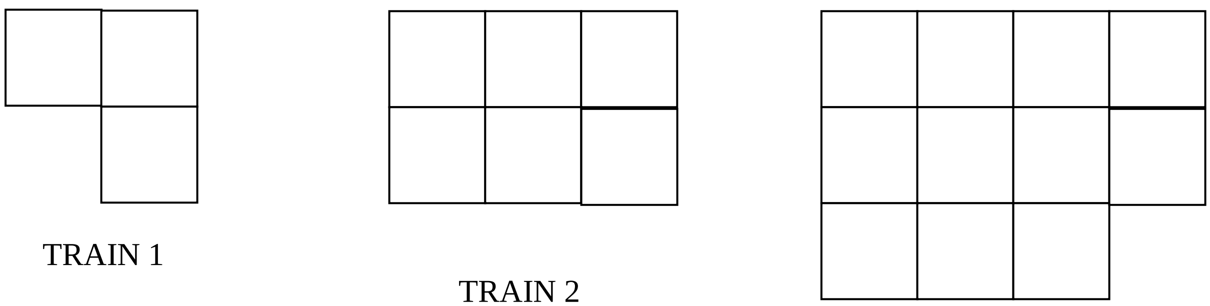
NAME ______________________

10. Build the next train. Write a rule for getting from one train to the next.

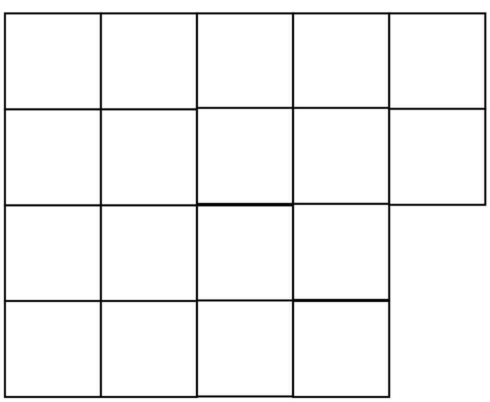


Train 1 Train 2 Train 3

11. Build the next train. How do you get from one train to the next? How many unit squares are in each train? Write a rule that relates the train number and the number of unit squares. Compare and contrast this rule with the rule in #3 and #4. If you have time, make a graph.



TRAIN 1 TRAIN 2 TRAIN 3



TRAIN 4

Question to think about:
Does the way this is drawn make a difference for making the next one?
What do **TRAIN 0** and **TRAIN -1** look

12. (IF YOU HAVE EXTRA TIME). Build the first 3 trains that fit the rule “the train number squared plus 3 times the train number minus 2.”