Rich Tasks and Productive Struggle
Effective Teaching with Principles to Action Institute, 2015

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Table Introductions

- Your name
- Your favorite number

John

5
How to play:

1. Players take turns rolling a number cube

2. After each roll, a player decides which column to place the digit.

3. That player then adds the value to his/her total.

4. The player who is closest to the target (in the last total) without going over the target wins.
Choose 1 to discuss with a partner

• How might you use this game in your mathematics classroom?
• How might you modify this game for your students?
• What would you look for while your students played the game?
## Target 1

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<tr>
<th>Tenths</th>
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# Decimals on a Hundred Chart

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Outcomes

• Examine productive and unproductive beliefs about teaching and learning mathematics.

• Identify the characteristics of tasks that promote reasoning.

• Consider the role of productive struggle in elementary mathematics.
About the Institute: Effective Teaching Practices

1. Implement tasks that promote reasoning and problem solving.
2-4. Use and connect mathematical representations.
2-4. Build procedural fluency from conceptual understanding.
2-4. Facilitate meaningful mathematical discourse.
5. Establish mathematics goals to focus learning.
5. Pose purposeful questions.
1. Elicit and use evidence of student thinking.
Support productive struggle in learning mathematics.
Thoughts About Teaching and Learning Mathematics

- Brainstorm thoughts or beliefs that different stakeholders have about teaching learning mathematics.
- Capture your ideas on your web.
- Describe a theme for your web.
Thoughts About Teaching and Learning Mathematics

- Examine ideas of other groups during the gallery walk.
- Ideas that appeared on your web.
- P = Productive belief
- U = Unproductive belief
Actual Student Beliefs

• Place holder for student beliefs. Collection in progress.
UNPRODUCTIVE Beliefs about Teaching and Learning Mathematics

Mathematics learning should focus on practicing procedures and memorizing basic number combinations.

All students need to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Students can learn to apply mathematics only after they have mastered the basic skills.

The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.

An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.
PRODUCTIVE Beliefs about Teaching and Learning Mathematics

Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.

The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Students can learn mathematics through exploring and solving contextual and mathematical problems.
Success for All by...

- Engaging with challenging tasks.
- Connecting new learning with prior knowledge.
- Acquiring conceptual and procedural knowledge.
- Constructing knowledge through discourse.
- Receiving meaningful and timely feedback.
- Developing metacognitive awareness.
Implement Tasks that Promote Reasoning and Problem Solving
What does rigor look like in mathematics classrooms?
MATHEMATICAL RIGOR

Conceptual Understanding

Procedural Fluency

Application
<table>
<thead>
<tr>
<th><strong>BLOOM’S TAXONOMY</strong></th>
<th><strong>WEBB’S DOK</strong></th>
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<tbody>
<tr>
<td><strong>KNOWLEDGE</strong></td>
<td><strong>RECALL</strong></td>
</tr>
<tr>
<td>The recall of specifics and universals, involving little more than bringing to mind the appropriate material.</td>
<td>Recall of a fact, information, or procedure (e.g., What are 3 critical skill cues for the overhand throw?)</td>
</tr>
<tr>
<td><strong>COMPREHENSION</strong></td>
<td><strong>SKILL/CONCEPT</strong></td>
</tr>
<tr>
<td>Ability to process knowledge on a low level such that the knowledge can be reproduced or communicated without a verbatim repetition.</td>
<td>Use of information, conceptual knowledge, procedures, two or more steps, etc.</td>
</tr>
<tr>
<td><strong>APPLICATION</strong></td>
<td><strong>STRATEGIC THINKING</strong></td>
</tr>
<tr>
<td>The use of abstractions in concrete situations.</td>
<td>Requires reasoning, developing a plan or sequence of steps; has some complexity; more than one possible answer</td>
</tr>
<tr>
<td><strong>ANALYSIS</strong></td>
<td><strong>EXTENDED THINKING</strong></td>
</tr>
<tr>
<td>The breakdown of a situation into its component parts.</td>
<td>Requires an investigation; time to think and process multiple conditions of the problem or task.</td>
</tr>
<tr>
<td><strong>SYNTHESIS AND EVALUATION</strong></td>
<td></td>
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</table>
How do tasks and teaching impact mathematics learning?
So what does THIS mean?

<table>
<thead>
<tr>
<th>Task Quality</th>
<th>Implementation</th>
<th>Results</th>
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</thead>
<tbody>
<tr>
<td>Low</td>
<td>High or Low</td>
<td>Low</td>
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<td>High</td>
<td>Low</td>
<td>Moderate</td>
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Research on Tasks

Not all tasks provide the same opportunities for student thinking and learning (Hiebert et al. 1997; Stein et al. 2009).

Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature (Boaler and Staples 2009; Hieber and Wearne 1993; Stein and Lane 1996)
What are characteristics of quality mathematics tasks?
What do high-level tasks look like?

1. Read each task.

1. Sort the tasks by their cognitive level (*Lower or Higher Level of Cognitive Demand*).

1. Be prepared to defend your reasoning.
What do they have in common?

• Review your sorting.
• What do higher-level tasks have in common?
• What do lower-level tasks have in common?
• Where do you see evidence of procedure, concept, and application in these tasks?
<table>
<thead>
<tr>
<th><strong>Lower-level demands</strong> <em>(memorization):</em></th>
<th><strong>Lower-level demands</strong> <em>(procedures without connections):</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>• reproducing previously learned facts, rules, formulas, definitions or committing them to memory</td>
<td>• are algorithmic</td>
</tr>
<tr>
<td>• Cannot be solved with a procedure</td>
<td>• require limited cognitive demand</td>
</tr>
<tr>
<td>• Have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions</td>
<td>• have no connection to the concepts or meaning that underlie the procedure</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Higher-level demands</strong> <em>(procedures with connections):</em></th>
<th><strong>Higher-level demands</strong> <em>(doing mathematics):</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>• use procedure for deeper understanding of concepts</td>
<td>• require complex non-algorithmic thinking</td>
</tr>
<tr>
<td>• broad procedures connected to ideas instead narrow algorithms</td>
<td>• require students to explore and understand the mathematics</td>
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<tr>
<td>• usually represented in different ways</td>
<td>• demand self-monitoring of one’s cognitive process</td>
</tr>
<tr>
<td>• require some degree of cognitive effort; procedures may be used but not mindlessly</td>
<td>• require considerable cognitive effort and may involve some level of anxiety b/c solution path isn’t clear</td>
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</tbody>
</table>

Lower-level demands (memorization):

| • reproducing previously learned facts, rules, formulas, definitions or committing them to memory |
| • cannot be solved with a procedure |
| • have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions |

Lower-level demands
(procedures without connections):

• are algorithmic
• require limited cognitive demand
• have no connection to the concepts or meaning that underlie the procedure
• focus on producing correct answers instead of understanding
• require no explanations

Higher-level demands (procedures with connections):

- use procedure for deeper understanding of concepts
- broad procedures connected to ideas instead narrow algorithms
- usually represented in different ways
- require some degree of cognitive effort; procedures may be used but not mindlessly

Higher-level demands (doing mathematics):

- require complex non-algorithmic thinking
- require students to explore and understand the mathematics
- demand self-monitoring of one’s cognitive process
- require considerable cognitive effort and may involve some level of anxiety b/c solution path isn’t clear

Are there any you want to move?

<table>
<thead>
<tr>
<th>Lower-Level Tasks</th>
<th>Higher-Level Tasks</th>
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How can you make it better?

• Select 2 lower-level tasks.
• Work with a partner to talk about how you could make these tasks higher-level.
• Record your ideas on the back of the task.
Bumper Cars

• Take 1 of the improved tasks and bump in to someone else.
• Share how you can improve the task.
• Trade tasks.
• Bump into someone else.
Support Productive Struggle in Mathematics
More Research on Tasks

• Tasks with high cognitive demands are the most difficult to implement well, and are often transformed into less demanding tasks during instruction (Stein, Grover, and Henningsen 1996; Stigle and Hiebert 2004)
Think-Pair-Share

• How do you know when a student is struggling?

• What do you do?
Which of the following pictures best describes your productive struggle?
Find it?...

Nope.
A Destructive Struggle

• Leads to frustration.
• Makes learning goals feel hazy and out of reach.
• Feels fruitless.
• Leaves students feeling abandoned and on their own.
• Creates a sense of inadequacy.

Productive Struggle

• Leads to understanding.
• Makes learning goals feel attainable and effort seem worthwhile.
• Yields results.
• Leads students to feelings of empowerment and efficacy.
• Creates a sense of hope.

Which One?
Productive Struggle

What do students look like when they are engaging in productive struggle?

What do Teachers look like when they are facilitating productive struggle?
How Can We Support the Development of Productive Struggle?

- Choose rigorous tasks!
- Determine what it looks like in students.
- Make it an explicit focus in classrooms. (Tell students we are doing this!)
- Focus on productive behaviors instead of intelligence.
- Communicate with families.
Mindset and Productive Struggle

Fixed Mindset
• Understanding, proficiency, ability are “set”
• You are good at something or you aren’t

Growth Mindset
• Understanding, proficiency, ability are developed regardless of your genes
• You become better as something as you work with it – as you struggle with it

Dweck, 2008
Why isn’t there a magic bullet for developing productive struggle?
Our personal growth...

1. I anticipate what students might struggle with during a lesson and I prepare to support them.

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2. I give students time to struggle with tasks and ask questions that scaffold thinking without doing the work for them.

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3. I help students realize that confusion, mistakes, misconceptions, and struggle are a part of learning.

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4. I praise students for their efforts

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5. My students struggle at times but they know breakthroughs come from struggle.

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6. My students ask questions that are related to their struggles to help them understand.

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7. My students persevere in solving problems. They don't give up.

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8. My students help one another without telling their classmates what the answer is or how to solve it.

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Session 1 Closing

What are 2 ideas from this session that resonate with you?
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.
Pose Questions, Increase Discourse and Problem Solving

Content: Size of A whole, Unit fractions, number lines

NCTM Common Core Institute, 2014

Beth McCord Kobett
Assistant Professor
Stevenson University
bkobett@stevenson.edu
Some of these Things Are Not Like Each Other!

\[
\begin{array}{c|c}
4/9 & 1/3 \\
\hline
1/4 & 2/6 \\
\end{array}
\]
Now make your own in pairs – *Which one doesn’t belong.* Think about your
Fraction Mingle

1. Take a fraction card from the table.

1. Stand up and begin Mingling.

1. When I give the signal, stop.

1. Begin discussing your fraction.
There are 25 sheep and 5 dogs in a flock. How old is the Shepherd?
Three out of four students will give a numerical answer to this problem.

Paired Interviews

• Interview each other.
• Talk about the best time you observed students engaged in discourse.
• What did you see?
• What did you notice?
• How did you feel?
• Then meet with another pair, share stories and identify the common themes among the stories.
Strategies for Increasing Discourse, improve questioning (yours and theirs, and problem solving)

- Interesting, rich tasks
- Notice, Wonder, Question, Engage in Tasks, and Solve Problems
- Explanation Game
“One of the great secrets to fostering deep learning is the ability to help students raise new kinds of questions that they will find fascinating.” — Ken Bain
“People are most likely to take a deep approach to their learning when they are trying to answer questions or solve problems that they have come to regard as important, intriguing, or just beautiful. One of the great secrets to fostering deep learning is the ability to help students raise new kinds of questions that they will find fascinating.”

Ken Bain
A little bit about questions...

• A child between the age of two and five, a child asks about 40,000 questions.

• During the shift of the three year span, there is a shift in the kinds of questions being asked from simple factual questions (like name of object) to explanations.

• At the same time, about a quadrillion connections are found in their brains (more than three times than an adult).

• The questions show how they are connecting stimuli and thoughts.
Let’s Ask Questions!

As we move through the activities, let’s continue to generate questions.
Notice and Wonder
People and Brownies!

What do you Notice?

• 2 people and 3 brownies
• 5 people and 6 brownies
• 6 people and 4 brownies
• 8 people and 5 brownies
People and Brownies!

What do you Wonder?

• 3 people and 4 brownies
• 4 people and 5 brownies
• 6 people and 4 brownies
• 8 people and 5 brownies
People and Brownies!

What Questions do you have?

• 3 people and 4 brownies
• 4 people and 5 brownies
• 6 people and 4 brownies
• 8 people and 5 brownies
Third Graders Questions about Brownies

• What kind of brownies?
• Are there nuts in there?
• Did you make them?
• Who gets the most?
• Who gets the least?
• How can we make them fair?
• Will two parts be the same as a bigger part?
• Does anyone get a whole brownie?
There is a shortage of red thread in the country...

- Stores are sold out. Companies are trying to find it everywhere.
- What might you notice about this?
- What might you wonder?
- What are the problems that might occur?
Flag Fractions

About what fractional part of the flag is red? How might you prove and explain your solution?
Each will group will get three country flags.

• Find the fractional part of red for each flag.
• Then order the flags from least to greatest.
• Show the value on a number line.
• Represent your group’s work on chart paper.
Facilitate meaningful mathematical discourse
Teacher and Student action (Flag Activity)

What might you see teachers do during the Flag lesson?

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
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</table>
Facilitate meaningful mathematical discourse
Teacher and Student actions

<table>
<thead>
<tr>
<th>What are <em>teachers</em> doing?</th>
<th>What are <em>students</em> doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</td>
<td>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole class discourse.</td>
</tr>
<tr>
<td>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</td>
<td>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</td>
</tr>
<tr>
<td>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</td>
<td>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others’ strategies, and describing the approaches used by others.</td>
</tr>
<tr>
<td>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</td>
<td>Identifying how different approaches to solving a task are the same and how they are different.</td>
</tr>
<tr>
<td>Statement</td>
<td>True/False</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>3 parts of size ¼ each.</td>
<td></td>
</tr>
<tr>
<td>3 parts of size ½ each.</td>
<td></td>
</tr>
<tr>
<td>Closer to 0 than to 1.</td>
<td></td>
</tr>
<tr>
<td>¼ more than ½.</td>
<td></td>
</tr>
<tr>
<td>1/3 more than 1/2.</td>
<td></td>
</tr>
<tr>
<td>3 parts in the Whole.</td>
<td></td>
</tr>
<tr>
<td>The unit fraction is 1/3.</td>
<td></td>
</tr>
<tr>
<td>The unit fraction is 1/4.</td>
<td></td>
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What does he tell us about his understanding of fractions?

“How do I share cookies?”
Miguel and Janie are each painting a mural. Janie says that she painted \( \frac{3}{4} \) of the mural and Miguel said that he painted \( \frac{3}{8} \) of the mural. Who has most of the mural painted? Use words, pictures, and representations to show your thinking.
Same number of parts but parts of different sizes!

Compare $\frac{3}{4}$ to $\frac{3}{8}$
Janie > Miguel
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments (NCTM, 2014, p. 29)

The question becomes... how do we provide opportunities for students to build shared understanding?
Five Practices to Promote Discourse (Smith and Stein, 2011)

1. **Anticipating** student responses prior to the lesson

2. **Monitoring** students’ work on and engagement with the tasks

3. **Selecting** particular students to present mathematical work

4. **Sequencing** students’ responses in a specific order for discussion

5. **Connecting** different students’ responses and connecting the student responses to key mathematical ideas.
# A Look inside a classroom

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### Facilitate meaningful mathematical discourse

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Pose Purposeful Questions

How do we plan, anticipate, and engage our students in questions?
Funneling - Involves using a set of questions to lead students to a desired procedure or conclusion, while giving limited attention to student responses that veer from the desired path. The teacher has decided on the path and leads the students along the path, not allowing them to make their own connections or build their own understanding (p. 37).

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3. **Explain it.** What could it be? What role or function might it serve? Why might it be there?
4. **Give reasons.** What makes you say that?
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During the Explanation Game, which types of questions did you ask and answer?
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<td>Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.</td>
<td>Expecting to be asked to explain, clarify, and elaborate on their thinking.</td>
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<td>Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.</td>
<td>Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly.</td>
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<td>Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion.</td>
<td>Reflecting on and justifying their reasoning, not simply providing answers.</td>
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<td>Allowing sufficient wait time so that more students can formulate and offer responses.</td>
<td>Listening to, commenting on, and questioning the contributions of their classmates.</td>
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If this ____ is ____ , what does the whole look like?

If this is 1/5, what does the whole look like?

If this is ¼, what does the whole look like?

Which whole is greater?
If this ____ is ____, what does the whole look like?

Try it with a partner. Choose a manipulative and a fraction.

What do you notice about the models?

What questions might you ask?
How do we compare fractions?

- What tasks can we use to stimulate discourse?
- What do you notice about her understanding about comparing fractions?

Order from least to greatest

\[
\frac{3}{8}, \frac{3}{15}, \frac{3}{2}, \frac{3}{16}
\]
How do we compare fractions?
Framework for Mathematics Teaching Questions

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Same number of parts but parts of different sizes!

Compare $\frac{3}{4}$ to $\frac{3}{8}$
Happy Days Preschool is having their annual tricycle race! We are now ten minutes into the race. The tricycles have traveled this far to the finishing line. Show where each tricycle is on the number line and then make a prediction about which tricycle will win.

What do you anticipate students might do?

What questions might you ask to promote discourse?
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

www.nctm.org
Pose Questions, Increase Discourse and Problem Solving

Content: Addition and Subtraction and Problem Solving

NCTM Common Core Institute, 2014

Beth McCord Kobett
Assistant Professor
Stevenson University
bkobett@stevenson.edu
The Take Away game

Using the cubes:
Make a 7 x 5 grid.

For: 2 players or 2 teams
Move 1: Player 1 removes one, two, or three cubes.

Move 2: Player 2 removes one, two, or three cubes.

Play continues until all the cubes are gone. The player that takes the last cube is the winner.

2 Adapted from: Powerful Problem Solving by Max Ray.
There are 25 sheep and 5 dogs in a flock. How old is the Shepherd?
Three out of four students will give a numerical answer to this problem.

Math Class

Have you ever been caught in a discourse loop like this?
<table>
<thead>
<tr>
<th>8 + 23</th>
<th>18 + 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>44 + 26</td>
<td>34 + 17</td>
</tr>
</tbody>
</table>
Now make your own in pairs – *Which one doesn’t belong.* Think about your...
Clarifying Questions Mingle

1. On a post it note. Explain how you would add $23 + 49 =

1. Stand up and begin Mingling.

1. When I give the signal, stop.

1. Say or read your explanation.

2. Others may ask one clarifying question.

Adapted from: Powerful Problem Solving by Max Ray.
Paired Interviews

• Interview each other.
• Talk about the best time you observed students engaged in discourse.
• What did you see?
• What did you notice?
• How did you feel?
• Then meet with another pair, share stories and identify the common themes among the stories.
Strategies for Increasing Discourse, improve questioning (yours and theirs, and problem solving)

- Interesting, rich tasks
- Notice, Wonder, Question, Engage in Tasks, and Solve Problems
- Games
- Explanation Game
“One of the great secrets to fostering deep learning is the ability to help students raise new kinds of questions that they will find fascinating.” – Ken Bain
“People are most likely to take a deep approach to their learning when they are trying to answer questions or solve problems that they have come to regard as important, intriguing, or just beautiful. One of the great secrets to fostering deep learning is the ability to help students raise new kinds of questions that they will find fascinating.”

Ken Bain
A little bit about questions...

- A child between the age of two and five, a child asks about 40,000 questions.
- During the shift of the three year span, there is a shift in the kinds of questions being asked from simple factual questions (like name of object) to explanations.
- At the same time, about a quadrillion connections are found in their brains (more than three times than an adult).
- The questions show how they are connecting stimuli and thoughts.
Let’s Ask Questions!

As we move through the activities, let’s continue to generate questions.
Notice, Wonder, and Question
Notice, Wonder, and Question
• How deep is the hole?
• How many people can fit in the hole?
• How tall is the boy?
• Can anyone else fit in there?
• Is he standing on something?
Notice, Wonder, and Question
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments (NCTM, 2014, p. 29)

The question becomes... how do we provide opportunities for students to build shared understanding?
Uncle Ronnie owns a t-shirt factory!

His factory is a giant mess!
He has t-shirts everywhere!
He needs to get organized!

• What are you noticing about what you are seeing, hearing or reading?

• What new things are you wondering about?
Uncle Ronnie owns a t-shirt factory!

His factory is a giant mess!
He has t-shirts everywhere!

What do you notice?
What questions do you have?
Questions about the T-shirt Factory

- What kind of t-shirts?
- Where is the factory?
- Did you make them?
- Who gets the most?
- Who gets the least?
- What can we do with the t-shirts?
- Is there a container store?
Aunt Betsy decided to help get his order together by having him package his t-shirts into groups of ten.

You are going to get a t-shirt order. Help Uncle Ronnie figure out how many groups of ten and how many single t-shirts you will have in your order.
How many of 10 t-shirts?

Of 10 t-shirts?
Jack has lots of shells. He has more than 40 but less than 60. When he counts them by twos, he has one left over. When counts them by fives, he has none left over.

The number of shells is more than half of 100. How many shells does Jack have? Show your thinking and reasoning.
Group Rotate and Share

- Two people will leave the group and rotate to the next group
- The other group members stay to explain and defend the work of the group.
Facilitate meaningful mathematical discourse
Teacher and Student action (Flag Activity)

What might you see teachers do during the shell lesson?
What might you see students do during the shell lesson?

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There are 24 feet in the house

What might you anticipate?

Analyze student work. How would you have students share? What questions might you ask? Be prepared to share your thinking.
Five Practices to Promote Discourse (Smith and Stein, 2011)

1. **Anticipating** student responses prior to the lesson

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Some children visited the Butterfly House at the zoo. They learned that the butterfly is made up 4 wings, one body, and two antennae. While they were there, they made models of the butterflies.
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How many wings, bodies and antennae would be needed to make 7 butterflies?

Show all work and explain your thinking.
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Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.
Windows, Dinos and Ants

Using the levels of classroom discourse, consider how you might score the small snippet we watch.
Sam has 4 tables for his birthday party. His mother puts the same number of things at each table.

He has 16 plates, 8 horns, 4 flowers, and 16 chairs.

What questions do you have?
Facilitate Meaningful Discourse

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<td>How might you prove that 51 is the solution?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>How do you know that the sum of two odd numbers will always be even?</td>
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<td></td>
<td></td>
<td>Why does plan A in the Smartphone Plans task start out cheaper but become more expensive in the long run?</td>
</tr>
<tr>
<td><strong>What are teachers doing?</strong></td>
<td><strong>What are students doing?</strong></td>
<td></td>
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<tr>
<td>---------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td></td>
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<tr>
<td>Advancing student understanding by asking questions that build on, but do not take over or</td>
<td>Expecting to be asked to explain, clarify, and elaborate on their thinking.</td>
<td></td>
</tr>
<tr>
<td>funnel, student thinking.</td>
<td>Thinking carefully about how to present their responses to questions clearly, without rushing to</td>
<td></td>
</tr>
<tr>
<td>Making certain to ask questions that go beyond gathering information to probing thinking</td>
<td>respond quickly.</td>
<td></td>
</tr>
<tr>
<td>and requiring explanation and justification.</td>
<td>Reflecting on and justifying their reasoning, not simply providing answers.</td>
<td></td>
</tr>
<tr>
<td>Asking intentional questions that make the mathematics more visible and accessible for</td>
<td>Listening to, commenting on, and questioning the contributions of their classmates.</td>
<td></td>
</tr>
<tr>
<td>student examination and discussion.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Allowing sufficient wait time so that more students can formulate and offer responses.</td>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Question type</strong></th>
<th><strong>Description</strong></th>
<th><strong>Examples</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gathering Information</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probing Thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Making the Mathematics Visible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Encouraging Reflection and Justification</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
How can two people share five pennies?
How can two children share 5 pennies?

1. Give 1 penny in way to a person then two children 2 pennies.
2. Give 2 pennies a way that the children will have 1 penny.
How can two children share 5 pennies?

You can’t split the 5 pennies between two people because 5 is an odd number. So you can get one more penny and the number becomes 6. Each kid would get 3 pennies.

6 ÷ 2 = 3
3 + 3 = 6
5 + 1 = 6
2 × 3 = 6.
How can two children share 5 pennies?

buy something and share

throw one in a water fountain
give one to your mom and dad. give one to your teacher.

2 + 2 + 1 = 5
How can two children share 5 pennies?

One child takes two and one, another takes three.  \[ x + y = 5 \]

I've been to a place that sells full bags for two sense.

Gum

Jordan
Jenny’s Birthday

Jenny is 7 years old today! How many birthday candles have been on her cake since she was born?
Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables. She has a supply of 31 legs.

1. Solve

2. What do you anticipate students might do?

3. What questions might you ask to promote discourse?

4. Let’s listen in on some students.
How do we create a place where thinking is valued and visible?

- Do students regularly engage in wonder?
- Do students ask lots and lots of questions?
- Do students exhibit the practices AS they engage in the content?
- Do students have an opportunity to collaborate and build on each other’s thinking?
“If we want to promote a culture of thinking, we must surround students with thinking, not as a special activity that we engage in on special occasions, but in the daily ordinariness of the classroom.”
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

www.nctm.org
Eliciting and Using Evidence of Student Thinking

Effective Teaching with Principles to Action Institute, 2015

John SanGiovanni
Elementary Mathematics Supervisor
Howard County Public School System
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Evidence of Student Thinking
Paired Interviews

• Interview the person sitting next to you and ask:
• What is the most surprising student evidence of learning you have collected?
• How did you collect this information? When did it happen?
• Why was it surprising?
• What did you do differently as a result of learning this information?
Welcome to the Clapping Institute!
Welcome to the Clapping Institute!

• Listen to my clapping.
• Evaluate my clapping on a scale of 1-5.
• Share with a partner why you scored me the way that you did.
Welcome to the Clapping Institute!

• A volunteer....

• Please leave the room. You will return to perform for us in a few moments.
Performance Criteria

How will we use each of these criteria?

– Volume (1-5)
– Appropriateness (1-5)
– Creativity (1-5)
Our Volunteer’s Performance

Talk with your neighbor.
Score our volunteer.
Please clap for us. We will score your performance on each of these categories.

- Volume (1-5)
- Appropriateness (1-5)
- Creativity (1-5)
Think · Pair · Share

– **Think:** What are your thoughts and insights on this activity? Have you ever been in the position of one of the volunteers? Record a few statements and be prepared to share with a partner in three minutes.

– **Pair:** Discuss your reflections with a partner in three minutes.

– **Share:** Share with the group.
Let’s Clap!

• You have to know what you’re looking for...
• They have to know what you’re looking for
• Plan for what you’re looking for
  – Anticipating student responses to mathematics content and tasks.
  – Determining what is evidence
Think-Pair-Share:
How should instruction and assessment be related?

1. \( \text{INSTRUCTION} \cap \text{ASSESSMENT} \)
2. \( \text{INSTRUCTION} \cup \text{ASSESSMENT} \)
3. \( \text{INSTRUCTION} \circ \text{ASSESSMENT} \)
4. \( \text{INSTRUCTION} \cap \text{ASSESSMENT} \)
Formative Assessment is Like Looking for Icebergs

- Summative Assessment
- Selected Response Items
- Observations
- Conversations
- Interviews
- Extended response tasks
- Journals and reflections
- Open-Ended Questions
- Portfolios
- Reports and projects
- Exit tickets
Formative Assessment is like GPS.

1. Where are we going?
2. Where are we?
3. How are we getting there?
Why Use Formative Assessment?

- To inform instruction and provide feedback to students (and you) on their learning.

The Big Picture

- Where are you? – formative assessment
- Where are you going? – standards
- How are you going to get there? – progressions
What Beliefs Might We Hold about Mathematics Assessment?

At your table:

Choose two belief statements and:

1. Decide whether the statement is a productive or unproductive belief.
2. Explain your reasoning.
3. Describe the point of view of someone you know about the same belief statement. How might have that belief statement developed?

<table>
<thead>
<tr>
<th>Beliefs About Mathematics Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select 2 statements about assessment. Write P if it is a productive belief or U if it is an unproductive belief. Be sure to explain your reasoning.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>P or U</th>
</tr>
</thead>
<tbody>
<tr>
<td>The primary purpose of assessment is accountability for students through report card marks or grades.</td>
<td>U</td>
</tr>
<tr>
<td>Only multiple-choice and other &quot;objective&quot; paper-and-pencil tests can measure mathematical knowledge reliably and accurately.</td>
<td>P</td>
</tr>
<tr>
<td>Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.</td>
<td>P</td>
</tr>
<tr>
<td>Assignment in the classroom is an interruption of the instructional process.</td>
<td>U</td>
</tr>
<tr>
<td>Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.</td>
<td>P</td>
</tr>
<tr>
<td>A single assessment can be used to make important decisions about students and teachers.</td>
<td>U</td>
</tr>
<tr>
<td>Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.</td>
<td>P</td>
</tr>
<tr>
<td>Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.</td>
<td>P</td>
</tr>
<tr>
<td>Assessment is something that is done to students.</td>
<td>U</td>
</tr>
<tr>
<td>Stopping teaching to review and take practice tests improves students’ performance on high-stakes tests.</td>
<td>U</td>
</tr>
<tr>
<td>Multiple data sources are needed to provide an accurate picture of teacher and student performance.</td>
<td>U</td>
</tr>
<tr>
<td>Ongoing review and distributed practice within effective instruction are productive test preparation strategies.</td>
<td>U</td>
</tr>
</tbody>
</table>

Unproductive Belief Statements

• The primary purpose of assessment is accountability for students through report card marks or grades.

• Assessment in the classroom is an interruption of the instructional process.

• Only multiple-choice and other “objective” paper-and-pencil tests can measure mathematical knowledge reliably and accurately.

• A single assessment can be used to make important decisions about students and teachers.

• Assessment is something that is done to students.

• Stopping teaching to review and take practice tests improves students’ performance on high-stakes tests.
Productive Beliefs Statements

• The primary purpose of assessment is to inform and improve the teaching and learning of mathematics.

• Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.

• Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.

• Multiple data sources are needed to provide an accurate picture of teacher and student performance.

• Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.

• Ongoing review and distributed practice within effective instruction are productive test preparation strategies.
Elena and her 3 friends ate 9 cookies. How many cookies did each friend eat?

Using the student work as your only evidence, what do you believe the student understands?
Did the student’s explanation of his thinking change your initial impression of his understanding?

How?
Let’s consider...

- What counts as evidence...
  - Prerequisite/progression of concept
  - Representations
  - Concepts/conceptual understanding
  - Possible procedures/symbols

- How you might collect evidence?
Task/Question to investigate:

- Interpreting their thinking
  - Do your students understand?
  - How do you know?

- Making In-The-Moment Decisions
  - How is their work similar/different than what you expected?
  - How do you reflect on what you expected?
  - How might you adapt what you expected?
  - Does anything surprise you?

- What might you change?
What Counts As Evidence
What Might Count As Evidence?

Compare. Explain how they could be compared without finding common denominators.
Making Use of Evidence: What Do We Notice?

Write $>,$ $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.

\[
\frac{5}{6} \quad > \quad \frac{2}{8}
\]

Tell how you compared them without common denominators.

I used pictures to help so I drew \(\frac{5}{6}\) and \(\frac{2}{8}\) and \(\frac{5}{6}\) covered more area.
Making Use of Evidence:
What Do We Notice:

Write $>$, $<$, or $=$ in the circle to compare the fractions. Explain how they could be compared without using common denominators.

Tell how you compared them without common denominators.

I compared them by seeing how much more you need to get 8 from 2 and 5 to 6 in $\frac{5}{6}$ you need 1 part and on $\frac{2}{8}$ you need 6 parts. That's how I compared.
Interpreting Student Thinking
Fractions on a Number Line

The point shows \( \frac{3}{4} \) on each number line. Write the missing endpoint for each number line in the box.
Fractions on a Number Line

How might our students find the missing endpoint?

What strategies might they use?

What misconceptions might they have?
Interpreting Student Thinking: What’s Happening Here?

1. The point shows ⅓ on each number line. Write the missing endpoint for each number line in the box.

   Explain how you found your endpoints.

   I counted the lines to see how many it was.

2. The point shows ⅓ on each number line. Write the missing endpoint for each number line in the box.

   Explain how you found your endpoints.
Interpreting Student Thinking: What’s Happening Here?

3. The point shows $\frac{3}{4}$ on each number line. Write the missing endpoint for each number line in the box.

\[0 \quad \frac{2}{4} \quad 1 \]

Explain how you found your endpoints.

I counted how many 4ths there were on each number line then how many whales there were in the box.

\[0 \quad \frac{3}{4} \quad 1\]

4. The point shows $\frac{3}{4}$ on each number line. Write the missing endpoint for each number line in the box.

\[0 \quad \frac{3}{4} \quad 1\]

Explain how you found your endpoints.

I lined up them all to equal $3\frac{1}{4}$.
Making In-the-moment Decisions
Interpreting Student Thinking

Students are solving $\frac{3}{4} + \frac{1}{2}$

- What might you expect to see?
- How might you react?
Reflecting on Evidence of Student Learning to Inform Next Steps
What Happens Next?

A student is solving $\frac{5}{6} + \frac{2}{5}$...

• What might you expect to see?
• What would you plan to do next for this student?
Putting When could \( \frac{1}{4} \) be bigger than \( \frac{1}{2} \)? Use words and pictures to explain your thinking.

What do you anticipate students might do, say, and represent?

What are some possible student misconceptions?

What kinds of student evidence do you expect?
When could \( \frac{1}{4} \) be bigger than \( \frac{1}{2} \)?

Use words and pictures to explain your answer.

\( \frac{1}{4} \) being bigger than \( \frac{1}{2} \) is impossible. It is not possible because:

\[
\frac{1}{2} \text{ is bigger than } \frac{1}{4}.
\]
When could 1/4 be bigger than 1/2?
Use words and pictures to explain your answer.

You can make 1/4 bigger than 1/2 when you have 2 pizzas. One person was bigger and one was smaller. Well, on the one that's bigger you would do 1/2 and on the smaller side you would do 1/2. But you would do 1/4 on the big piece so it's bigger.
When could $\frac{1}{4}$ be bigger than $\frac{1}{2}$? Use words and pictures to explain your answer.

$\frac{1}{4}$ could be bigger than $\frac{1}{2}$ when the shape that was bigger than the other shape.
\( \frac{1}{4} \) of my desk is bigger than \( \frac{1}{2} \) of my colorless box.

\[ \frac{1}{4} \]

\[ \frac{1}{2} \]

\[ \text{desk} = 18 \text{ inches} \]
\[ \frac{4.5}{18.0} \]
\[ \frac{16}{20} \]
\[ \frac{1}{4} \text{ of desk} = 4.5 \text{ inches} \]

pencil box = 8 inches
Implement tasks that promote reasoning and problem solving.

Facilitate meaningful mathematical discourse.

Establish mathematics goals to focus learning.

Use and connect mathematical representations.

Pose purposeful questions.

Build procedural fluency from conceptual understanding.

Elicit and use evidence of student thinking.

Support productive struggle in learning mathematics.
<table>
<thead>
<tr>
<th>“In a Nutshell”</th>
<th>Design a LOGO for the practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Big Ideas about the Practice</td>
<td>What are teacher behaviors of the practice?</td>
</tr>
<tr>
<td>What are student behaviors of the practice?</td>
<td></td>
</tr>
</tbody>
</table>
Institute Closing:
Personal reflection on practices

• Which is the easiest for you to implement?

• Which is most challenging?

• How might you use your strength to develop your challenge?
Institute Closing: Content Debrief

What might you **Start** as a result of the Institute?

What might you **Stop** as a result of the Institute?

What might you **Keep** as a result of the Institute?
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