Beliefs about Teaching and Learning Mathematics

<table>
<thead>
<tr>
<th>Beliefs about teaching and learning mathematics</th>
<th>Unproductive beliefs</th>
<th>Productive beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</td>
<td>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</td>
<td></td>
</tr>
<tr>
<td>All students need to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.</td>
<td>All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</td>
<td></td>
</tr>
<tr>
<td>Students can learn to apply mathematics only after they have mastered the basic skills.</td>
<td>Students can learn mathematics through exploring and solving contextual and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td>The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.</td>
<td>The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</td>
<td></td>
</tr>
<tr>
<td>The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.</td>
<td>The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.</td>
<td></td>
</tr>
<tr>
<td>An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.</td>
<td>An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</td>
<td></td>
</tr>
</tbody>
</table>

Principles to Action, Ensuring Mathematics Success for All (NCTM, 2014) p 11
## Characteristics of Mathematical Tasks

### Levels of Demands

<table>
<thead>
<tr>
<th>Lower-level demands (memorization):</th>
<th>Lower-level demands (procedures without connections):</th>
</tr>
</thead>
<tbody>
<tr>
<td>• reproducing previously learned facts, rules, formulas, definitions or committing them to memory</td>
<td></td>
</tr>
<tr>
<td>• Cannot be solved with a procedure</td>
<td></td>
</tr>
<tr>
<td>• Have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions</td>
<td></td>
</tr>
<tr>
<td>• are algorithmic</td>
<td></td>
</tr>
<tr>
<td>• require limited cognitive demand</td>
<td></td>
</tr>
<tr>
<td>• have no connection to the concepts or meaning that underlie the procedure</td>
<td></td>
</tr>
<tr>
<td>• focus on producing correct answers instead of understanding</td>
<td></td>
</tr>
<tr>
<td>• require no explanations</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Higher-level demands (procedures with connections):</th>
<th>Higher-level demands (doing mathematics):</th>
</tr>
</thead>
<tbody>
<tr>
<td>• use procedure for deeper understanding of concepts</td>
<td></td>
</tr>
<tr>
<td>• broad procedures connected to ideas instead narrow algorithms</td>
<td></td>
</tr>
<tr>
<td>• usually represented in different ways</td>
<td></td>
</tr>
<tr>
<td>• require some degree of cognitive effort; procedures may be used but not mindlessly</td>
<td></td>
</tr>
<tr>
<td>• require complex non-algorithmic thinking</td>
<td></td>
</tr>
<tr>
<td>• require students to explore and understand the mathematics</td>
<td></td>
</tr>
<tr>
<td>• demand self-monitoring of one’s cognitive process</td>
<td></td>
</tr>
<tr>
<td>• require considerable cognitive effort and may involve some level of anxiety b/c solution path isn’t clear</td>
<td></td>
</tr>
</tbody>
</table>

Productive Struggle Reflection Survey

Rate the frequency of each statement in your classroom.

1. I anticipate what students might struggle with during a lesson and I prepare to support them.

1 2 3 4 5 6 7 8 9 10
Never Always

2. I give students time to struggle with tasks and ask questions that scaffold thinking without doing the work for them.

1 2 3 4 5 6 7 8 9 10
Never Always

3. I help students realize that confusion and errors are a natural part of learning. We talk about mistakes, misconceptions, and struggles.

1 2 3 4 5 6 7 8 9 10
Never Always

4. I praise students for their efforts more frequently than the correct answers.

1 2 3 4 5 6 7 8 9 10
Never Always

5. My students struggle at times but they know breakthroughs come from struggle.

1 2 3 4 5 6 7 8 9 10
Never Always

6. My students ask questions that are related to their struggles to help them understand.

1 2 3 4 5 6 7 8 9 10
Never Always

7. My students persevere in solving problems. They don’t give up.

1 2 3 4 5 6 7 8 9 10
Never Always

8. My students help one another without telling their classmates what the answer is or how to solve it.

1 2 3 4 5 6 7 8 9 10
Never Always
Analyzing Mathematical Tasks: Grades K – 2

Categorize each of the Grade K-2 tasks below as High Level Cognitive Demand or Low Level Cognitive Demand.\textsuperscript{1, 2} Then, develop a list of criteria that describe the tasks in each category.

**Task A**

Manipulatives or Tools Available: Counters, interlocking cubes

Write and solve a number story for $26 - 8$.

Show how you solved the problem using words, counters, tallies, or pictures.

**Task B**

Manipulatives or Tools Available: None

In the number 28

what digit is in the **ones** place? _______

what digit is in the **tens** place? _______

**Task C**

Manipulatives or Tools Available: None

Use the graph to answer the questions.

1. How many students picked red as their favorite color?

2. Did more students like blue or purple?

3. Which color did only one student choose as a favorite color?

---

\textsuperscript{1} Adapted from Smith, Stein, Arbaugh, Brown, and Mossgrove, Characterizing the Cognitive Demand of Mathematical Tasks: A Task Sorting Activity, Professional Development Guidebook for Perspectives on the Teaching of Mathematics, NCTM, 2004.

\textsuperscript{2} Since these are K-2 tasks, assume that they would be read or presented orally to students, not just presented in written form.
Analyzing Mathematical Tasks: Grades K – 2

Task D

Manipulatives or Tools Available: counters, interlocking cubes

Some people were riding on a bus. At the bus stop, 7 more people get on. Now there are 13 people on the bus. How many people were on the bus before the bus stop?

Task E

Manipulatives or Tools Available: interlocking cubes

Complete each fact family. Add or subtract. The first one is started for you.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 6 + 9 = ____</td>
<td>2. 8 + 7 = ____</td>
<td>3. 4 + 8 = ____</td>
</tr>
<tr>
<td>9 + 6 = ____</td>
<td>7 + 8 = ____</td>
<td>8 + 4 = ____</td>
</tr>
<tr>
<td>15 – 9 = ____</td>
<td>15 – 7 = ____</td>
<td>12 – 8 = ____</td>
</tr>
<tr>
<td>15 – 6 = ____</td>
<td>15 – 8 = ____</td>
<td>12 – 4 = ____</td>
</tr>
</tbody>
</table>

Task F

Manipulatives or Tools Available: None

These puzzles show parts of number grids. Complete the puzzles.
Task G

Manipulatives or Tools Available: Interlocking cubes (optional)

1. How many legs are on the people in our classroom? Show how you figured it out.

2. How could you tell someone how to figure out the number of legs for any number of people? Can you think of a rule? For example, how would you finish this sentence: “To find the number of legs in a group of people, you . . .”

3. What if there were 26 legs? How many people would there be?

Task H

Manipulatives or Tools Available: None

Complete the addition facts in one minute or less.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>+ 9</td>
<td>+ 9</td>
<td>+ 5</td>
<td>+ 5</td>
<td>+ 4</td>
<td>+ 9</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>+ 2</td>
<td>+ 3</td>
<td>+ 4</td>
<td>+ 3</td>
<td>+ 1</td>
<td>+ 4</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>5</td>
<td>9</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>+ 9</td>
<td>+ 5</td>
<td>+ 4</td>
<td>+ 3</td>
<td>+ 4</td>
<td></td>
</tr>
</tbody>
</table>

Task I

Manipulatives or Tools Available: Counters, interlocking cubes

Solve each story problem.

1. Four bees were buzzing around a flower. Three more bees were at the hive. They flew over to join the others at the flower. Then how many bees were buzzing around the flower?

2. Now there were seven bees buzzing around the flower. Two bees left the flower and flew back to the hive. How many bees were still buzzing around the flower?

You’ve solved two problems about bees. How are the stories different?
Task J

Manipulatives or Tools Available: none

Rosa is going to measure her book. First she will measure using crayons. Then she will measure using paper clips.

Will it take more crayons or paper clips to measure her book? Why do you think so?

The length of Rosa’s book is 2 crayons.

How many paper clips long is Rosa’s book? How do you know?

Task K

Manipulatives or Tools Available: base-ten blocks, tens and ones mat

Write the tens and ones. Circle the number that is greater. The first one is started for you.

1. 56 __5____ tens
   6____ ones

2. 74 ____ tens
   ____ones

3. 31 ____ tens
   ____ones

Task L

Manipulatives or Tools Available: red and blue crayons or cubes; colored pencils; crayons or markers

I want to put 8 crayons in my box. I want to have a mix of red and blue crayons. How many of each color could I have? How many blues? How many reds?

Find as many different ways as you can.
Analyzing Mathematical Tasks: Grades K – 2

Task M

Manipulatives or Tools Available: base-ten blocks, tens and ones mat

<table>
<thead>
<tr>
<th>Show this many</th>
<th>Subtract this many</th>
<th>Do you need to regroup? (Yes or No)</th>
<th>Solve.</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>8</td>
<td></td>
<td>45 – 8 = _____</td>
</tr>
<tr>
<td>27</td>
<td>5</td>
<td></td>
<td>27 – 5 = _____</td>
</tr>
<tr>
<td>32</td>
<td>4</td>
<td></td>
<td>32 – 4 = _____</td>
</tr>
<tr>
<td>44</td>
<td>8</td>
<td></td>
<td>44 – 8 = _____</td>
</tr>
<tr>
<td>49</td>
<td>6</td>
<td></td>
<td>49 – 6 = _____</td>
</tr>
<tr>
<td>34</td>
<td>4</td>
<td></td>
<td>34 – 4 = _____</td>
</tr>
</tbody>
</table>

Task N

Manipulatives or Tools Available: none

Write a number sentence. Solve.

1. Jesse had 9 marbles. Louisa had 7 marbles. How many marbles did they have in all?

2. Jesse and Louisa had 16 marbles. Louisa took away 7 marbles. How many were left?
### Classification of Tasks

<table>
<thead>
<tr>
<th>Task</th>
<th>Low Level</th>
<th>High Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criteria</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mathematics Survey

1. To be good in math, you need to .......... because ........
2. Math is hard when ..... 
3. Math is easy when ..... 
4. How can math help you?
5. The best thing about math is ...
6. If you have trouble solving a problem in math, what do you do?

<table>
<thead>
<tr>
<th>Trends Implied by the Surveys</th>
<th>Positive Attitudes, Dispositions, Beliefs</th>
<th>Instructional Plans for Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics is a solitary, silent endeavor.</td>
<td>Collaboration and communication contribute to mathematical understanding.</td>
<td>Structure group tasks; make children’s strategies public; encourage children to note others’ contributions to their learning</td>
</tr>
<tr>
<td>The teacher is in charge of imparting knowledge. The rewards for developing mathematical expertise are external and are often postponed until the future.</td>
<td>Mathematics involves learners in constructing meaning for themselves. The rewards for developing expertise are intrinsic.</td>
<td>Encourage interaction, revisiting, extending (Schwartz 1996); involve students through student-authored problems, mathematics journals, mathematics “publications:</td>
</tr>
<tr>
<td>Problems are solved in a swift, prescribed manner.</td>
<td>Problems are solved through flexible use of multiple strategies. The time required to solve problems depends on the complexity of the problem.</td>
<td>Encourage strategy sharing, problem-posing investigations, extended explorations, mathematics journals (Whitin &amp; Whitin 2000)</td>
</tr>
<tr>
<td>Mathematics is unrelated to other subjects.</td>
<td>Mathematics has real-life application across the curriculum and in contexts outside school.</td>
<td>Emphasize content-related problems (e.g., science), problems inspired by children’s literature, student-authored problems</td>
</tr>
</tbody>
</table>

©Phyllis E. Whitin, NCTM Teaching Children Mathematics, April 2007
FLUENCY INTERVIEW

1. Explain how to use the “count on” strategy for 3 + 9. (S)

2. Jhordan explains that 6 + 7 is 12. Is he correct? Explain how you know. (A)

3. What strategy did you use to solve 6 + 8? (S)

4. What strategy did you use to solve 9 + 3? (E)

5. Jasmine solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Does this strategy always work? (F)

6. Which facts do you “just know”? For which facts do you use a strategy? (E)

©Gina Kling and Jennifer M. Bay-Williams April 2014 issue of Teaching Children Mathematics
### Addition Drill: Find the Sums

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>80 + 15 =</td>
<td>73 + 21 =</td>
<td>89 + 10 =</td>
<td>78 + 11 =</td>
</tr>
<tr>
<td>46 + 12 =</td>
<td>14 + 71 =</td>
<td>22 + 47 =</td>
<td>24 + 42 =</td>
</tr>
<tr>
<td>26 + 70 =</td>
<td>11 + 28 =</td>
<td>47 + 32 =</td>
<td>83 + 13 =</td>
</tr>
<tr>
<td>69 + 10 =</td>
<td>87 + 10 =</td>
<td>21 + 72 =</td>
<td>26 + 21 =</td>
</tr>
<tr>
<td>82 + 11 =</td>
<td>38 + 10 =</td>
<td>55 + 40 =</td>
<td>26 + 72 =</td>
</tr>
<tr>
<td>68 + 31 =</td>
<td>43 + 33 =</td>
<td>69 + 30 =</td>
<td>45 + 54 =</td>
</tr>
<tr>
<td>85 + 13 =</td>
<td>46 + 30 =</td>
<td>48 + 11 =</td>
<td>24 + 70 =</td>
</tr>
<tr>
<td>20 + 27 =</td>
<td>17 + 30 =</td>
<td>76 + 20 =</td>
<td>12 + 21 =</td>
</tr>
<tr>
<td>49 + 10 =</td>
<td>30 + 11 =</td>
<td>43 + 16 =</td>
<td>70 + 26 =</td>
</tr>
<tr>
<td>49 + 50 =</td>
<td>18 + 11 =</td>
<td>80 + 16 =</td>
<td>45 + 22 =</td>
</tr>
</tbody>
</table>
Sort and Group Basic Facts

Choose some facts to group. Write your facts that belong together in the circle and then tell why you grouped them.

Why did you group these facts together?
One Less

Fluency Station: Skill: One-digit subtraction
CCSSM: 1.OA.C.6 and 2.OA.B2

One Less

The goal of this game is to find two cards where the number on one card is one less than the number on the other card.

Number of Players: Two Players

Materials:
- Numeral cards 1 – 10 (four of each)

How to Play:
- Players sit side by side. The cards are shuffled and placed facedown in a stack.
- Player 1 takes ten cards and places them faceup in a line between both players.
- Player 1 looks for two cards with a difference of one. When he finds two such cards, he says, “I can take these cards because ____ is one less than ____.”
- Player 2 adds two more cards to the faceup line. (The line always has ten cards.) She hunts for two cards with a difference of one.
- If there are no cards in the line with a difference of one, another card is added to the line.
- Players alternate turns until all the facedown cards have been added to the line and all possible “one-less” combinations have been made.

Variations: “Two Less” and “Three Less” are played in a similar manner.

Teacher Question:
Can you prove to me that one less than ___ is ____?
NOTE: Counters or a 0 to 10 number line might be very helpful so that children can visualize what they are being asked to do.
More: A Game of Differences

Fluency Station: Skill: One-digit subtraction; Variation – Two-digit subtraction
CCSSM: 1.OA.C.6 and 2.OA.B2, Variation – 2.NBT.B.5

The goal of this game is to practice finding differences using randomly selected numbers.

**Number of Players:** Two Players

**Materials:**
- Numeral cards 1 – 10 (four of each)
- Counters

**How to Play:**
- The cards are shuffled and put facedown in a stack. Each player turns over one card. Players decide who has the greater number, and then figure out how much “more” that player has. The player who has “more” takes the quantity of counters that equals the difference between the two players’ numbers.

Example:
Player 1 turns over a 3. Player 2 turns over a 9. Player 2 has the greater number, which is 6 more than Player 1’s, so Player 2 takes 6 counters.

Play continues until all the cards have been drawn. Players count their counters, and the player with more counters wins the game.

**Variation:** Each player turns over two cards and makes the greatest two-digit number possible. The players then determine who has the greater number, and using pencil and paper, figure out the difference between the two numbers. The player who has “more” gives himself points equal to the difference between the two numbers. When all the cards in the facedown stack are gone, the player with more points wins the game.

Example: Player 1 draws a 6 and 7, and makes 76. Player 2 draws a 1 and a 3, and makes 31. Player 1 has the greater number, in this case, 45 more, so Player 1 scores 45 points for this round.

**Teacher Questions:**
What have you noticed while playing this game?
Convince me that ___ is that much more than ____.
When you take a lot of counters in a turn, what do you notice about the two numbers drawn?
What do you notice about the amount of counters you get when both numbers are close together?

*Mastering Basic Math Skills: Games for Kindergarten through Second Grade* by Bonnie Adama Britt ©National Council of Teachers of Mathematics 2014
Spin, Circle, and Solve!

1. Spin the Strategy Spinner

2. Circle all of the problems on the page that you can solve using that strategy. (Don’t solve them yet.)

3. Solve the problems you circled.
Spin, Circle, and Solve Strategy Spinner

©Putting Essential Understanding of Addition and Subtraction into Practice Pre-K -2, National Council of Teachers of Mathematics, 2014
Ring Facts

Students play in groups of 2 to 6.

Materials: A Ring Facts game board for each player; one to two sets of basic fact cards (addition and subtractions); smiley-face stickers or buttons for the players to use to mark their preselected numbers on their game board.

Set up: Distribute game boards and sticky notes or buttons to all players, and stack basic fact cards facedown on each table, within everyone’s reach.

Goal: Be the first player to draw rings around five preselected numbers on a game board after those numbers have occurred as sums or differences for problems on basic fact cards.

Rules: 1. Each player chooses five numbers to “ring” on his or her game board and puts a sticker or button on the corner of each of those numbered squares.

2. Players take turns turning over cards, with each player displaying the problem on his or her card and reading it aloud to the other players. All the players find the sum or difference to complete the fact, and then each player looks to see whether he or she marked that sum or difference on his or her game board. If so, the player “rings” the sum by drawing a circle around it. For example, if player 1 turns over the problem 4 + 1, all the players find the sum, 5, and then look to see whether they preselected 5. If they did, the “ring” the sum.

3. The first player to ring all five previously marked numbers wins the game.

©Putting Essential Understanding of Addition and Subtraction into Practice Pre-K -2, National Council of Teachers of Mathematics, 2014
Collect 20 Together

Materials:
One dot cube
30 counters
Players: 2
Object: With a partner, collect 20 counters

How to Play:
1. To start, one player rolls the dot cube. What number did you roll?
   Take that many counters.
2. Take turns rolling the dot cube. Take that many counters and add them to the collection.
3. After each turn, check the total number of counters in your collection. The game ends when you have 20 counters.

Variations:
   a. At the end of the game, determine how many more than 20 counters you have.
   b. Play Collect 40 Together.
   c. Play with three people.
   d. For each turn, write the number rolled and the total number of counters you have so far.
Subtracting Fractions

\[
\begin{align*}
2 - 1 &= \quad 1 - \frac{7}{10} &= \quad 1 - \frac{5}{6} = \\
\frac{2}{2} - \frac{1}{2} &= \quad 4 - 2 &= \quad 1 \frac{5}{6} - \frac{1}{6} = \\
1 - \frac{1}{2} &= \quad 4\frac{3}{4} - 2\frac{3}{4} &= \quad 1 \frac{1}{6} - \frac{5}{6} = \\
5 - 1 &= \quad 1 \frac{1}{3} - 2\frac{1}{3} &= \quad 1 \frac{1}{12} - 7\frac{1}{12} = \\
5\frac{5}{5} - 1\frac{5}{5} &= \quad 1 \frac{1}{3} - 1\frac{3}{3} &= \quad 1 \frac{1}{12} - 7\frac{1}{12} = \\
1 - \frac{1}{5} &= \quad 1 - \frac{3}{8} &= \quad 1 \frac{4}{15} - 13\frac{15}{15} = \\
1 - \frac{2}{5} &= \quad 1 - \frac{7}{8} &= \quad 3 - 1 = \\
1 - \frac{4}{5} &= \quad 1 \frac{7}{8} - 3\frac{3}{8} &= \quad 3\frac{3}{8} - 1\frac{3}{8} = \\
1 - \frac{3}{5} &= \quad 1 \frac{3}{8} - 7\frac{7}{8} &= \quad 1 - 1\frac{3}{8} = \\
1 - \frac{1}{4} &= \quad 7 - 4 &= \quad 8 - 1 = \\
1 - \frac{3}{4} &= \quad 7\frac{5}{4} - 4\frac{5}{4} &= \quad 8\frac{8}{4} - 1\frac{8}{4} = \\
1 - \frac{1}{10} &= \quad 1 \frac{2}{5} - 4\frac{5}{5} &= \quad 1 - 1\frac{1}{8} = \\
1 - \frac{9}{10} &= \quad 1 \frac{4}{5} - 2\frac{2}{5} &= \quad 5\frac{4}{4} - 3\frac{4}{4} = \\
1 - \frac{3}{10} &= \quad 1 - 1\frac{6}{6} &= \quad 1 \frac{1}{4} - 3\frac{4}{4} = \\
\end{align*}
\]
1. Show how to use a number bond to decompose the difference between 16/9 – 5/9. Record your answer as a mixed number.


3. Solve ½ + 1/3. What strategy did you use?

4. Dhruva has 3/8 of a medium pepperoni pizza. His dad gives him 2/8 more of a medium pepperoni pizza. How much of a medium pepperoni pizza does he have now? Explain how you got your answer.

5. Jasmine solved 1 – 1/3 by changing it in her mind to 3/3 – 1/3. Why do you think she did this?
Adding Fractions

Examine the problems below. Which of these problems can be solved by adding \( \frac{1}{2} + \frac{1}{3} \)? Show work to support your answer to each problem.

1. Gabriel pours \( \frac{1}{2} \) of a cup of sand into an empty box. Then he pours \( \frac{1}{3} \) of a cup of sand into the box. How many cups of sand are in the box now?

2. Jayla has a full glass of water. She pours \( \frac{1}{2} \) cup of water from the glass into an empty bowl. Then Jayla pours in another \( \frac{1}{3} \) of the water from the glass into the bowl. How many cups of water are in the bowl now?

3. \( \frac{1}{2} \) of the boys in the class are wearing tennis shoes. \( \frac{1}{3} \) of the girls in the class are wearing tennis shoes. What fraction of the class is wearing tennis shoes?

4. \( \frac{1}{2} \) of the children at Polk Elementary School say they would like to visit the aquarium. \( \frac{1}{3} \) of the children at Polk Elementary School say they would like to visit the children’s museum. What fraction of the children at Polk Elementary School would like to visit the aquarium or the children’s museum?
Subtracting Fractions

Examine the problems below. Which of these problems can be solved by subtracting \( \frac{2}{3} - \frac{1}{2} \)? Show work to support your answer to each problem.

1. Starting at his house, Dhruva bikes \( \frac{2}{3} \) of a mile down the street. Then he turns around and bikes \( \frac{1}{2} \) mile back toward his house. How far down the street is Dhruva from his house?

2. \( \frac{2}{3} \) of the children at Polk Elementary rode the ferris wheel at the state fair. \( \frac{1}{2} \) of the children at Polk Elementary rode the carousel. What fraction of the children rode the ferris wheel, but not the carousel?

3. Maggie pours \( \frac{2}{3} \) cup of milk into her mug. Then she pours out \( \frac{1}{2} \) of the milk that is in her mug into the sink. How many cups of milk are in Maggie’s mug now?

4. \( \frac{2}{3} \) of the kingdom of Reyna is forestland. \( \frac{1}{2} \) of the neighboring kingdom of Lexia is forestland. How much more forestland is there in Reyna than in Lexia?

Fraction Addition and Subtraction

Make a diagram to represent each problem below. How are your diagrams similar? How are they different? What type of problem (join, separate, part-part-whole, or comparison) does each problem represent?

1. James has 2 ½ pages full of stickers. He gives ¾ of a page to his sister Josie. How many pages of stickers does James have now?

2. James has 2 ½ pages full of stickers. His sister Josie has ¾ of a page of stickers. How many more pages of stickers does James have than Josie?

3. James wants to fill 2 ½ pages in his book with stickers. Right now he has enough stickers to fill ¾ of a page. How many more pages full of stickers does James need?

4. James has 2 ½ pages full of superhero stickers. He has ¾ of a page of sports stickers. How many pages of stickers does James have altogether?
Examining Student Work

Erin has $\frac{3}{4}$ of a pound of chocolate candies and $\frac{2}{3}$ of a pound of mint candies. How many pounds of candy does she have altogether?

**Student A:** I used fraction circles for the $\frac{3}{4}$ and $\frac{2}{3}$. Because the pieces are different sizes, they can’t be added together. Also, if you could put them together, the result would be more than one circle.

**Student B:** I used pattern blocks with 2 hexagons as the whole, so this

represents 1. That means $\frac{3}{4}$ is 3 red trapezoids and $\frac{2}{3}$ is 4 blue rhombuses. So $\frac{3}{4} + \frac{2}{3} = \frac{7}{10}$. Erin has $\frac{7}{10}$ of a pound of candy.

**Student C:** I used rectangles as a whole, and I divided each rectangle into 12 equal pieces.

So $\frac{3}{4} + \frac{2}{3} = \frac{17}{24}$. Erin has $\frac{17}{24}$ of a pound of candy.

**Student D:** I know that 3 and 4 both multiply to 12, so I replaced $\frac{3}{4}$ with $\frac{9}{12}$ and $\frac{2}{3}$ with $\frac{8}{12}$.

Then $\frac{3}{4} + \frac{2}{3}$ becomes $\frac{9}{12} + \frac{8}{12}$. Take $\frac{3}{12}$ from the $\frac{8}{12}$ and put it with the $\frac{9}{12}$ to make $\frac{12}{12}$.

You still have $\frac{5}{12}$ left from the $\frac{8}{12}$. So $\frac{3}{4} + \frac{2}{3} = \frac{12}{12} + \frac{5}{12} = \frac{17}{12}$. Erin has $\frac{17}{12}$ pounds of candy.

**Student E:** Both of these fractions are a little less than 1, so the sum will be less than 2. Take 2 minus $\frac{1}{4}$, which leaves $1\frac{3}{4}$. Now take away $\frac{1}{3}$. $1\frac{3}{4} $ is the same as $1.75 - \frac{1}{3} = 1.466\ldots$. So Erin has 1.466 pounds of candy.

Clapping Institute Award

is presented to

for

Signature

Date
Beliefs About Mathematics Assessment

Select 2 statements about assessment. Write P if it is a productive belief or U if it is a unproductive belief. Be sure to explain your reasoning.

<table>
<thead>
<tr>
<th>Statement</th>
<th>P/U Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>The primary purpose of assessment is accountability for students through report card marks or grades.</td>
<td>Only multiple-choice and other “objective” paper-and-pencil tests can measure mathematical knowledge reliably and accurately.</td>
</tr>
<tr>
<td>Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.</td>
<td>Assessment in the classroom is an interruption of the instructional process.</td>
</tr>
<tr>
<td>Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.</td>
<td>A single assessment can be used to make important decisions about students and teachers.</td>
</tr>
<tr>
<td>Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.</td>
<td>Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.</td>
</tr>
<tr>
<td>Assessment is something that is done to students.</td>
<td>Stopping teaching to review and take practice tests improves students’ performance on high-stakes tests.</td>
</tr>
<tr>
<td>Multiple data sources are needed to provide an accurate picture of teacher and student performance.</td>
<td>Ongoing review and distributed practice within effective instruction are productive test preparation strategies.</td>
</tr>
</tbody>
</table>

# Putting it Together

What are three topics from the institute that are interesting to you?

Choose a task from the institute or choose one from the menu. How might you adapt it for your students?

How might you collect evidence?

What would you expect understanding to look like? What would students need to understand to complete the task?

What misconceptions/misunderstanding do you anticipate?

What would you do next?
## Beliefs About Mathematics Assessment

<table>
<thead>
<tr>
<th>Unproductive Beliefs</th>
<th>Productive Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>The primary purpose of assessment is accountability for students through report card marks or grades.</td>
<td>The primary purpose of assessment is to inform and improve the teaching and learning of mathematics.</td>
</tr>
<tr>
<td>Assessment in the classroom is an interruption of the instructional process.</td>
<td>Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.</td>
</tr>
<tr>
<td>Only multiple-choice and other “objective” paper-and-pencil tests can measure mathematical knowledge reliably and accurately.</td>
<td>Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.</td>
</tr>
<tr>
<td>A single assessment can be used to make important decisions about students and teachers.</td>
<td>Multiple data sources are needed to provide an accurate picture of teacher and student performance.</td>
</tr>
<tr>
<td>Assessment is something that is done to students.</td>
<td>Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.</td>
</tr>
<tr>
<td>Stopping teaching to review and take practice tests improves students’ performance on high-stakes tests.</td>
<td>Ongoing review and distributed practice within effective instruction are productive test preparation strategies.</td>
</tr>
</tbody>
</table>