

Beliefs about Teaching and Learning Mathematics

Beliefs about teaching and learning mathematics	
Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
All students need to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Principles to Action, Ensuring Mathematics Success for All (NCTM, 2014) p 11

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Characteristics of Mathematical Tasks

Levels of Demands

<p style="text-align: center;"><u>Lower-level demands</u> <u>(memorization):</u></p> <ul style="list-style-type: none"> • reproducing previously learned facts, rules, formulas, definitions or committing them to memory • Cannot be solved with a procedure • Have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions 	<p style="text-align: center;"><u>Lower-level demands</u> <u>(procedures without connections):</u></p> <ul style="list-style-type: none"> • are algorithmic • require limited cognitive demand • have no connection to the concepts or meaning that underlie the procedure • focus on producing correct answers instead of understanding • require no explanations
<p style="text-align: center;"><u>Higher-level demands</u> <u>(procedures with connections):</u></p> <ul style="list-style-type: none"> • use procedure for deeper understanding of concepts • broad procedures connected to ideas instead narrow algorithms • usually represented in different ways • require some degree of cognitive effort; procedures may be used but not mindlessly 	<p style="text-align: center;"><u>Higher-level demands</u> <u>(doing mathematics):</u></p> <ul style="list-style-type: none"> • require complex non-algorithmic thinking • require students to explore and understand the mathematics • demand self-monitoring of one's cognitive process • require considerable cognitive effort and may involve some level of anxiety b/c solution path isn't clear

Leinwand, S., D. Brahier, and D. Huinker . *Principles to Action*. Reston, VA: National Council of Teachers of Mathematics, 2014 (pg 18)

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Productive Struggle Reflection Survey

Rate the frequency of each statement in your classroom.

1. I anticipate what students might struggle with during a lesson and I prepare to support them.

1	2	3	4	5	6	7	8	9	10
Never									Always

2. I give students time to struggle with tasks and ask questions that scaffold thinking without doing the work for them.

1	2	3	4	5	6	7	8	9	10
Never									Always

3. I help students realize that confusion and errors are a natural part of learning. We talk about mistakes, misconceptions, and struggles.

1	2	3	4	5	6	7	8	9	10
Never									Always

4. I praise students for their efforts more frequently than the correct answers.

1	2	3	4	5	6	7	8	9	10
Never									Always

5. My students struggle at times but they know breakthroughs come from struggle.

1	2	3	4	5	6	7	8	9	10
Never									Always

6. My students ask questions that are related to their struggles to help them understand.

1	2	3	4	5	6	7	8	9	10
Never									Always

7. My students persevere in solving problems. They don't give up.

1	2	3	4	5	6	7	8	9	10
Never									Always

8. My students help one another without telling their classmates what the answer is or how to solve it.

1	2	3	4	5	6	7	8	9	10
Never									Always

Productive Struggle Images



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Purposeful Questions

What Are Teachers Doing?	What Are Students Doing?

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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Levels of Classroom Discourse

Teacher Role	Questioning	Explaining Mathematical Thinking	Mathematical Representations	Building Student Responsibility within the Community
Level 0				
Level 1				
Level 2				
Level 3				

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35(2), 81–116.

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Five Practices for Classroom Discourse

(Smith and Stein, 2011)

Anticipating	
Monitoring	
Selecting	
Sequencing	
Connecting	

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Explanation Game

1. **Take it:** Remove one item at a time from the bag. (Draw a picture, or label it).
2. **Name it.** Name a feature or aspect of the object you notice.
3. **Explain it.** What could it be? What role or function might it serve? Why might it be there?
4. **Give reasons.** What makes you say that? Support your ideas.
5. **Generate alternatives.** What other ideas do you have?

Ritchhart, R., Church, M., & Morrison, K. (2011). *Making thinking visible: How to promote engagement, understanding, and independence for all learners*. John Wiley & Sons.

Framework for Mathematics Teaching Questions

Question type	Description	Examples
Gathering Information		
Probing Thinking		
Making the Mathematics Visible		
Encouraging Reflection and Justification		

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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Encouraging Discourse and Questioning

Notice

Wonder


Question

Engage in Tasks

Solve Problems

Reflect

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Tricycle Race

Happy Days Preschool is having their annual tricycle race! We are now ten minutes into the race. The tricycles have traveled this far to the finishing line. Show where each tricycle is on the number line and then make a prediction about which tricycle will win.

 $\frac{1}{2}$  $\frac{1}{4}$  $\frac{5}{6}$  $\frac{2}{3}$ 

Miguel and Janie

Miguel and Janie are each painting a mural. Janie says that she painted $\frac{3}{4}$ of the mural and Miguel said that he painted $\frac{3}{8}$ of the mural. Who has most of the mural painted? Use words, pictures, and representations to show your thinking.



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What Does the Whole Look Like?


Create your own using any materials at your table.

If this _____ is $\frac{1}{5}$, what does the whole look like?

If this _____ is $\frac{1}{4}$, what does the whole look like?

Which whole is greater?

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Sharing Brownies

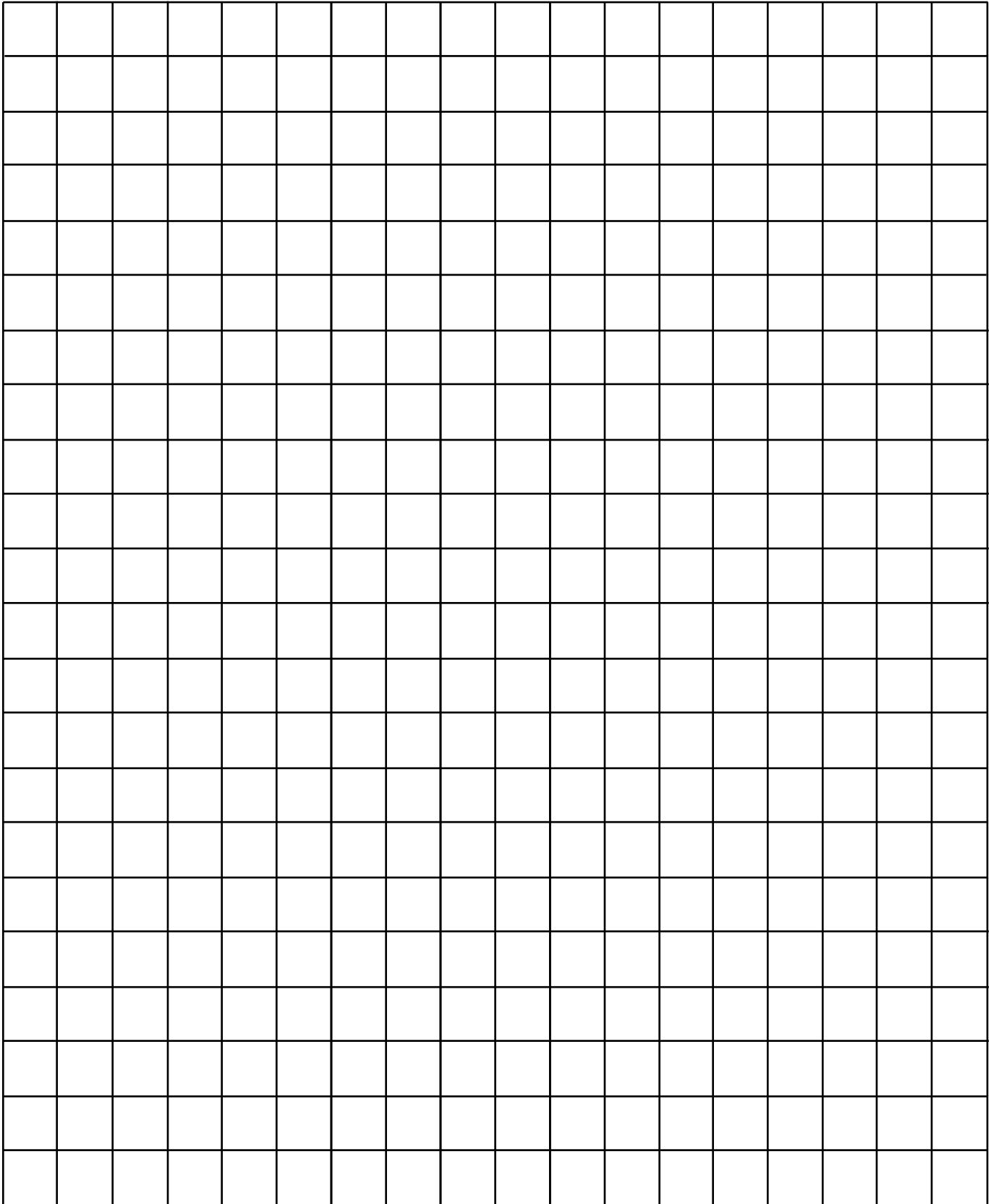
2 people and 3 brownies	5 people and 6 brownies
6 people and 4 brownies	8 people and 5 brownies

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1-CENTIMETER GRID PAPER



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Purposeful Questions

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Encouraging Discourse and Questioning

Notice:

Wonder:


Question:

Task:

Solve Problem:

Reflect:

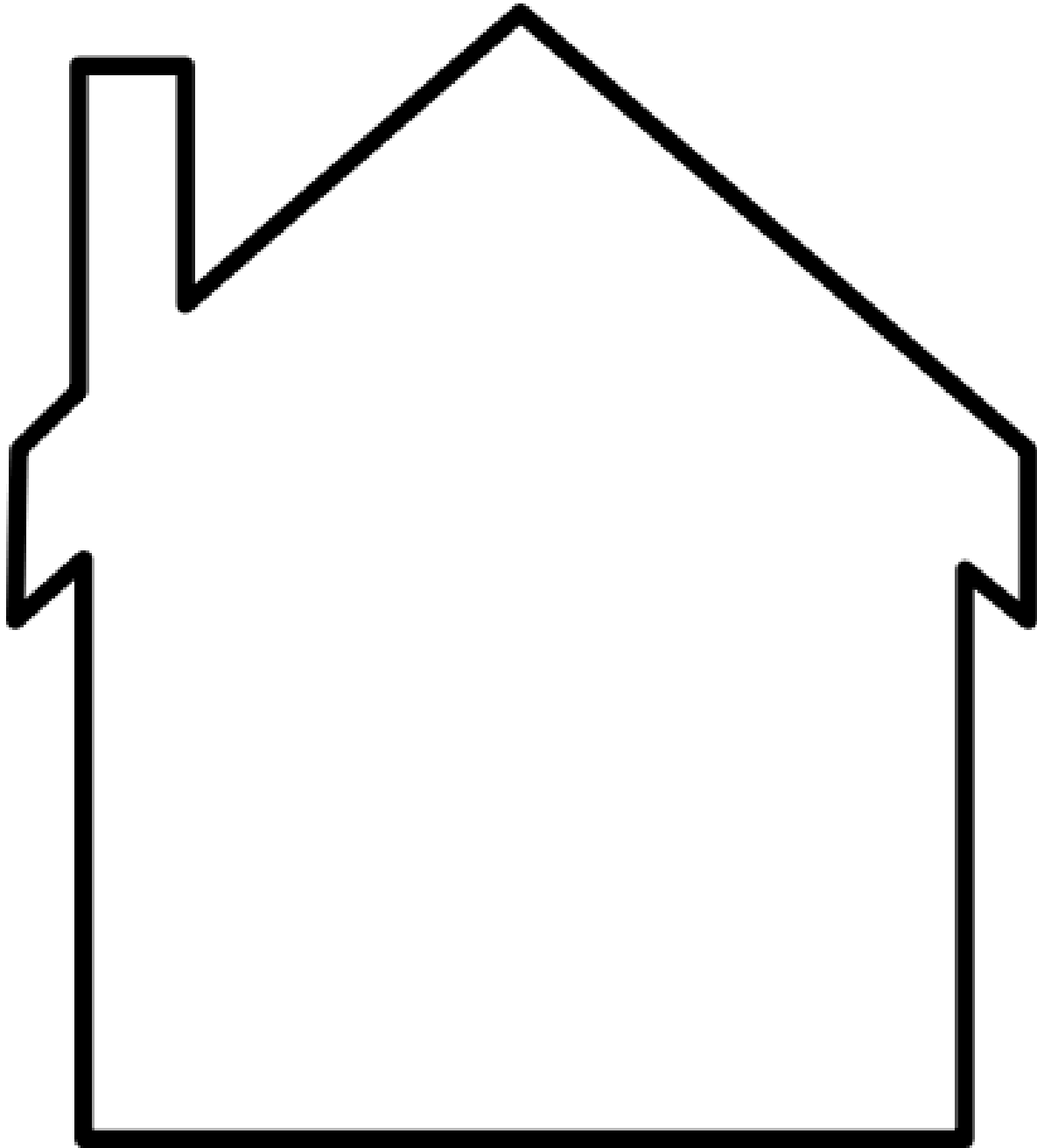
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Who is in the House?



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T-Shirt



Aunt Betsy decided to help get his order together by having him package his t-shirts into groups of ten.

You are going to get a t-shirt order. Help Uncle Ronnie figure out how many groups of ten and how many single t-shirts you will have in your order.

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Jenny's Birthday

Happy 7th Birthday!



Jenny is 7 years old today. How many candles have been on her birthday cake since she was born?

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Clapping Institute Award ★

is presented to

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for

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.....
Signature

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Date

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Beliefs About Mathematics Assessment

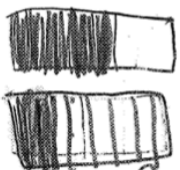
Select 2 statements about assessment. Write **P** if it is a productive belief or **U** if it is a unproductive belief. Be sure to explain your reasoning.

The primary purpose of assessment is accountability for students through report card marks or grades.		Only multiple-choice and other "objective" paper-and-pencil tests can measure mathematical knowledge reliably and accurately.	
Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.		Assessment in the classroom is an interruption of the instructional process.	
Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.		A single assessment can be used to make important decisions about students and teachers.	
Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.		Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.	
Assessment is something that is done to students.		Stopping teaching to review and take practice tests improves students' performance on high-stakes tests.	
Multiple data sources are needed to provide an accurate picture of teacher and student performance.		Ongoing review and distributed practice within effective instruction are productive test preparation strategies.	

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author

D Write $>$, $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.

$\frac{5}{6}$ $\frac{2}{8}$



Tell how you compared them without common denominators.

First, The denominator in $\frac{2}{8}$ is 8.
 The denominator in $\frac{5}{6}$ is 6.
 So I compared 8 and 6 and 8 is greater than 6. So $\frac{2}{8}$ is greater than $\frac{5}{6}$.

D Write $>$, $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.

$\frac{5}{6}$ $\frac{2}{8}$

Tell how you compared them without common denominators.

I compared them by looking at both denominators first. For $\frac{2}{8}$ it was 8. For $\frac{5}{6}$ it was 6. 8 is greater than 6, 2 is not greater than 5, but that is not what matters. What matters is the denominator, not the numerators. So, $\frac{2}{8}$ is greater than $\frac{5}{6}$.

D Write $>$, $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.



$\frac{5}{6}$ $\frac{2}{8}$



Tell how you compared them without common denominators.

$\frac{2}{8}$ is not a half and $\frac{5}{6}$ is half over.

D Write $>$, $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.

$\frac{5}{6}$ $\frac{2}{8}$

Tell how you compared them without common denominators.

I compared them by seeing how much more you need to get 8 from 2 and 5 to 6 in $\frac{5}{6}$ you need 1 part and on $\frac{2}{8}$ you need 6 parts that's how I compared.

D Write $>$, $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.



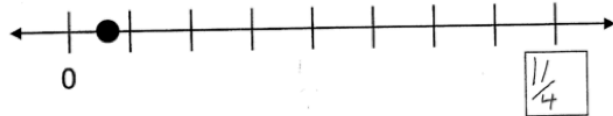
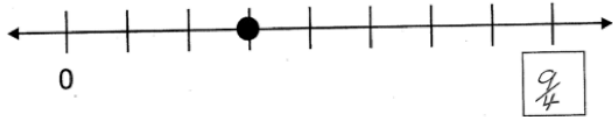
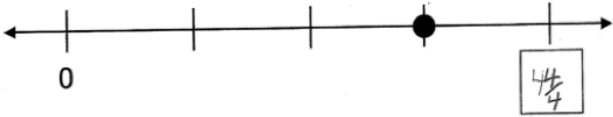
$\frac{5}{6}$ $\frac{2}{8}$



Tell how you compared them without common denominators.

I used pictures to help, so I drew $\frac{5}{6}$ and $\frac{2}{8}$ and $\frac{5}{6}$ covered more area.

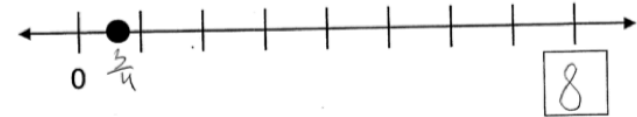
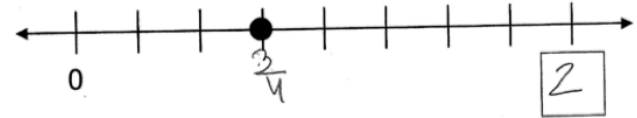
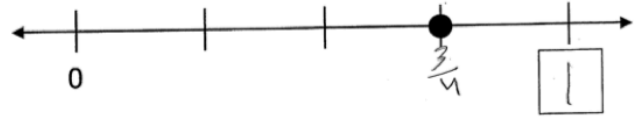
1. The point shows $\frac{3}{4}$ on each number line. Write the missing endpoint for each number line in the box.



Explain how you found your endpoints.

I counted the lines to see how many it was

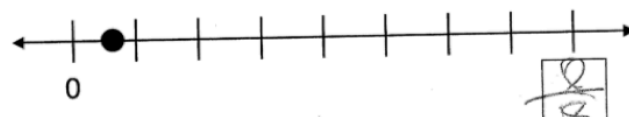
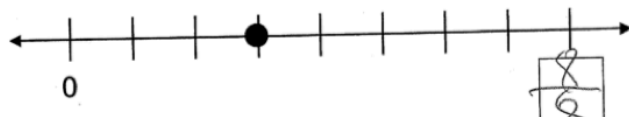
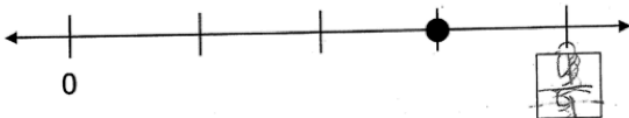
3. The point shows $\frac{3}{4}$ on each number line. Write the missing endpoint for each number line in the box.



Explain how you found your endpoints.

I counted how many 4ths there were in each number line then how many wholes that was and wrote the number of wholes there were in the box

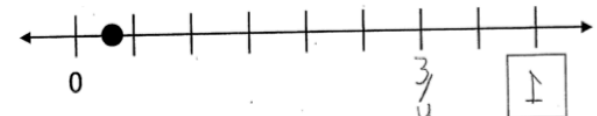
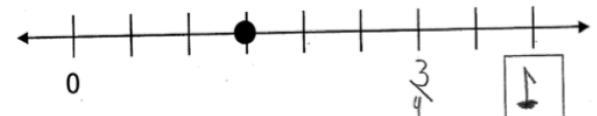
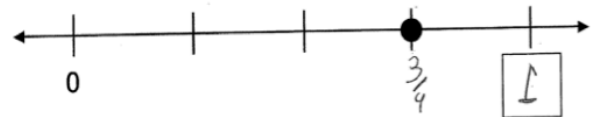
2. The point shows $\frac{3}{4}$ on each number line. Write the missing endpoint for each number line in the box.



Explain how you found your endpoints.

Well, I first counted how many lines then how many lines were there would be in the box.

4. The point shows $\frac{3}{4}$ on each number line. Write the missing endpoint for each number line in the box.



Explain how you found your endpoints.

I lined up them all to equal 3/4