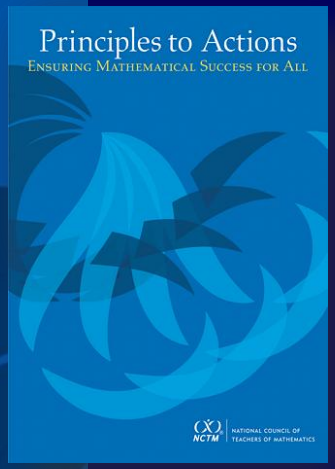
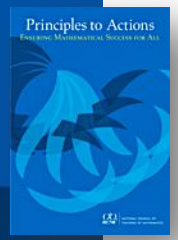


Effective Mathematics Teaching Practices that Support Learning for *All* Students: A Focus on Elementary School

DeAnn Huinker
University of Wisconsin–Milwaukee
huinker@uwm.edu

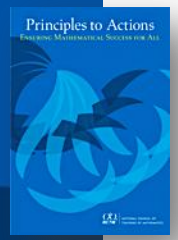
Effective Teaching with *Principles to Actions*:
Implementing College- and Career-Readiness Standards
An NCTM Interactive Institute, Charleston, SC

February 6, 2015



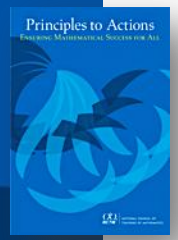
Our Agenda

- Overview of the *Teaching and Learning Principle* and the "8" mathematics teaching practices.
- Examine a short case of a third grade teacher implementing a lesson with his students.
- Relate the eight teaching practices to the case.
- Closing thoughts and reflections.



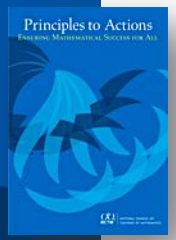
A time when I felt realy smart was when my teacher Ms. Robinson gave my class a hard problem that took us a long time to solv it.

Second Grade Student



I felt really smart in math when I used my number line

Second Grade Student

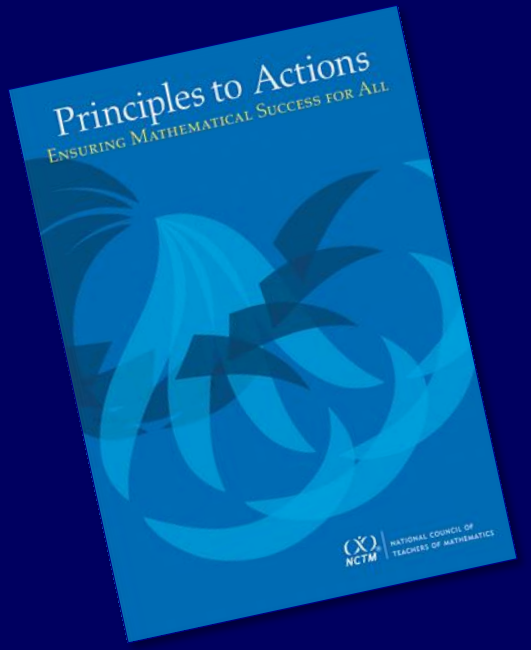


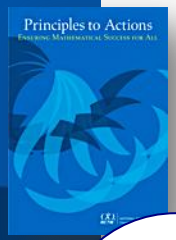
Professional Learning Goals

We are learning to:

Recognize and strengthen those aspects of our ***teaching practice*** *that provide* ***high leverage*** *in furthering students' success in mathematics and confidence in themselves as capable mathematical learners.*

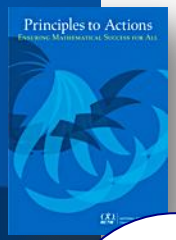
Teaching and Learning Principle





Guiding Principles for School Mathematics

- Teaching and Learning
- Access and Equity
- Curriculum
- Tools and Technology
- Assessment
- Professionalism



Guiding Principles for School Mathematics

Teaching and Learning

Access and Equity

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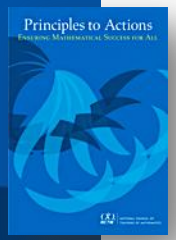
Tools and Technology

Assessment

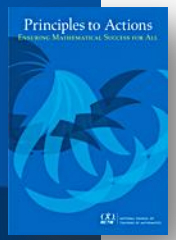
Professionalism



Overarching Message

Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.

Principles to Actions (NCTM, 2014, p. 4)



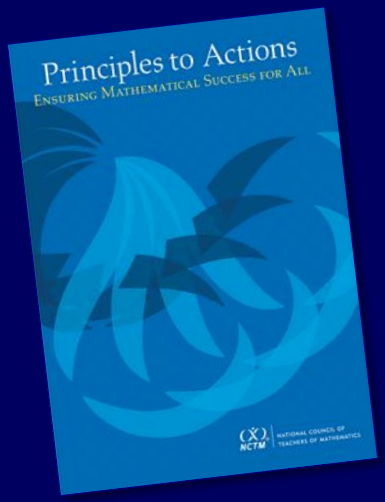
Why Focus on Teaching?

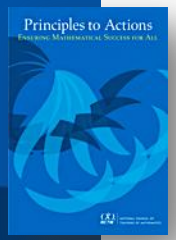
Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum."

(Ball & Forzani, 2011, p. 17)

Ball, D. L, & Forzani, F. M. (2011). Building a common core for learning to teach, and connecting professional learning to practice. *American Educator, 35*(2), 17-21.

Effective Mathematics Teaching Practices



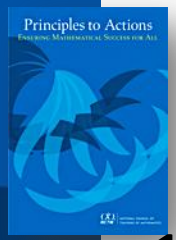


High-leverage, Effective Mathematics Teaching Practices

"Those practices at the heart of the work of teaching that are most likely to affect student learning."

(Ball & Forzani, 2010, p 45)

Ball, D. L, & Forzani, F. M. (2010). Teaching skillful teaching. *Educational Leadership, 68*(4), 40-45.



Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build procedural **fluency** from conceptual understanding.
7. Support productive **struggle** in learning mathematics.
8. Elicit and use **evidence** of student thinking.

Task: The Band Concert

The Band Concert

Make a quick sketch or diagram of the situation.

How might Grade 3 students approach this task?

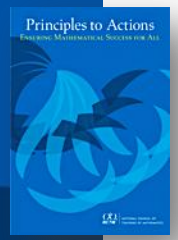
The third-grade class is responsible for setting up the chairs for their spring band concert. In preparation, they need to determine the total number of chairs that will be needed and ask the school's engineer to retrieve that many chairs from the central storage area.

The class needs to set up 7 rows of chairs with 20 chairs in each row, leaving space for a center aisle.

How many chairs does the school's engineer need to retrieve from the central storage area?

Math Goals	What might be the math learning goals?
Tasks & Representations	What representations might students use in reasoning through and solving the problem?
Discourse & Questions	How might we question students and structure class discourse to advance student learning?
Fluency from Understanding	How might we develop student understanding to build toward aspects of procedural fluency?
Struggle & Evidence	How might we check in on student thinking and struggles and use it to inform instruction?

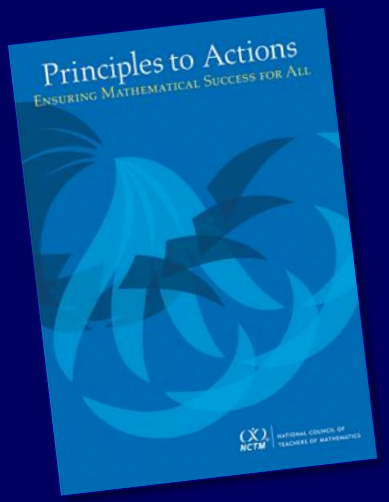
Case of Mr. Harris and the Band Concert Task



The Case of Mr. Harris and the Band Concert Task

- **Read** the Case of Mr. Harris and study the strategies used by his students.
- **Make note** of what Mr. Harris did before or during instruction to support his students' developing understanding of multiplication.
- **Talk** with a neighbor about the "Teaching Practices" Mr. Harris is using and how they support students' progress in their learning.

Relating the Case to the Mathematics Teaching Practices



Math Teaching Practice 1

Establish mathematics goals to focus learning.

Formulating clear, explicit learning goals sets the stage for everything else.

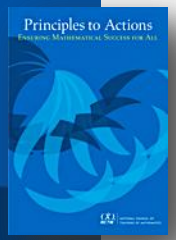
(Hiebert, Morris, Berk, & Janssen, 2007, p. 57)

Establish mathematics goals to focus learning

Learning Goals should:

- Clearly state what it is students are to learn and understand about mathematics as the result of instruction.
- Be situated within learning progressions.
- Frame the decisions that teachers make during a lesson.

Daro, Mosher, & Corcoran, 2011; Hattie, 2009; Hiebert, Morris, Berk, & Jensen., 2007; Wiliam, 2011



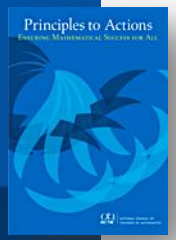
Let's consider the math goals and how they focused student learning with the Band Concert task.

Band Concert

Set up 7 rows of chairs with 20 chairs in each row, with a center aisle.

Case:

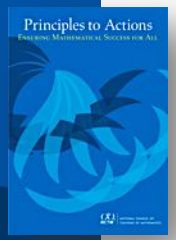
Lines 4-10



Common Core State Standards

Standard 3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Standard 3.NBT. 3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.



Math Goal

Students will recognize the structure of multiplication as equal groups within and among different representations, focusing on identifying the number of equal groups and the size of each group within collections or arrays.

Student-friendly version ...

We are learning to represent and solve word problems and explain how different representations match the story situation and the math operations.

Keep the math goal in mind as we delve into the case, and consider how it guided and informed the actions and decisions of Mr. Harris during the lesson.

Math Learning Goal

Students will recognize the structure of multiplication as equal groups within and among different representations—identify the number of equal groups and the size of each group within collections or arrays.

Math Teaching Practice 2

Implement tasks that promote reasoning and problem solving.

Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature.

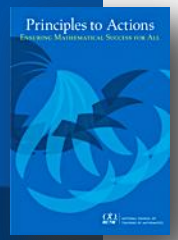
(Boaler & Staples. 2008; Stein & Lane. 1996)

Implement tasks that promote reasoning and problem solving

Mathematical tasks should:

- Allow students to explore mathematical ideas or use procedures in ways that are connected to understanding concepts.
- Build on students' current understanding and experiences.
- Have multiple entry points (access).
- Allow for varied solution strategies.

Boaler & Staples, 2008; Hiebert et al., 1997; Stein, Smith, Henningsen, & Silver, 2009



Turn and Talk

In what ways did the implementation of the task allow for multiple entry points (access) and engage students in reasoning and problem solving?

Case:

Lines 26-30

Lines 37-40

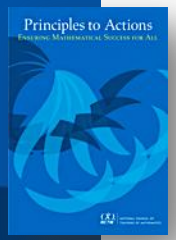
Math Teaching Practice 3

Use and connect mathematical representations.

Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas.

(National Research Council, 2001, p. 94)

National Research Council [NRC]. (2001). *Adding it up: Helping children learn mathematics* Washington, DC: National Academy Press.



"The teaching of representational competence should lie at the center of classroom practice in math and science."

(Collins, 2011, p. 105)

Collins, A. (2011). Representational competence: A Commentary on the Greeno analysis. In T. Koschmann (Ed.), *Theories of learning and research into instructional practice* (pp. 105-112). New York: Springer.

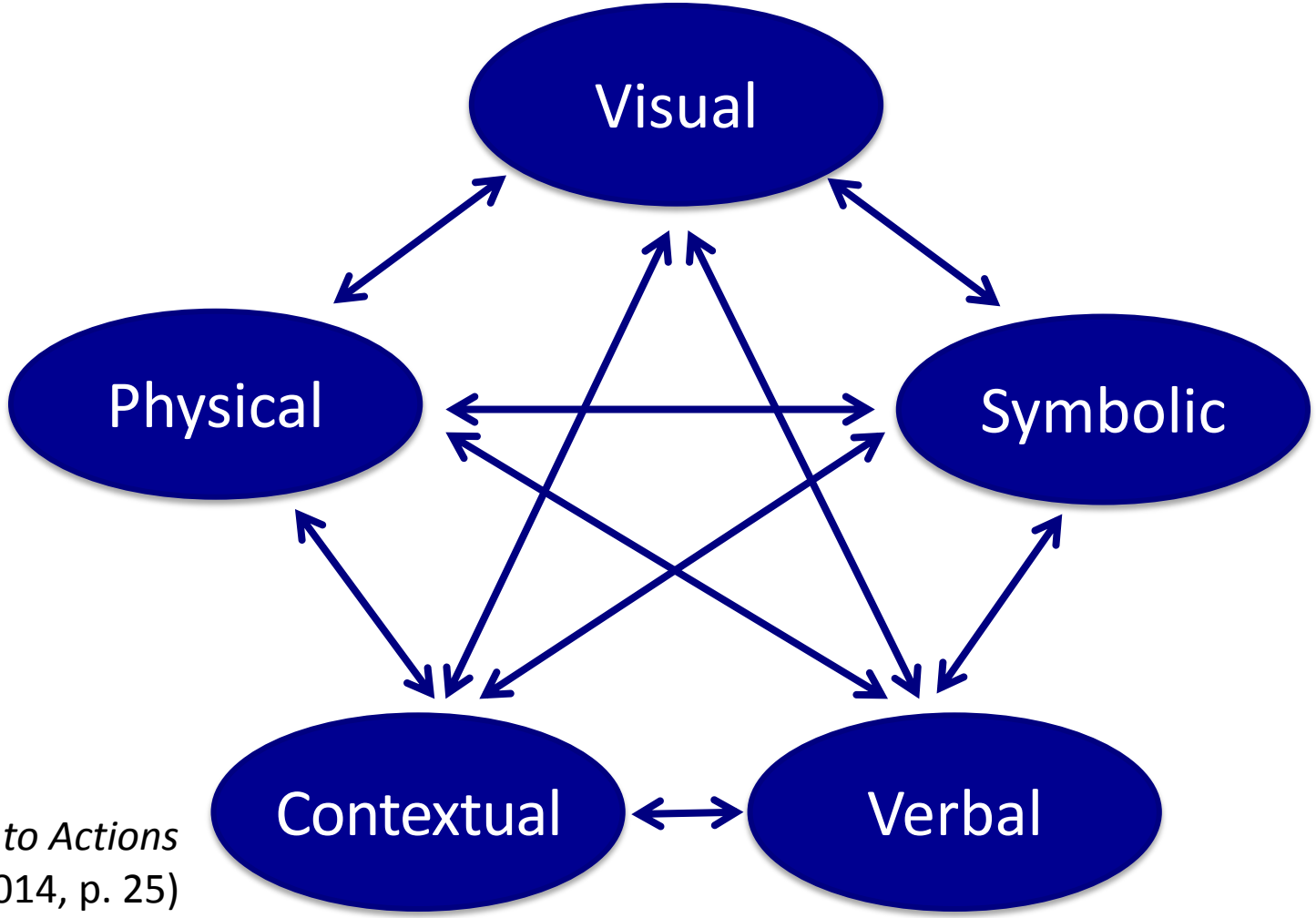
Use and connect mathematical representations

Different Representations should:

- Be introduced, discussed, and connected.
- Be used to focus students' attention on the structure of mathematical ideas by examining essential features.
- Support students' ability to justify and explain their reasoning.

Lesh, Post, & Behr, 1987; Marshall, Superfine, & Canty, 2010; Tripathi, 2008; Webb, Boswinkel, & Dekker, 2008

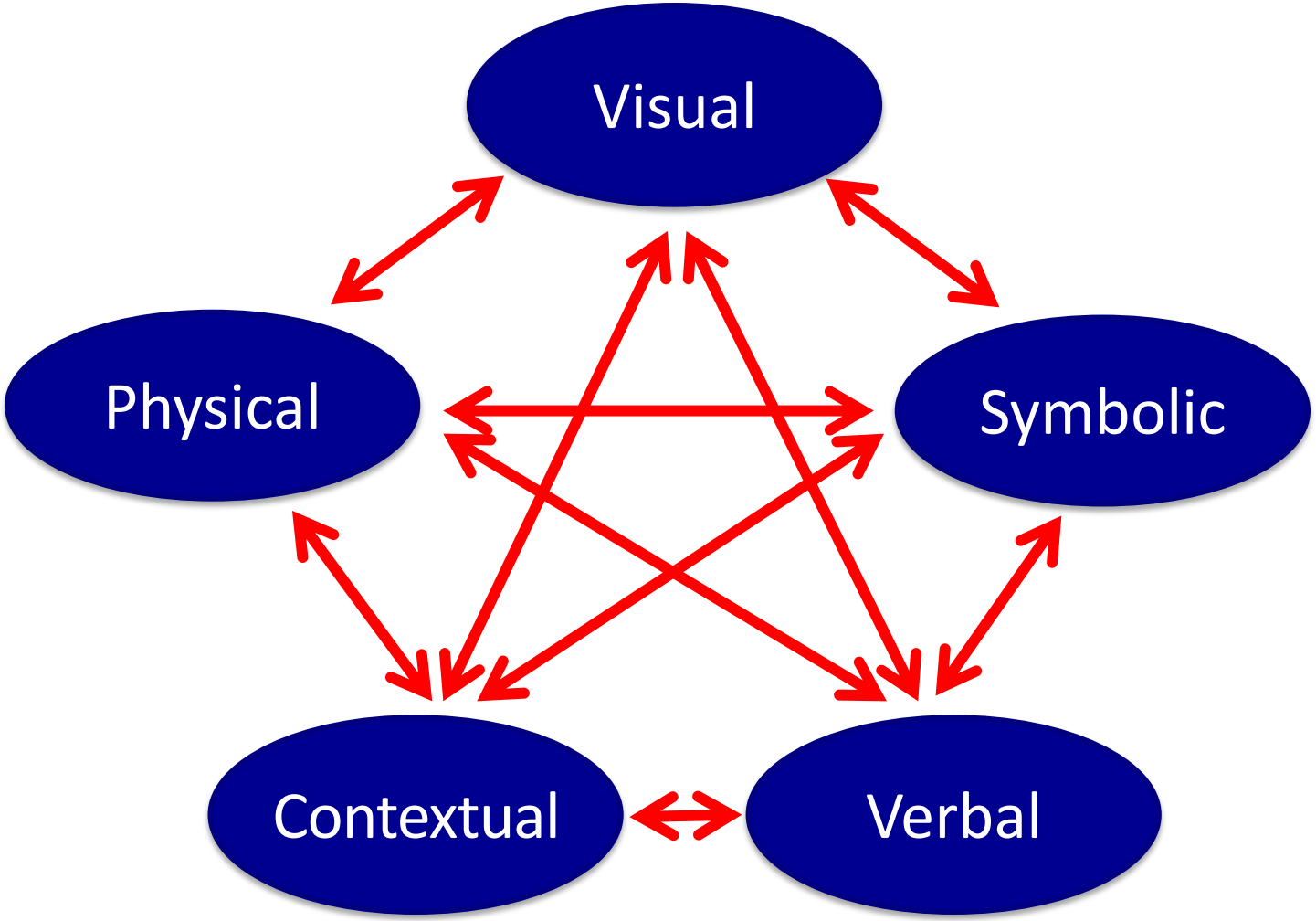
Important Mathematical Connections between and within different types of representations



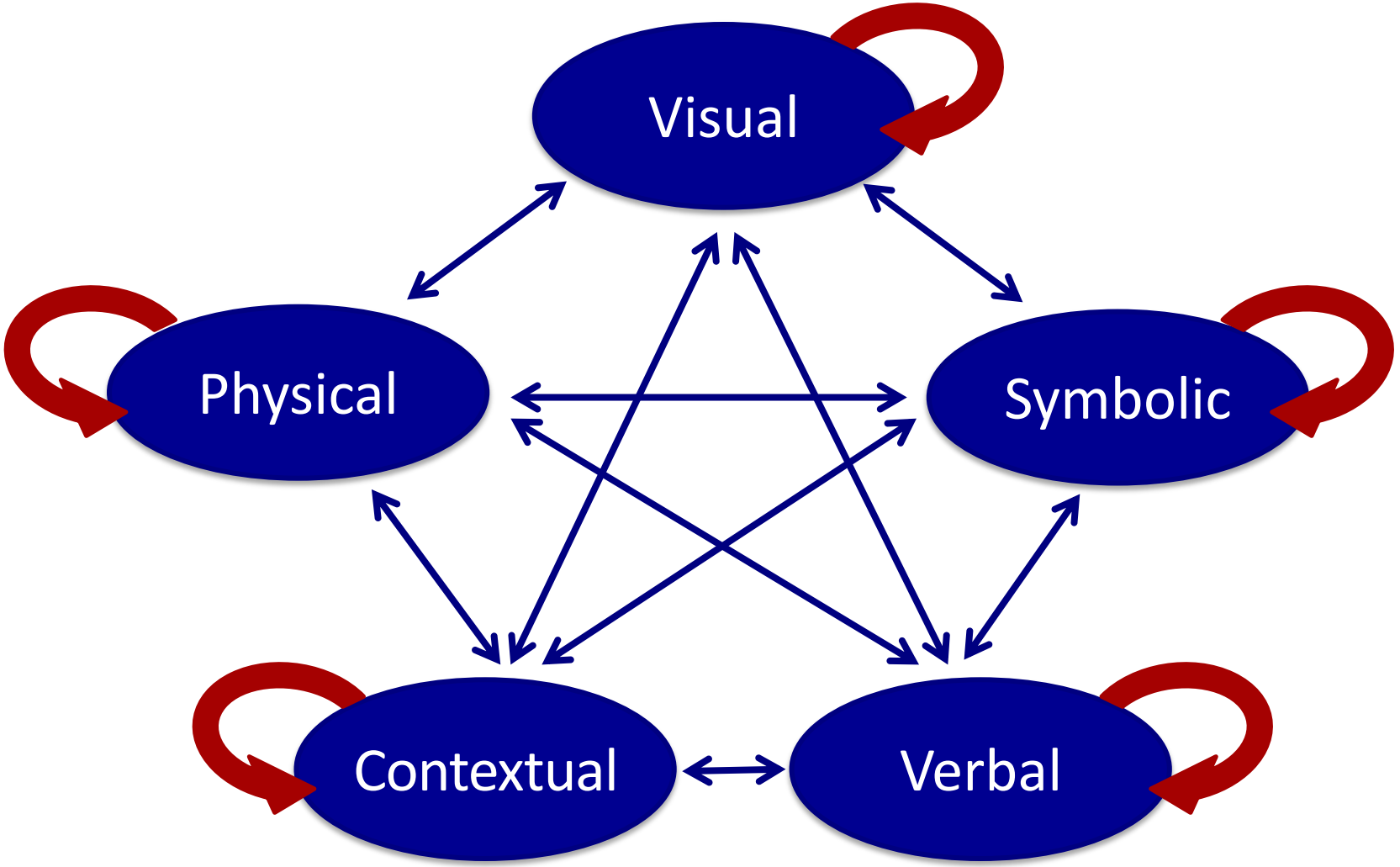
Principles to Actions
(NCTM, 2014, p. 25)

Adapted from: Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Important Mathematical Connections between and within different types of representations



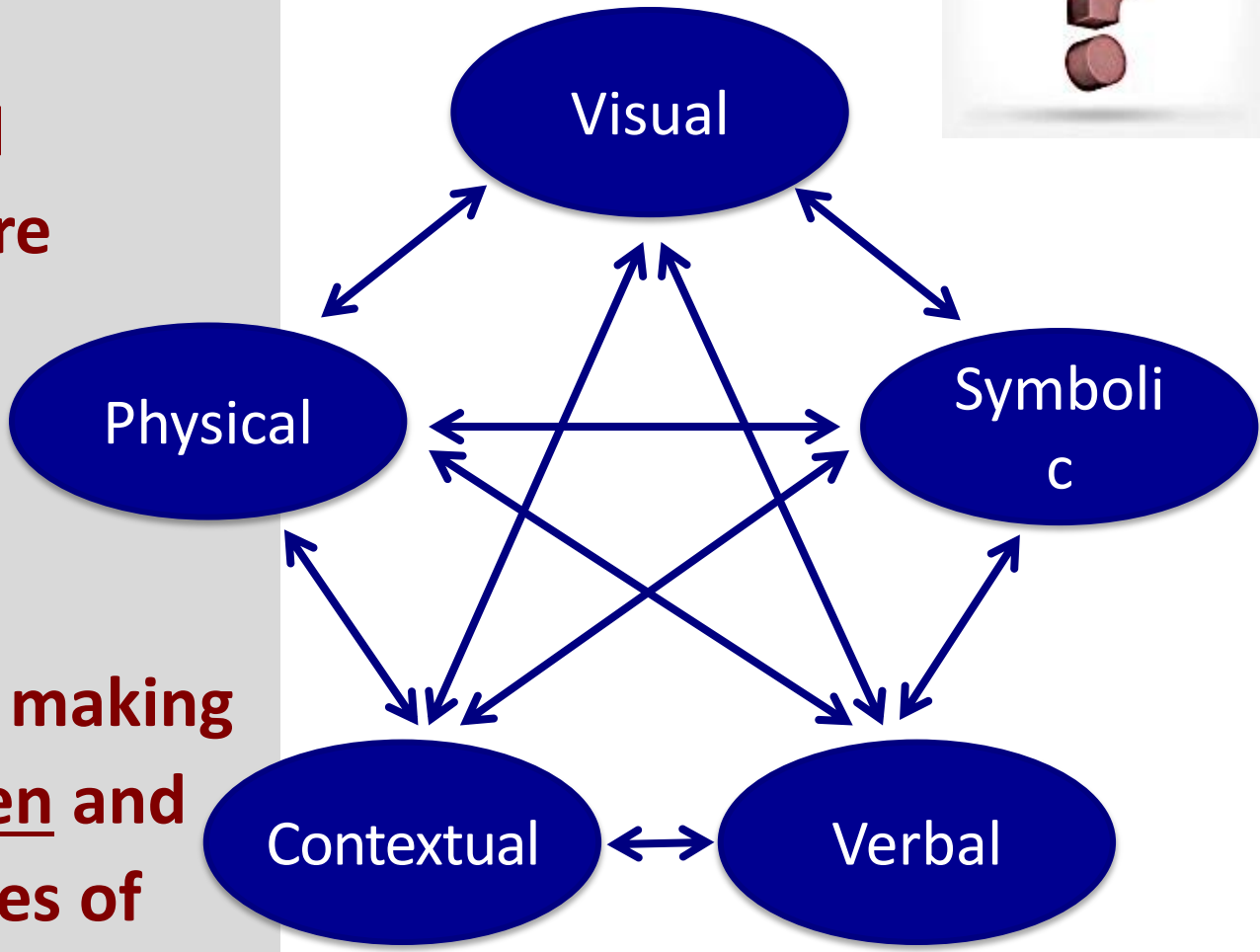
Important Mathematical Connections between and within different types of representations



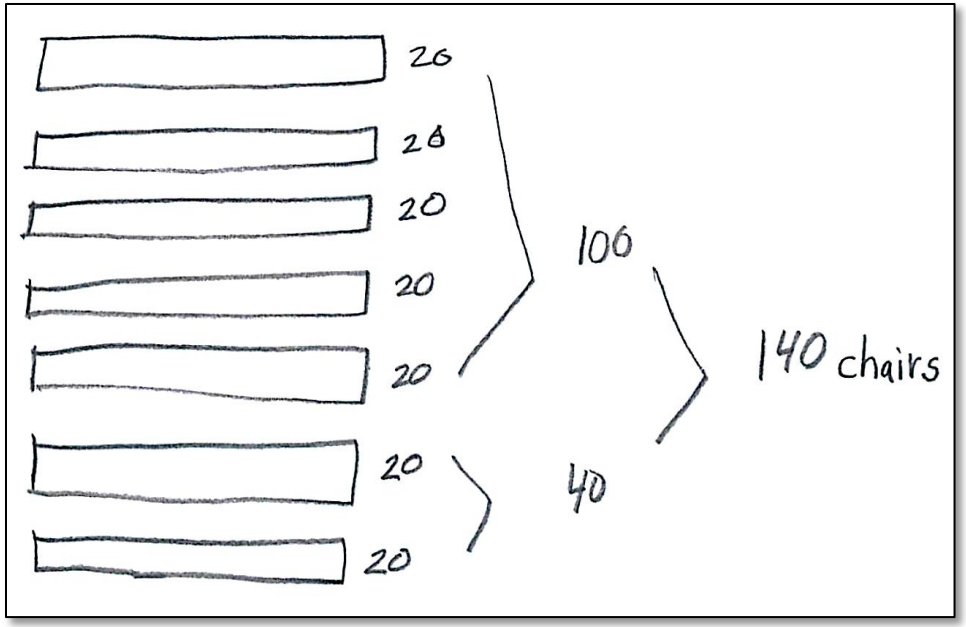
Turn and Talk

What mathematical representations were students working with in the lesson?

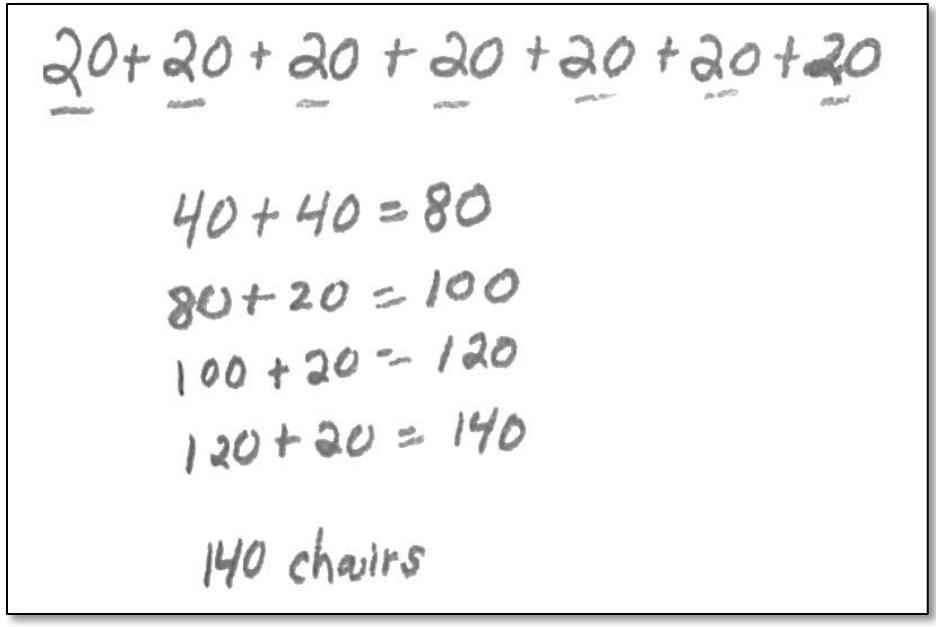
How did Mr. Harris support students in making connections <u>between</u> and <u>within</u> different types of representations?



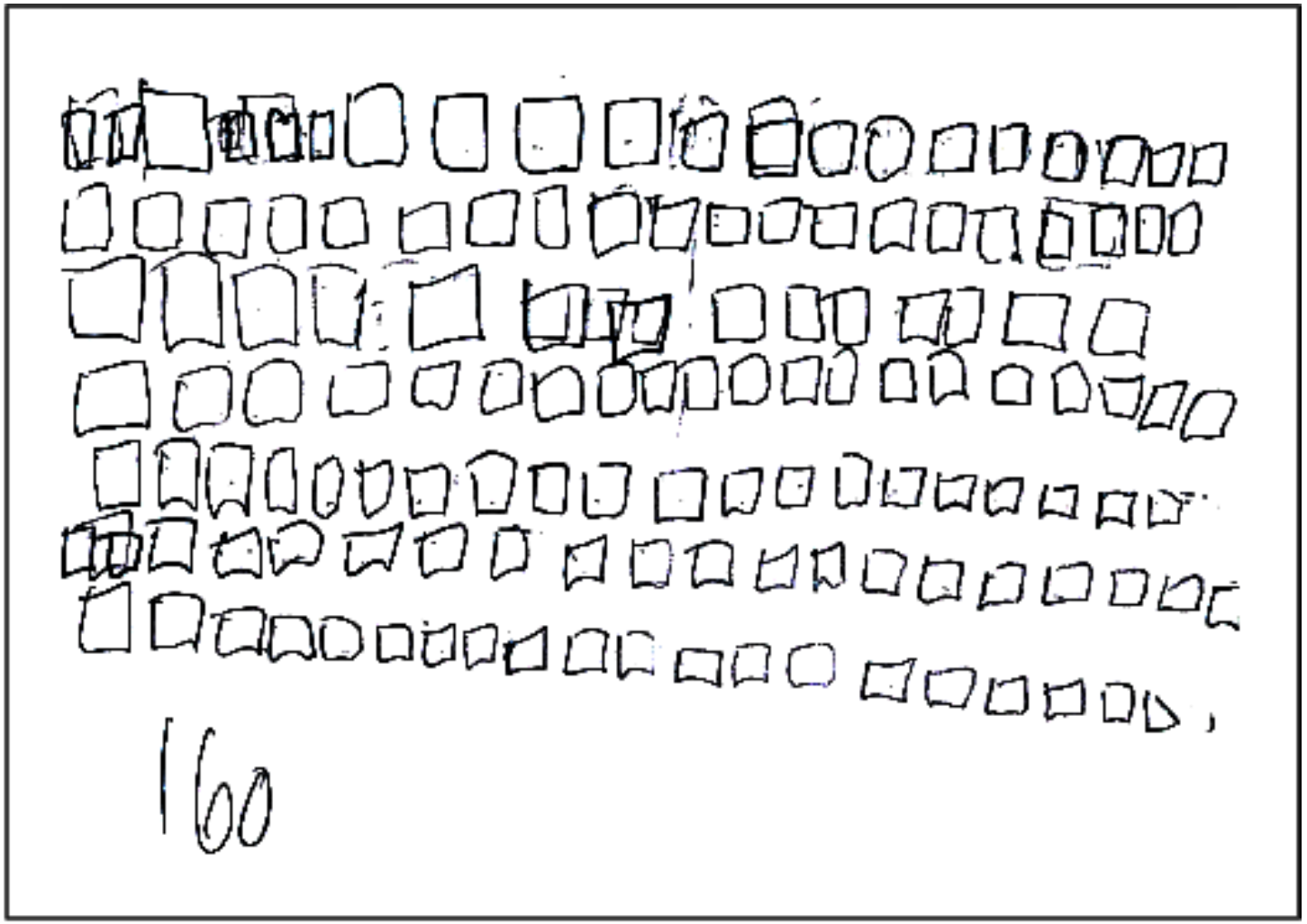
Lines 42-50: Pairs of students compared their representations.



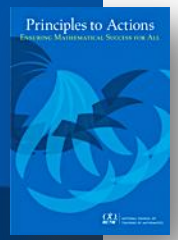
Jasmine



Kenneth



Molly



Representational Competence

The ability to use representations meaningfully to communicate mathematical ideas and to solve problems.

Marshall and colleagues suggest three strategies:

1. Encourage purposeful selection of representations.
2. Engage in dialogue about explicit connections among representations.
3. Alternate the direction of the connections made among representations.

Principles to Actions (NCTM, 2014, p. 26)

Marshall, A. M., Superfine, A. C., & Canty, R. S. (2010). Star students make connections. *Teaching Children Mathematics, 17*(1), 38-47.

Math Teaching Practice 4	Facilitate meaningful mathematical discourse.
Math Teaching Practice 5	Pose purposeful questions.

Facilitate meaningful mathematical discourse

Mathematical Discourse should:

- Build on and honor students' thinking.
- Let students share ideas, clarify understandings, and develop convincing arguments.
- Engage students in analyzing and comparing student approaches.
- Advance the math learning of the whole class.

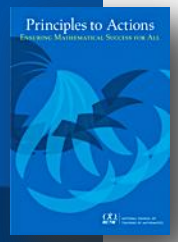
Carpenter, Franke, & Levi, 2003;
Fuson & Sherin, 2014; Smith & Stein, 2011

Pose purposeful questions

Effective Questions should:

- Reveal students' current understandings.
- Encourage students to explain, elaborate, or clarify their thinking.
- Make the targeted mathematical ideas more visible and accessible for student examination and discussion.

Boaler & Brodie, 2004; Chapin & O'Connor, 2007; Herbel-Eisenmann & Breyfogle, 2005



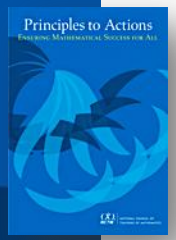
Questions

What do you notice about the questions that Mr. Harris asked on lines 32-35?

What purpose did his questions serve?

Whole Class Discourse

How did Mr. Harris structure the class discussion (lines 51-56) to advance student learning?



Purposeful Questions...

Lines 32-35

"How does your drawing show 7 rows?"

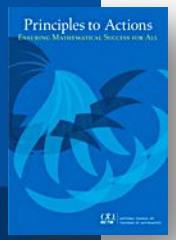
"How does your drawing show that there are 20 chairs in each row?

"How many twenties are you adding, and why?"

"Why are you adding all those twenties?

Math Learning Goal

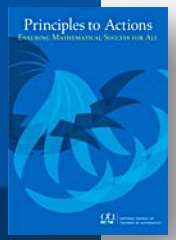
Students will recognize the structure of multiplication as equal groups within and among different representations—identify the number of equal groups and the size of each group within collections or arrays.



Structuring Mathematical Discourse...

During the whole class discussion of the task, Mr. Harris was strategic in:

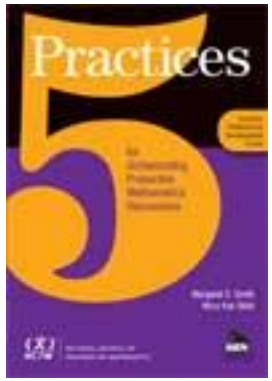
- Selecting specific student representations and strategies for discussion and analysis.
- Sequencing the various student approaches for analysis and comparison.
- Connecting student approaches to key math ideas and relationships.



1. Anticipating
2. **Monitoring**
3. **Selecting**
4. **Sequencing**
5. **Connecting**

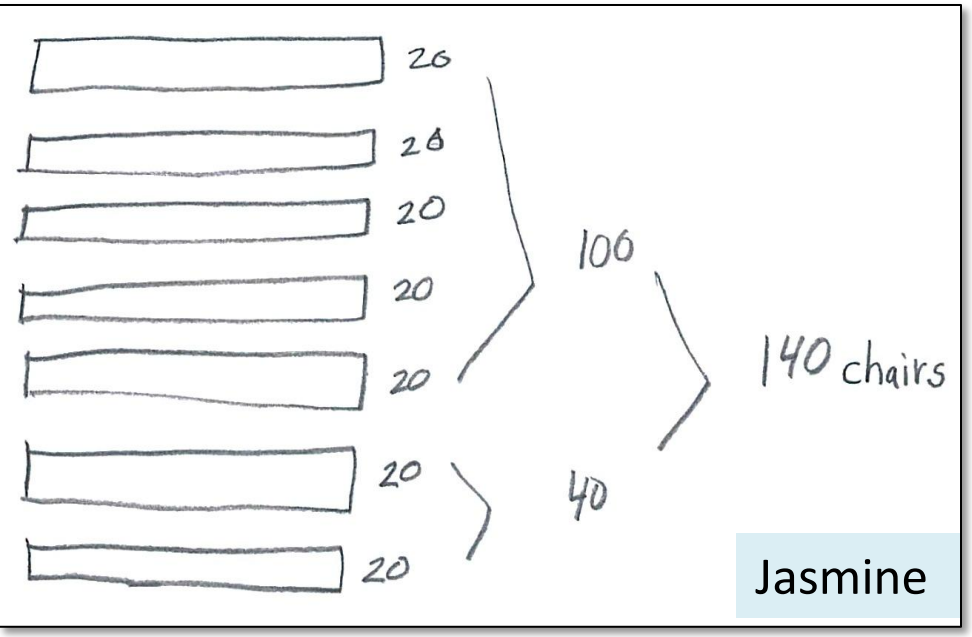
5 Practices for Orchestrating Productive Mathematics Discussions

(Smith & Stein, 2011)

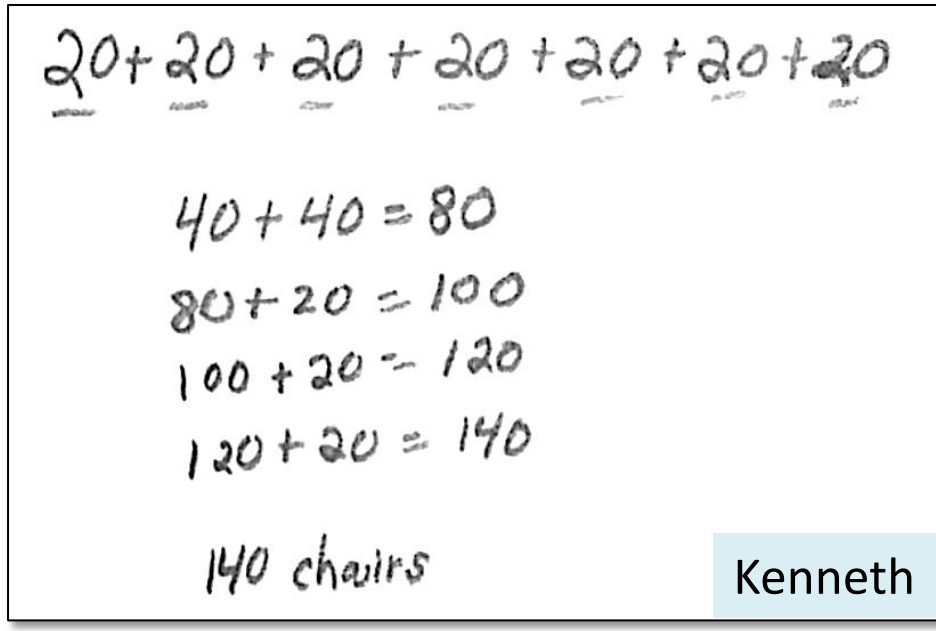


Smith, M. S., & Stein, M. K. (2011). Five practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.

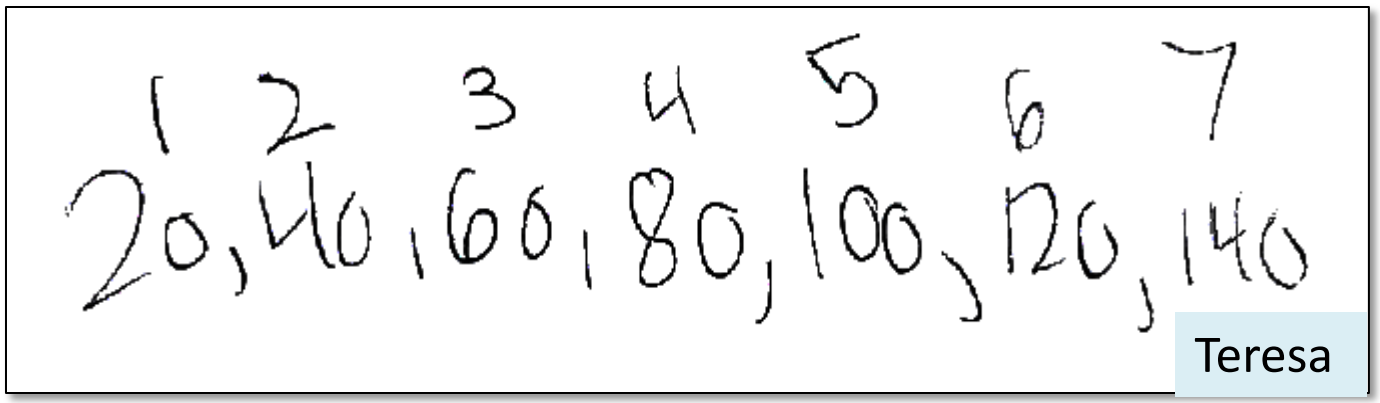
Selecting, Sequencing, Connecting



Jasmine



Kenneth



Teresa

Math Teaching Practice 6

Build procedural fluency from conceptual understanding.

A rush to fluency undermines students' confidence and interest in mathematics and is considered a cause of mathematics anxiety.

(Ashcraft 2002; Ramirez et al. 2013)

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*(5), 181-185

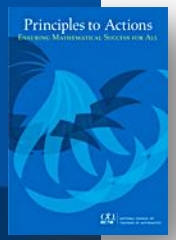
Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development,* 14(2), 187-202..

Build procedural fluency from conceptual understanding

Procedural Fluency should:

- Build on a foundation of conceptual understanding.
- Over time (months, years), result in known facts and generalized methods for solving problems.
- Enable students to flexibly choose among methods to solve contextual and mathematical problems.

Baroody, 2006; Fuson & Beckmann, 2012/2013;
Fuson, Kalchman, & Bransford, 2005; Russell, 2006



Turn and Talk

In what ways did this lesson help students begin to develop a foundation of conceptual understanding for building toward procedural fluency?

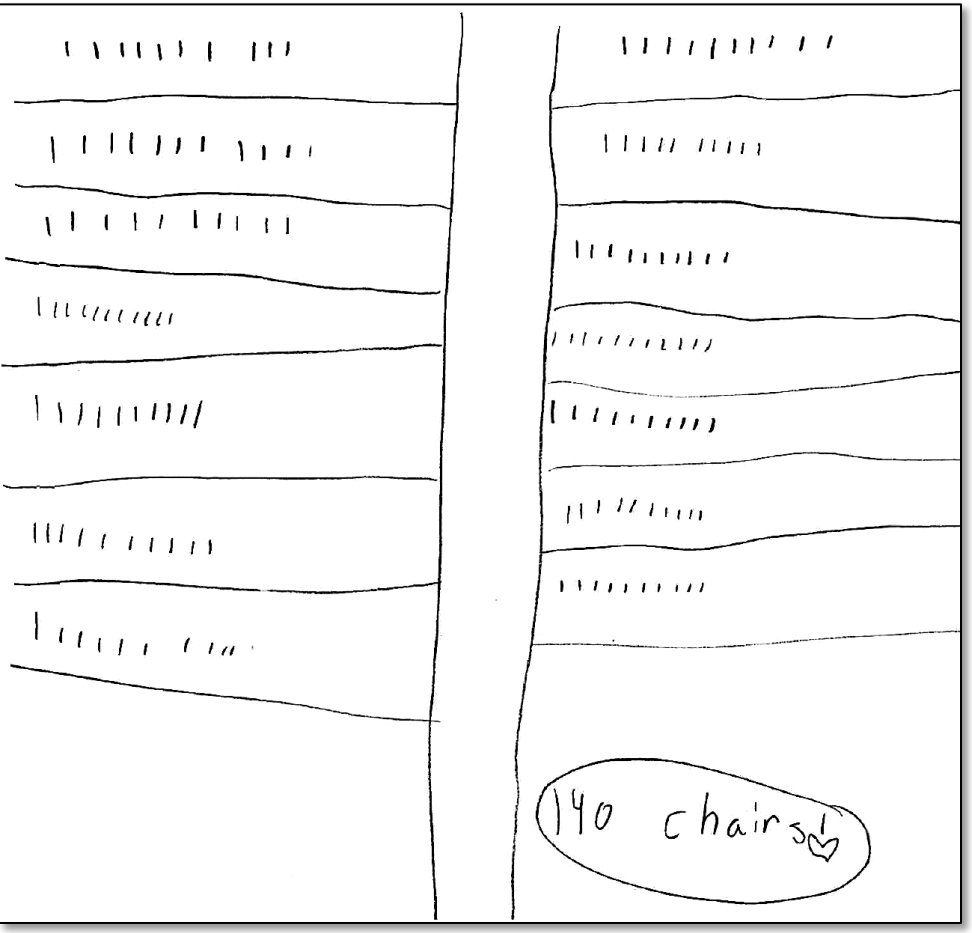
Lines 9-10: Choosing multiples of ten: 7 x 20.

Lines 61-66: Relating skip counting to equal groups.

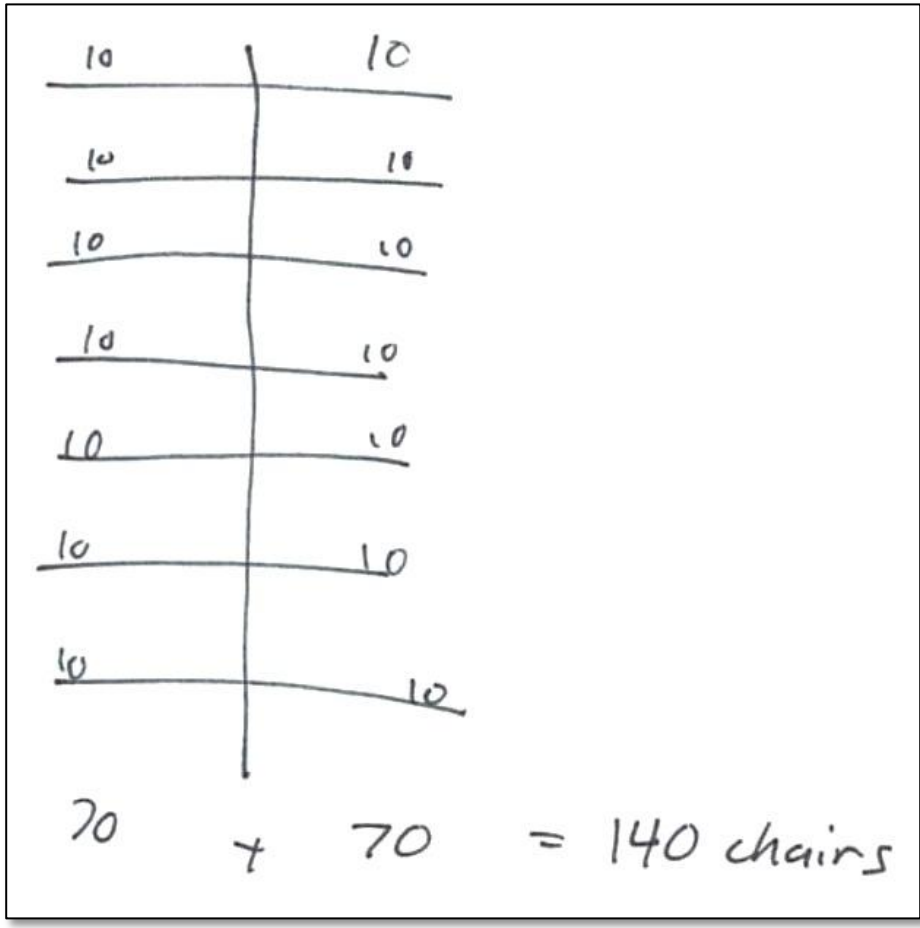
Lines 69-71: Connect the equation to their work.

Lines 77-80: Focusing students' attention on decomposing quantities and informally on the distributive property.

Where might Mr. Harris take his students mathematically in subsequent lessons using this student work to build fluency?



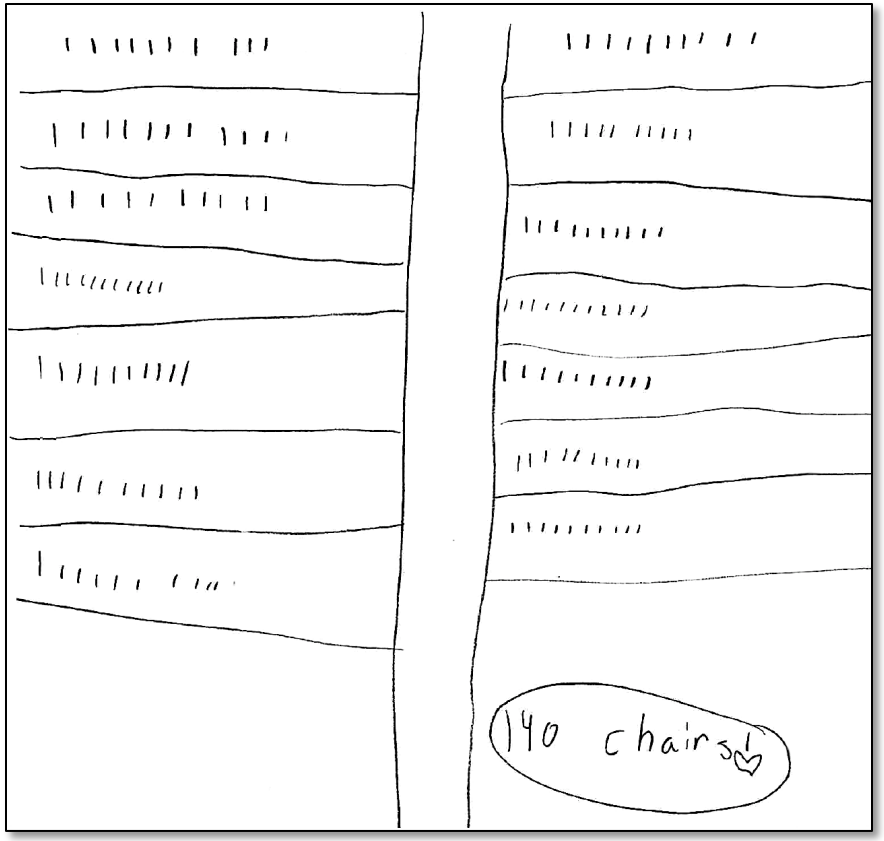
Tyrell



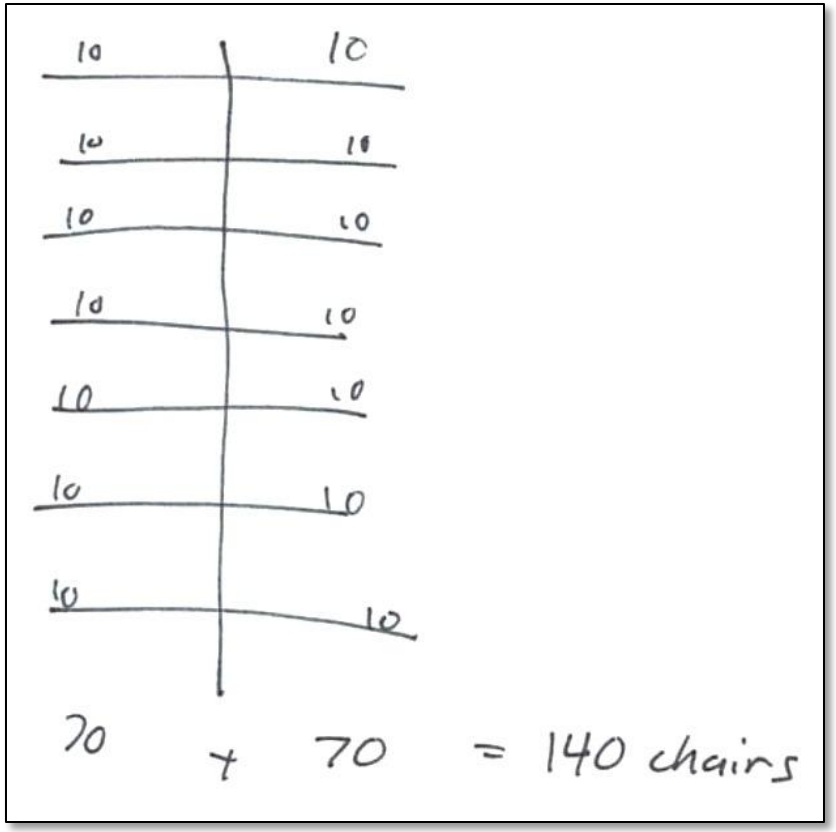
Ananda

7 x 20 = 140

7 tens + 7 tens = 14 tens = 140



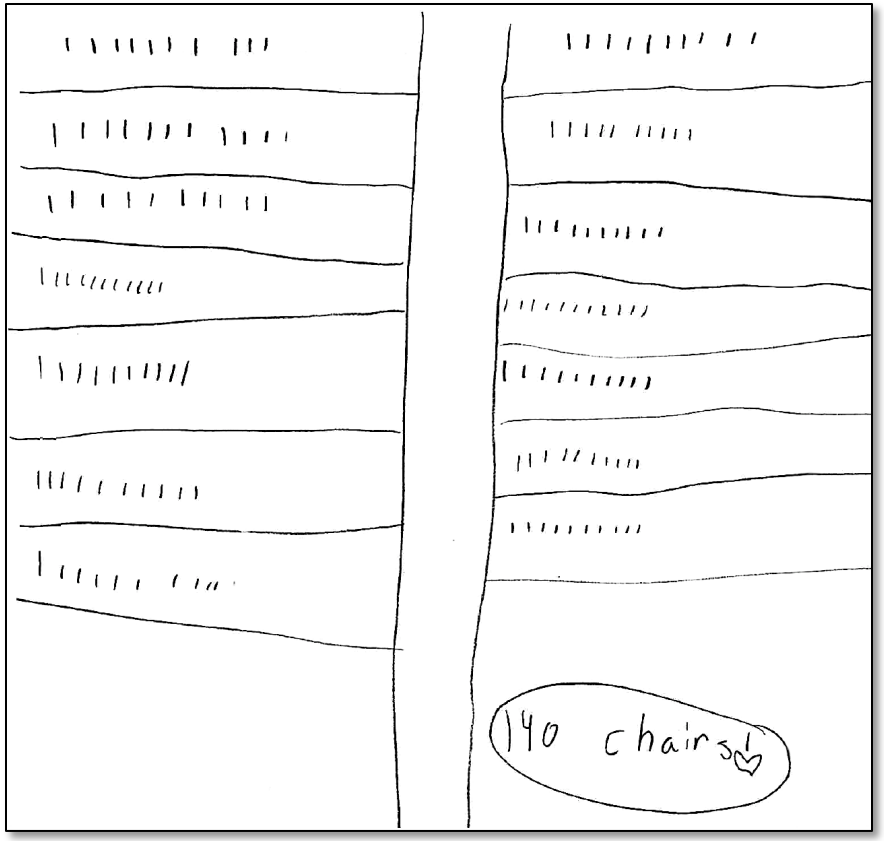
Tyrell



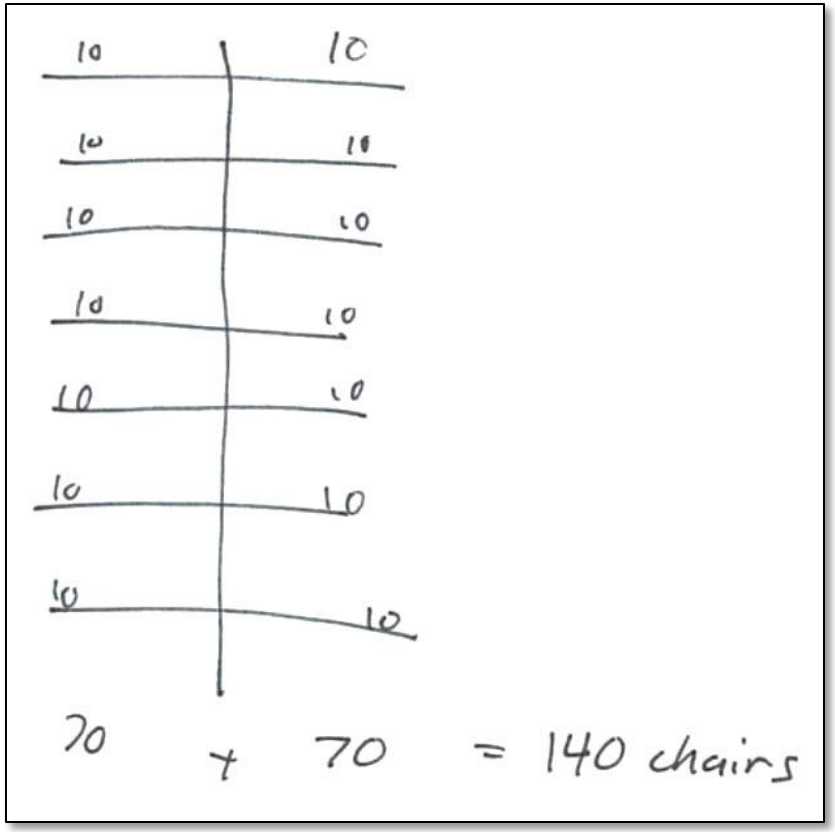
Ananda

Students decomposed the 20 into 10 + 10, then found each part separately, and then recomposed to find the total.

7 x 20 = 7 x (10 + 10) = 70 x 10 + 7 x 10 = 70 + 70 = 140



Tyrell



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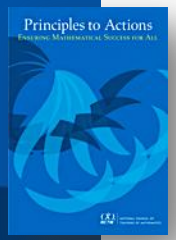
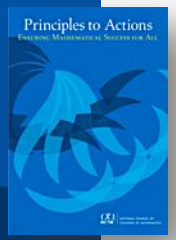


"Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems."

Principles to Actions (NCTM, 2014, p. 42)

Math Teaching Practice 7	Support productive struggle in learning mathematics.
Math Teaching Practice 8	Elicit and use evidence of student thinking.



Let's consider how Mr. Harris checked in on student thinking and struggles and used it to inform his instruction.

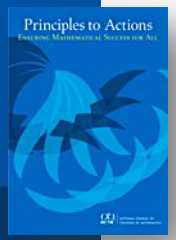
Note how he supported his students' struggles without "taking over" the thinking for them.

Support productive struggle in learning mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding.
- Develop students' capacity to persevere in the face of challenge.
- Help students realize that they are capable of doing well in mathematics with effort.

Black, Trzesniewski, & Dweck, 2007; Dweck, 2008; Hiebert & Grouws, 2007; Kapur, 2010; Warshauer, 2011



Mindsets for Learning Mathematics

Fixed mindset

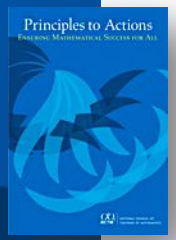
These students are more likely to give up when they encounter difficulties because they believe that learning mathematics should come naturally (easily and quickly).

Growth mindset

These students are likely to persevere through a struggle because they see challenging work as an opportunity to learn and grow.

Principles to Actions (NCTM, 2014, p. 50)

Dweck, C. (2006). *Mindsets: The new psychology of success*. New York: Random House.



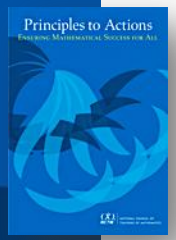
Valuing of Mistakes

An important and powerful aspect of teachers' practice concerns the ways in which they treat mistakes in mathematics classrooms...

Students need to move from viewing mistakes as learning failures to viewing them **as learning achievements**.
(*Mistakes show that students tried and highlight focus points for learning*.)

(Boaler, 2013, p. 149)

Boaler, J. (2013, March). Ability and mathematics: The mindset revolution that is reshaping education. *FORUM*, 55(1), 143-152.



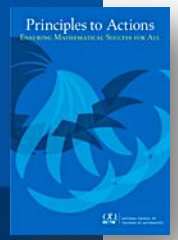
Mistakes and Brain Growth

When students ***think about why something is wrong***, new synaptic connections are sparked that cause the brain to grow.

When students do the work correctly, there is no brain growth.

(Moser et al. 2011)

Moser, J., H. S. Schroder, C. Heeter, T. P. Moran & Y. H. Lee. (2011). Mind your Errors: Evidence for a neural mechanism linking growth mindset to adaptive post error adjustments. *Psychological Science, 22*, 1484–9.



That was my favorite wrong answer today!!

--Gary Luck, High School Teacher

If you are not struggling, you are not learning.

– Susan Carter, First Grade Teacher

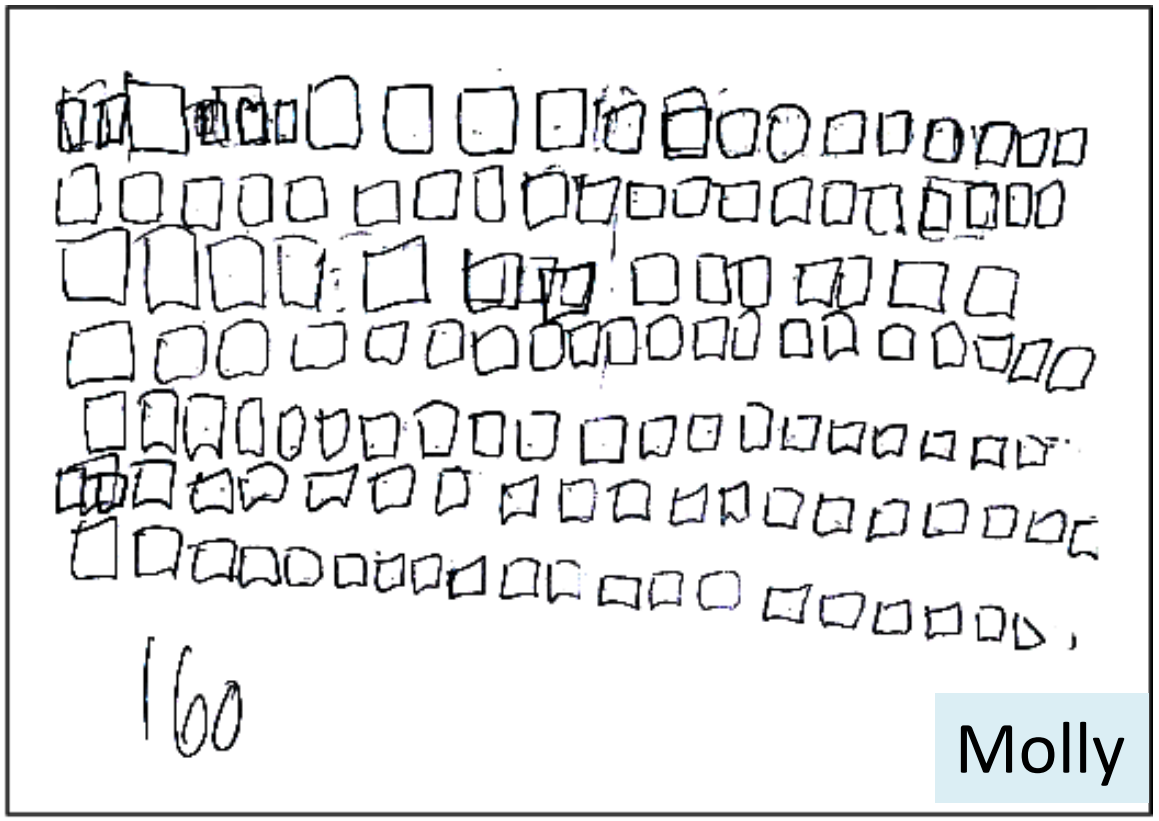
My Favorite No! (Warm-up Routine)

--Leah Alcala, Eighth Grade Teacher

Video: https://www.teachingchannel.org/videos/class-warm-up-routine

Principles to Actions (NCTM, 2014, p. 50)

Carter, S. (2008). Disequilibrium and questioning in the primary classroom: Establishing routines that help students learn. *Teaching Children Mathematics*, *15*(3), 134-137.



Allow grapple...

Support effort...

Build toward understanding and fluency...

Math Teaching Practice 8

Elicit and use evidence of student thinking.

Math Learning Goal

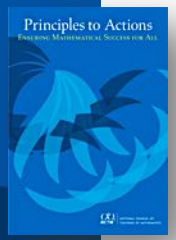
Students will recognize the structure of multiplication as equal groups within and among different representations—identify the number of equal groups and the size of each group within collections or arrays.

Elicit and use evidence of student thinking

Evidence should:

- Provide a window into students' thinking.
- Help the teacher determine the extent to which students are reaching the math learning goals.
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Chamberlin, 2005; Jacobs, Lamb, & Philipp, 2010; Sleep & Boerst, 2010; van Es, 2010' Wiliam, 2007



Case: Mr. Harris & the Band Concert Task

Throughout the lesson, Mr. Harris was eliciting and using evidence of student thinking.

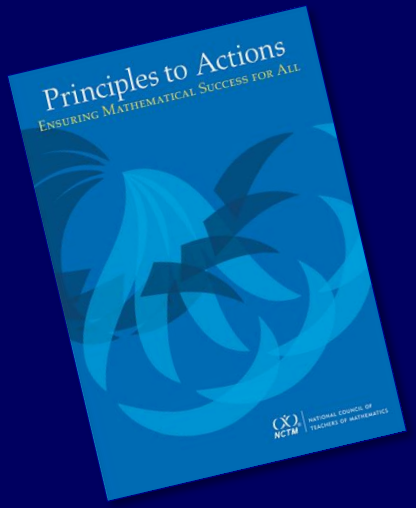
Lines 32-35: Purposeful questioning as they worked individually.

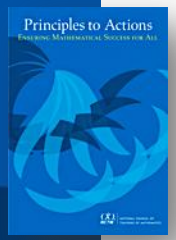
Lines 42-50: When students worked in pairs to compare their representations.

Lines 58-74: Whole class discussion.

Lines 78-80: Student written reflection (exit slip).

Closing Thoughts

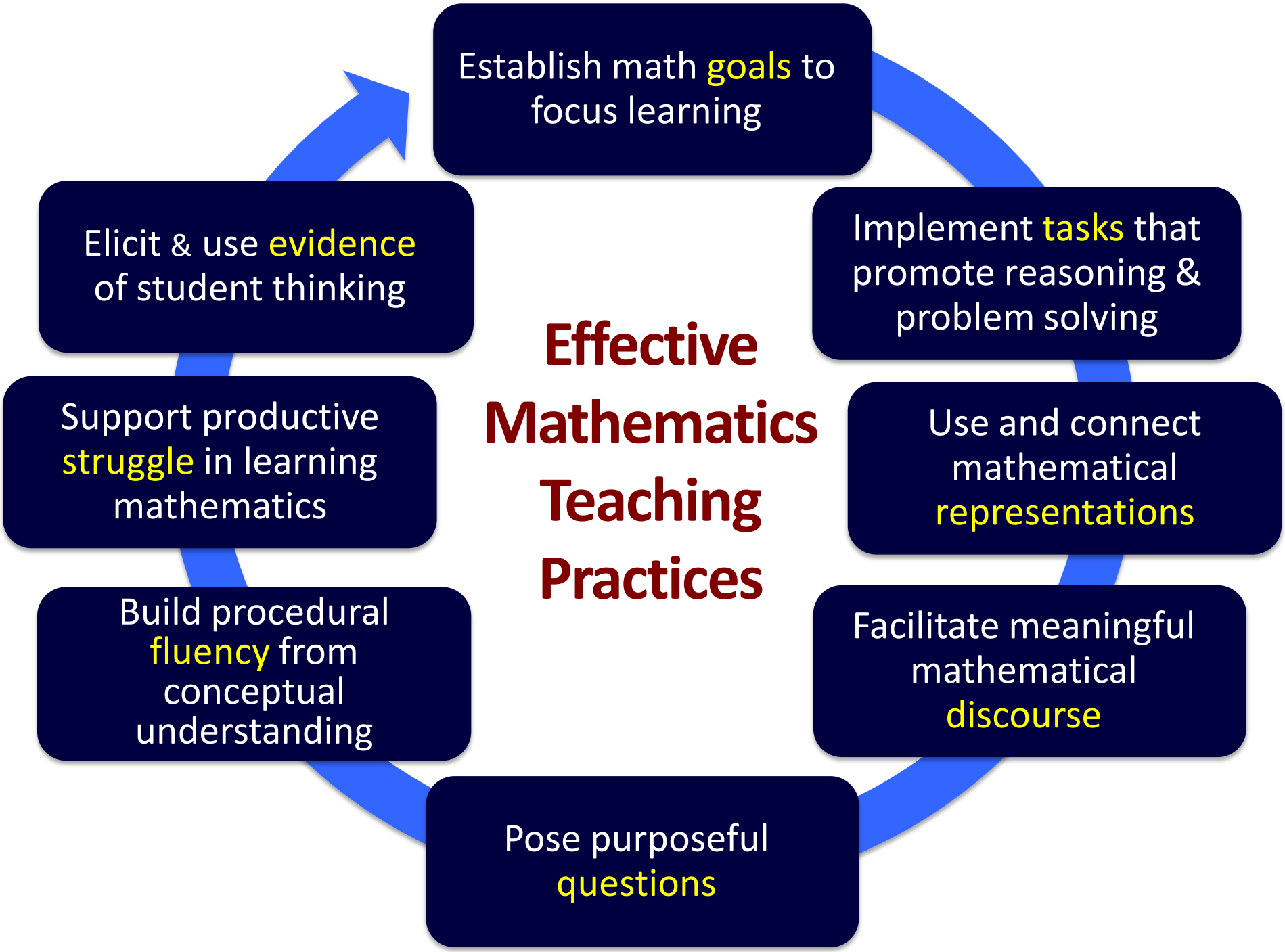


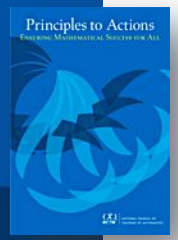


NCTM's Core Set of Effective Mathematics Teaching Practices

"Although the important work of teaching is not limited to the eight Mathematics Teaching Practices, this core set of research-informed practices is offered as a ***framework for strengthening the teaching and learning of mathematics***."

Principles to Actions (NCTM, 2014, p. 57)


Establish math goals to focus learning
Implement tasks that promote reasoning & problem solving
Use and connect mathematical representations
Facilitate meaningful mathematical discourse
Pose purposeful questions
Build procedural fluency from conceptual understanding
Support productive struggle in learning mathematics
Elicit & use evidence of student thinking
Effective Mathematics Teaching Practices



I Feld very smart in math

Wed I work hord

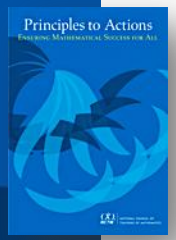
And keep going and going

And do your best

Second Grade Student

Thank You!

Dr. DeAnn Huinker
University of Wisconsin–Milwaukee
huinker@uwm.edu



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