Rich Tasks and Productive Struggle
Effective Teaching with Principles to Actions Institute, 2017

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@Latrendak
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<th>Table Introductions</th>
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<tr>
<td>• Your name</td>
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Latrenda
12
How to play:

1. Players take turns rolling a number cube

2. After each roll, a player decides which column to place the digit.

3. That player then adds the value to his/her total.

4. The player who is closest to the target (in the last total) without going over the target wins.
Choose 1 to discuss with a partner

• How might you use this game in your mathematics classroom?
• How might you modify this game for your students?
• What would you look for while your students played the game?
## Target 1

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# Decimals on a Hundred Chart

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Session Goals

- Examine productive and unproductive beliefs about teaching and learning mathematics.
- Identify the characteristics of tasks that promote reasoning.
- Consider the role of productive struggle in elementary mathematics.
Effective Teaching Practices

1. Implement tasks that promote reasoning and problem solving.

2-4. Use and connect mathematical representations.

2-4. Build procedural fluency from conceptual understanding.

5. Elicit and use evidence of student thinking.

Facilitate meaningful mathematical discourse.

Establish mathematics goals to focus learning.

2-4. Pose purposeful questions.


About the Institute: Effective Teaching Practices
Thoughts About Teaching and Learning Mathematics

• Brainstorm thoughts or beliefs that different stakeholders have about teaching and learning mathematics.
• Capture your ideas on your web.
• Describe a theme for your web.
Thoughts About Teaching and Learning Mathematics

- Examine ideas of other groups during the gallery walk.
- √ ideas that appeared on your web.
- P = Productive belief
- U = Unproductive belief
Actual Student Beliefs

• Students in grades 3-5 responded to this mathematics survey. The students were from self-contained gifted, academic magnet, and regular education classrooms. Students participating included ELL, ESS, and other underrepresented groups. Examine the student responses at your table.

• Turn and talk to your partner:
  – What trends did you notice in the student responses?
  – Select one student paper.
  – What do you know about this student’s attitudes towards mathematics?
  – What do you know about this student’s math environment (home, school, etc.)?
<p>| Mathematics learning should focus on practicing procedures and memorizing basic number combinations. | The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests. |
| All students need to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems. | An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused. |
| Students can learn to apply mathematics only after they have mastered the basic skills. |  |
| The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems. |  |</p>
<table>
<thead>
<tr>
<th>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</th>
<th>The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</th>
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<tr>
<td>All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</td>
<td>The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.</td>
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<td>An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</td>
<td>Students can learn mathematics through exploring and solving contextual and mathematical problems.</td>
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Success for All by...

• Engaging with challenging tasks.
• Connecting new learning with prior knowledge.
• Acquiring conceptual and procedural knowledge.
• Constructing knowledge through discourse.
• Receiving meaningful and timely feedback
• Developing metacognitive awareness.
Implement Tasks that Promote Reasoning and Problem Solving
What does rigor look like in mathematics classrooms?
MATHEMATICAL RIGOR

- Conceptual Understanding
- Procedural Fluency
- Application
**BLOOM’S TAXONOMY**

**KNOWLEDGE**
The recall of specifics and universals, involving little more than bringing to mind the appropriate material.

**COMPREHENSION**
Ability to process knowledge on a low level such that the knowledge can be reproduced or communicated without a verbatim repetition.

**APPLICATION**
The use of abstractions in concrete situations.

**ANALYSIS**
The breakdown of a situation into its component parts.

**SYNTHESIS AND EVALUATION**
Putting together elements & parts to form a whole, then making value judgments about the method.

**WEBB’S DOK**

**RECALL**
Recall of a fact, information, or procedure (e.g., What are 3 critical skill cues for the overhand throw?)

**SKILL/CONCEPT**
Use of information, conceptual knowledge, procedures, two or more steps, etc.

**STRATEGIC THINKING**
Requires reasoning, developing a plan or sequence of steps; has some complexity; more than one possible answer

**EXTENDED THINKING**
Requires an investigation; time to think and process multiple conditions of the problem or task.

Can we just look at the verbs to determine the DOK?
The Depth of Knowledge is NOT determined by the verb, but the context in which the verb is used and the depth of thinking required.
Depth of Knowledge is not verb dependent. Depth of Knowledge is based on the cognitive complexity of the standard.

<table>
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<tr>
<th>Verb</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
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<tr>
<td>List</td>
<td>List the order of operations</td>
<td>List the order of operations next to each step you take completing this problem</td>
<td>List the steps you took to solve this problem and how you knew if they were correct or not. If not, explain your new thinking to get to the next step.</td>
<td>List possible experiments that could be conducted to demonstrate the concept of “order” in mathematics. Give several different examples and contexts.</td>
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Reflect:

How do tasks and teaching impact mathematics learning?
Research on Tasks

• Not all tasks provide the same opportunities for student thinking and learning (Hiebert et al. 1997; Stein et al. 2009).

• Student learning is **greatest** in classrooms where the tasks **consistently encourage high-level student thinking and reasoning** and **least** in classrooms where the tasks **are routinely procedural in nature** (Boaler and Staples 2009; Hieber and Wearne 1993; Stein and Lane 1996)
What are characteristics of quality mathematics tasks?
So what does *THIS* mean?

What do high-level tasks look like?

1. Read each task.

2. Sort the tasks by their cognitive level (*Lower or Higher Level of Cognitive Demand*).

3. Be prepared to defend your reasoning.
## Levels of Demands

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<th><strong>Lower-level demands</strong></th>
<th><strong>Higher-level demands</strong></th>
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<td><strong>Lower-level demands</strong></td>
<td><strong>Lower-level demands</strong></td>
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<td><strong>(memorization):</strong></td>
<td><strong>(procedures without connections):</strong></td>
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<tr>
<td>• reproducing previously learned facts, rules, formulas, definitions or committing them to memory</td>
<td>• are algorithmic</td>
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<tr>
<td>• Cannot be solved with a procedure</td>
<td>• require limited cognitive demand</td>
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<tr>
<td>• Have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions</td>
<td>• have no connection to the concepts or meaning that underlie the procedure</td>
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<td></td>
<td>• focus on producing correct answers instead of understanding</td>
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<td>• require no explanations</td>
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<th><strong>Higher-level demands</strong></th>
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<td><strong>(procedures with connections):</strong></td>
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<td><strong>(doing mathematics):</strong></td>
</tr>
<tr>
<td>• use procedure for deeper understanding of concepts</td>
<td>• require complex non-algorithmic thinking</td>
</tr>
<tr>
<td>• broad procedures connected to ideas instead narrow algorithms</td>
<td>• require students to explore and understand the mathematics</td>
</tr>
<tr>
<td>• usually represented in different ways</td>
<td>• demand self-monitoring of one’s cognitive process</td>
</tr>
<tr>
<td>• require some degree of cognitive effort; procedures may be used but not mindlessly</td>
<td>• require considerable cognitive effort and may involve some level of anxiety b/c solution path isn’t clear</td>
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What do they have in common?

• Review your sorting.
• What do higher-level tasks have in common?
• What do lower-level tasks have in common?
• Where do you see evidence of procedure, concept, and application in these tasks?
Lower-level demands (memorization):

- reproducing previously learned facts, rules, formulas, definitions or committing them to memory
- cannot be solved with a procedure
- have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions

Lower-level demands 
(procedures without connections):

• are algorithmic
• require limited cognitive demand
• have no connection to the concepts or meaning that underlie the procedure
• focus on producing correct answers instead of understanding
• require no explanations

Higher-level demands (procedures with connections):

• use procedure for deeper understanding of concepts
• broad procedures connected to ideas instead narrow algorithms
• usually represented in different ways
• require some degree of cognitive effort; procedures may be used but not mindlessly

Higher-level demands (doing mathematics):

- require complex non-algorithmic thinking
- require students to explore and understand the mathematics
- demand self-monitoring of one’s cognitive process
- require considerable cognitive effort and may involve some level of anxiety b/c solution path isn’t clear

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How can you make it better?

• Select 2 lower-level tasks.
• Work with a partner to talk about how you could make these tasks higher-level.
• Record your ideas on the back of the task.
• Take 1 of the improved tasks and bump in to someone else.
• Share how you can improve the task.
• Trade tasks.
• Bump into someone else.
Support Productive Struggle in Mathematics
More Research on Tasks

• Tasks with high cognitive demands are the most difficult to implement well, and are often transformed into less demanding tasks during instruction (Stein, Grover, and Henningsen 1996; Stigle and Hiebert 2004)
Turn and Talk

• How do you know when a student is struggling?

• What do you do?
Which of the following pictures best describes your productive struggle?
Find it?...

Nope.
Which One?

A Destructive Struggle
- Leads to frustration.
- Makes learning goals feel hazy and out of reach.
- Feels fruitless.
- Leaves students feeling abandoned and on their own.
- Creates a sense of inadequacy.

Productive Struggle
- Leads to understanding.
- Makes learning goals feel attainable and effort seem worthwhile.
- Yields results.
- Leads students to feelings of empowerment and efficacy.
- Creates a sense of hope.
## Think – Pair – Share

| What do students look like when they are engaging in productive struggle? | What do teachers look like when they are facilitating productive struggle? |
Let’s Take A Look at Two Classrooms

• Read the scenario about the two teachers and discuss the questions below with your tablemates.
• What beliefs are evident in Ms. Flahive’s and Ms. Ramirez’s classrooms?
• What impact do those beliefs have on students’ opportunities to grapple with the mathematical ideas and relationships in the problem?
Lessons Learned

• Ms. Flahive’s students learn that if you struggle and are vocal about your confusion, the teacher will ultimately tell you what to do.

• Ms. Ramirez’s students learn that if you struggle and are at an impasse, the teacher will provide some assistance – but in the end you have to figure things out for yourself.
Productive Struggle: Student Behaviors

- Struggling at times but know that learning comes from confusion and struggle.
- Asking questions to help them understand the task
- Persevering in solving problems and realizing that it is ok to say, “I don’t know how to proceed here,” but it is not ok to give up.
- Helping one another without telling their classmates what the answer is or how to solve the problem.

Principles to Action (2014) pg 52
Productive Struggle: 
Teacher Behaviors

• Anticipating what students might struggle with and being prepared to support them productively.
• Giving students time to struggle, and asking questions that scaffold students’ thinking without stepping in.
• Helping students realize that confusion and errors are a part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.
• Praising students for their efforts in making sense and perseverance.

Principles to Action (2014) pg 52
How Can We Support the Development of Productive Struggle?

- Choose rigorous tasks!
- Determine what it looks like in students.
- Make it an explicit focus in classrooms. (Tell students we are doing this!)
- Focus on productive behaviors instead of intelligence.
- Communicate with families.
Mindset and Productive Struggle

Fixed Mindset

• Understanding, proficiency, ability are “set”
• You are good at something or you aren’t

Growth Mindset

• Understanding, proficiency, ability are developed regardless of your genes
• You become better at something as you work with it – as you struggle with it

Dweck, 2008
Why isn’t there a magic bullet for developing productive struggle?
1. I anticipate what students might struggle with during a lesson and I prepare to support them.

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2. I give students time to struggle with tasks and ask questions that scaffold thinking without doing the work for them.

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3. I help students realize that confusing mistakes, misconceptions, and struggles lead to learning.

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4. I praise students for their efforts.

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5. My students struggle at times but they know breakthroughs come from struggle.

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6. My students ask questions that are related to their struggles to help them understand.

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7. My students persevere in solving problems. They don’t give up.

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8. My students help one another without telling their classmates what the answer is or how to solve it.

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What are 2 ideas from this session that resonate with you?
Disclaimer

The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.
Building Procedural Fluency Through Conceptual Understanding

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Latrendakpd@gmail.com
@Latrendak
Game of Nine Cards

• **Materials:** Nine cards numbered 1 – 9.
• **Object:** Be the first person to identify three cards in your hand that add up to 15.
  ▪ Take turns selecting cards.
  ▪ *Note:* You may have more than 3 cards in your hand, but you must use exactly 3 cards to make a sum of 15. (For instance, if you have 2, 3, 5 and 7 in your hand, you would win because $3 + 5 + 7 = 15$. You don’t need to use the 2.)
  ▪ Play the game with a partner. HAVE FUN!!!!

http://illuminations.nctm.org/DeepSeaDuelLP/
Play to Win?
If you play first, should you choose an even or an odd number?
- If your opponent plays first and picks an even number, what number should you choose to avoid a loss?
- Is there a “best” card to choose?
- What role does fluency play in being successful at this game?
- How would you differentiate the game to meet your students’ needs?
Session Goals:

• Explore strategies for helping students build procedural fluency from conceptual understanding.
• Explore strategies for using problem solving experiences to promote fluency development.
• Explore the use of student-generated strategies and common strategies to promote fluency development.
Building Procedural Fluency Through Conceptual Understanding
Commit and Toss

• You’ll need an index card and a pen or marker.
• You have 45 seconds to write down your thoughts on fluency. What does it mean to be fluent? Why is it important to build procedures from conceptual understanding?
• When you hear the signal, toss your card as far as you can.
• Pick up a card (not your own) and share and discuss with a partner.
What is Fluency?

- **Basic Fact Fluency** – “the efficient, appropriate, and flexible application of single-digit calculation skills and ... an essential aspect of mathematical proficiency”. (Baroody, 2006)
- **Computational Fluency** – “the efficient, appropriate and flexible application of single-digit and multi-digit calculation skills” (Baroody, 2006)
- **Procedural Fluency** – “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately”. (CCSSI 2010, p. 6)
Fluency

Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.

Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems. This sequence is beneficial whether students are building toward fluency with single- and multi-digit computation with whole numbers or fluency with, for example, fraction operations, proportional relationships, measurement formulas, or algebraic procedures.

Three Phases

• Children typically progress through 3 phases in mastering the basic number combinations (single-digit addition and multiplication combinations and their complementary subtraction and division combinations).
  – Counting strategies – including object or verbal counting to derive an answer
  – Reasoning strategies – using known facts and relationships to solve an unknown combination
  – Mastery – achieving automaticity in basic fact combinations
<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Counting</strong></td>
<td><strong>Counting objects</strong></td>
</tr>
<tr>
<td>Direct modeling (counting</td>
<td>Separating from</td>
</tr>
<tr>
<td>objects and fingers)</td>
<td>Separating to</td>
</tr>
<tr>
<td>• Counting all</td>
<td>• Adding on</td>
</tr>
<tr>
<td>• Counting on from first</td>
<td></td>
</tr>
<tr>
<td>• Counting on from larger</td>
<td></td>
</tr>
<tr>
<td>Counting abstractly</td>
<td></td>
</tr>
<tr>
<td>• Counting all</td>
<td></td>
</tr>
<tr>
<td>• Counting on from first</td>
<td></td>
</tr>
<tr>
<td>• Counting on from larger</td>
<td></td>
</tr>
<tr>
<td><strong>Reasoning</strong></td>
<td><strong>Properties</strong></td>
</tr>
<tr>
<td>• $a + 0 = a$</td>
<td>• $a - 0 = a$</td>
</tr>
<tr>
<td>• $a + 1 = \text{next whole number}$</td>
<td>• $a - 1 = \text{previous whole number}$</td>
</tr>
<tr>
<td>• Commutative property</td>
<td></td>
</tr>
<tr>
<td>Known-fact derivations</td>
<td>Inverses/complement of known</td>
</tr>
<tr>
<td>(e.g., $5 + 6 = 5 + 5 + 1$;</td>
<td>addition facts (e.g., $12 - 5$</td>
</tr>
<tr>
<td>$7 + 6 = 7 + 7 - 1$)</td>
<td>is known because $5 + 7 = 12$)</td>
</tr>
<tr>
<td><strong>Redistributed derived facts</strong></td>
<td></td>
</tr>
<tr>
<td>(e.g., $7 + 5 = 7 + (3 + 2)$</td>
<td><strong>Redistributed derived facts</strong></td>
</tr>
<tr>
<td>$= (7 + 3) + 2 = 10 + 2 = 12$</td>
<td>(e.g., $12 - 5 = (7 + 5) - 5$</td>
</tr>
<tr>
<td></td>
<td>$= 7 + (5 - 5) = 7$)</td>
</tr>
</tbody>
</table>

*From First-Grade Basic Facts: An Investigation into Teaching and Learning of an Accelerated, High-Demand Memorization Standard, Journal for Research in Mathematics Education © 2008 NCTM*
The Addition Strings Task

Examine the student work on the task below.
Solve the set of addition problems. Each time you solve a problem, try to use the previous problem to solve the next problem.

\[ 7 + 3 = \_\_\_ \]
\[ 17 + 3 = \_\_\_ \]
\[ 27 + 3 = \_\_\_ \]
\[ 37 + 3 = \_\_\_ \]
\[ 37 + 5 = \_\_\_ \]

What pattern do you notice?

Show how you might represent student reasoning with a drawing or on a number line so that students could visually see the relationships among the quantities.
Conceptual Understanding and Procedural Fluency

When procedures are connected with the underlying concepts, students have better retention of the procedures and are more able to apply them in new situations (Fuson, Kalchman, and Bransford 2005). Martin (2009, p. 165) describes some of the reasons that fluency depends on and extends from conceptual understanding:

To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.

Let’s Take a Look at Building Procedural Fluency from Conceptual Understanding in a First Grade Class

Jennifer DiBrienza’s First Grade Class:


Turn and Talk to a partner: Describe the teacher actions you observed that support students building procedural fluency from conceptual understanding.
Build Procedural Fluency from Conceptual Understanding

Teacher Actions

• Ask students to discuss and explain why the procedures that they are using work to solve particular problems.

• Use visual models to support students’ understanding of general methods.

Student Actions

• Demonstrate flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.

• Strive to use procedures appropriately and efficiently.
Strategies Anyone?

• Conceptual understanding of strategies is an important component of basic fact knowledge.
• Providing students with opportunities to use student-developed or student-derived strategies to learn basic facts can help students make sense of the facts. This allows students to use what they know about number combinations and relationships among facts to make connections to those concepts based on their knowledge of strategies.
• These types of activities allow students to progress to computational fluency thus laying the foundation for teachers to provide students with opportunities to build procedural fluency.
• Development of reasoning strategies help students:
  – Move from counting to more efficient methods to recall facts quickly and accurately.
  – Make sense of their thinking and the strategies of others.
Critiquing the Reasoning of Others

• Jose said that he decided to put these two facts together. Do you agree that they belong together? Why or why not?
  6 + 5 and 6 + 6

• One student grouped the following facts:
  9 + 1 = 10, 2 + 8 = 10 and 7 + 3 = 10. If you had chosen to group these facts, what “rule” would you have used?
Analyzing Student Strategies

• A student grouped together the following expressions: 5 + 6, 7 + 6, 5 + 8, and 9 + 6. What reasoning might the student have used to sort these combinations of addends and group them together?

• Turn and Talk to a partner and discuss your thoughts.
Reasoning Strategies

• Two strategies for helping students transition from counting to reasoning strategies are to:
  – Use story problems
  – Explicitly teach reasoning strategies
Build Procedural Fluency from Conceptual Understanding

**Teacher Actions**

- Provide students with opportunities to use their own reasoning strategies and methods for solving problems.
- Connect student-generated strategies and methods to more efficient procedures as appropriate.

**Student Actions**

- Make sure that they understand and can explain the mathematical basis for the procedures that they are using.
- Determine whether specific approaches generalize to a broad class of procedures.
A Tale of Two Tests

Think-Pair-Share

Timed Test vs. Fluency Interview

• How are the two tests different?

• If given a choice, which test would you prefer to take? Why?
Teacher Actions

- Provide students with opportunities for distributed practice of procedures.

Student Actions

- Strive to use procedures appropriately and efficiently.
Using Games to Reinforce Fluency Development

• Games provide an excellent opportunity to keep students engaged! They increase student involvement, encourage student-to-student interaction, and improve communication.

• Use games and engaging activities that allow students to choose from a variety of learned strategies in order to help them become more efficient at choosing strategies.

• Games provide teachers with an opportunity to provide students with distributed practice in order to promote retention of facts.

• Games allow students to participate in engaging, efficient, and self-motivating activities to reinforce math fluency.
Fluency Stations

Complete at least three of the fluency stations. Respond to the focus questions after completing each activity.

• Collect 20 Together
• Collect 25 Together
• One Less
• More – A Game of Differences
• Spin, Circle, and Solve!
• Sort and Group Basic Facts
• Ring Facts
Focus Questions

Answer the following questions as you explore each fluency activity:

• How might students use counting and/or reasoning strategies?

• How could this activity help students build procedural fluency?

• How would you modify, extend, and/or differentiate the activity to address student needs (varying ability levels, grade levels, etc.)?
3- 2- 1- Reflection

To help my students build procedural fluency through conceptual understanding ...

• 3 things I will start doing
• 2 things I want to learn more about
• 1 thing I will keep doing

Share your reflections with a partner.
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.
Facilitating Discourse and Purposeful Questions
Effective Teaching with Principles to Actions Institute, 2017

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Latrendakpd@gmail.com
@Latrendak
The Take Away Game

Using the cubes:
Make a 7 x 5 grid.

For: 2 players or 2 teams
Move 1: Player 1 removes one, two, or three cubes.

Move 2: Player 2 removes one, two, or three cubes.

Play continues until all the cubes are gone. The player that takes the last cube is the winner.

2 Adapted from: Powerful Problem Solving by Max Ray.
Turn and Talk to a Partner

- How might you use this game in your mathematics classroom?
- How might you modify this game for your students?
- What questions would you ask students about their actions during this game?
There are 25 sheep and 5 dogs in a flock. How old is the Shepherd?
Three out of four students will give a numerical answer to this problem.

Effective Teaching Practices

- Implement tasks that promote reasoning and problem solving.
- Facilitate meaningful mathematical discourse.
- Establish mathematics goals to focus learning.
- Use and connect mathematical representations.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Elicit and use evidence of student thinking.
- Support productive struggle in learning mathematics.
Session Goals

• Develop capacity for facilitating discourse.

• Identify types and patterns of questions in mathematics classes.
Facilitating Meaningful Discourse
Math Class

Have you ever been caught in a discourse loop like this?
Paired Interviews

• Interview each other.
• Talk about the best time you observed students engaged in discourse.
• What did you see?
• What did you notice?
• How did you feel?
• Then meet with another pair, share stories and identify the common themes among the stories.
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments (NCTM, 2014, p. 29)

The question becomes... how do we provide opportunities for students to build shared understanding?
Jack has lots of shells. He has more than 40 but less than 60. When he counts them by twos, he has one left over. When he counts them by fives, he has none left over.

The number of shells is more than half of 100. How many shells does Jack have? Show your thinking and reasoning.
Group Rotate and Share

• Two people will leave the group and rotate to the next group

• The other group members stay to explain and defend the work of the group.
Facilitate Meaningful Mathematical Discourse
Teacher and Student Actions

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What might you see teachers do during the shell lesson?

What might you see students do during the shell lesson?
## Facilitate meaningful mathematical discourse

### Teacher and student actions

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</td>
<td>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</td>
</tr>
<tr>
<td>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</td>
<td>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</td>
</tr>
<tr>
<td>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</td>
<td>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others’ strategies, and describing the approaches used by others.</td>
</tr>
<tr>
<td>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</td>
<td>Identifying how different approaches to solving a task are the same and how they are different.</td>
</tr>
</tbody>
</table>
Five Practices for Orchestrating Productive Classroom Discussions

1. **Anticipating** student responses prior to the lesson
2. **Monitoring** students’ work on and engagement with the tasks
3. **Selecting** particular students to present mathematical work
4. **Sequencing** students’ responses in a specific order for discussion
5. **Connecting** different students’ responses and connecting the student responses to key mathematical ideas.

(Smith and Stein, 2011)
Let’s Mingle

- On a post it note. Explain how you would add $23 + 49 = \phantom{a}$.
- Stand up and begin Mingling.
- When I give the signal, stop and find a partner.
- Say or read your explanation.
- Partners may ask one clarifying question.

Adapted from: Powerful Problem Solving by Max Ray.
## Which types of questions did you ask and answer?

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gathering Information</td>
<td>Students recall facts, definitions, or procedures.</td>
<td>What do we know about Darren’s donuts?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When you write an equation, what does the equal sign tell you?</td>
</tr>
<tr>
<td>Probing Thinking</td>
<td>Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.</td>
<td>Can you tell me how your picture and number sentence helped you solve the problem?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Can you tell me how you figured out how many donuts Darren has and how many donuts Tamara has?</td>
</tr>
<tr>
<td>Making the Mathematics Visible</td>
<td>Students discuss mathematical structures and make connections among mathematical ideas and relationships.</td>
<td>What does your number sentence show about the donut problem?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>How do you know that both Tamara and Darren have 7 donuts?</td>
</tr>
<tr>
<td>Encouraging Reflection and Justification</td>
<td>Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.</td>
<td>How do you know that the sum of two odd numbers will always be even?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>What is another number sentence you can write to tell about Tamara’s donuts?</td>
</tr>
</tbody>
</table>

Pose Purposeful Questions
Pose Purposeful Questions

How do we plan, anticipate, and engage our students in questions?
The Donuts Task

• Dion chooses 3 chocolate donuts and 4 vanilla donuts. Draw a picture and write an equation to show Dion’s donuts.

• Tamika has 4 vanilla donuts and 3 chocolate donuts. Draw a picture and write an equation to show Tamika’s donuts.

• Tamika claims that she has more donuts than Dion. Who has more donuts, Dion or Tamika? Draw a picture and write an equation to show how you know who has more donuts.
The Donuts Task

• What might students notice about this task?
• What kinds of questions might students ask about this task?
• What could we expect to see students doing?
• What could we expect to see the teacher doing?
Focus for Viewing

• Select a partner. One of you will be partner A and the other person will be partner B.

• As you watch the video clip, pay attention to the student and teacher actions related to the use of purposeful questions.
  – What are the students doing? (Partner A)
  – What is the teacher doing? (Partner B)
  – Make notes on the recording sheet as you view the video clip.
Let’s Take a Look at Purposeful Questioning in a Kindergarten Class

• Amanda Smith’s Kindergarten Class

• Use your recording sheet to make note of the types of questions Mrs. Smith asked her students.

• Be prepared to share your observations with a partner.
# Posing Purposeful Questions

## Teacher and Student Actions – Turn and Talk

**What is the teacher doing?**

**What are the students doing?**

<table>
<thead>
<tr>
<th>What are <em>teachers</em> doing?</th>
<th>What are <em>students</em> doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner A</td>
<td>Partner B</td>
</tr>
</tbody>
</table>

**Bonus Question:**
Did you make any observations about teacher and student actions relating to meaningful mathematics discourse?
<table>
<thead>
<tr>
<th><strong>What are teachers doing?</strong></th>
<th><strong>What are students doing?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.</td>
<td>Expecting to be asked to explain, clarify, and elaborate on their thinking.</td>
</tr>
<tr>
<td>Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.</td>
<td>Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly.</td>
</tr>
<tr>
<td>Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion.</td>
<td>Reflecting on and justifying their reasoning, not simply providing answers.</td>
</tr>
<tr>
<td>Allowing sufficient wait time so that more students can formulate and offer responses.</td>
<td>Listening to, commenting on, and questioning the contributions of their classmates.</td>
</tr>
</tbody>
</table>
Funneling - Involves using a set of questions to lead students to a desired procedure or conclusion, while giving limited attention to student responses that veer from the desired path. The teacher has decided on the path and leads the students along the path, not allowing them to make their own connections or build their own understanding (p. 37).

Focusing - Involves the teacher in attending to what the students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates. This teacher is open to the task being explored in multiple ways (p.37)
<table>
<thead>
<tr>
<th>Teacher role</th>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Mathematical representations</th>
<th>Building student responsibility within the community</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>Teacher is at the front of the room and dominates conversation. Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only. Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.</td>
<td>Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.</td>
<td>Representations are missing, or teacher shows them to students.</td>
<td>Culture supports students keeping ideas to themselves or just providing answers when asked.</td>
</tr>
<tr>
<td>Level 1</td>
<td>Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only. Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions. Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing. Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing. Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to re-state in their own words what another student has said.</td>
</tr>
<tr>
<td>Level 2</td>
<td>Teacher facilitates conversation between students, and encourages students to ask questions of one another. Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher. Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers. Students label their math drawings so that others are able to follow their mathematical thinking.</td>
<td>Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers. Students label their math drawings so that others are able to follow their mathematical thinking.</td>
<td>Students label their math drawings so that others are able to follow their mathematical thinking.</td>
<td>Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.</td>
</tr>
<tr>
<td>Level 3</td>
<td>Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others. Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse. Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher. Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
<td>Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse. Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher. Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
<td>Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
<td>Students believe that they are math leaders and can help shape the thinking of others. They help shape others’ math thinking in supportive, collegial ways and accept the same support from others.</td>
</tr>
</tbody>
</table>
How do we create a place where thinking is valued and visible?

• Do students regularly engage in wonder?
• Do students ask lots and lots of questions?
• Do students exhibit the practices AS they engage in the content?
• Do students have an opportunity to collaborate and build on each other’s thinking?
REFLECTION

• To facilitate meaningful mathematics discourse and pose purposeful questions:
  – Identify two points for application in your school setting.
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Elicit and Use Evidence of Student Thinking
Effective Teaching with Principles to Actions Institute, 2017

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Think-Pair-Share:
How should instruction and assessment be related?
Formative Assessment is Like Looking for Icebergs

Observations

Interviews

Conversations

Extended response tasks

Journals and reflections

Reports and projects

Summative Assessment

Selected Response Items

Open-Ended Questions

Portfolios

Exit tickets
Formative Assessment is like GPS.

1. Where are we going?
2. Where are we?
3. How are we getting there?
Why Use Formative Assessment?

- To inform instruction and provide feedback to students (and you) on their learning.

The Big Picture

- Where are you? – formative assessment
- Where are you going? – standards
- How are you going to get there? – progressions
Assessment Belief Statements

Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.

The primary purpose of assessment is accountability for students through report card marks or grades.

The primary purpose of assessment is to inform and improve the teaching and learning of mathematics.

Assessment in the classroom is an interruption of the instructional process.

Assessment is something that is done to students.

Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.

Multiple data sources are needed to provide an accurate picture of teacher and student performance.

Stopping teaching to review and take practice tests improves students’ performance on high-stakes tests.

Only multiple-choice and other “objective” paper-and-pencil tests can measure mathematical knowledge reliably and accurately.

Ongoing review and distributed practice within effective instruction are productive test preparation strategies.

A single assessment can be used to make important decisions about students and teachers.
What Beliefs Might We Hold about Mathematics Assessment?

At your table:

Choose two belief statements and:

1. Decide whether the statement is a productive or unproductive belief.
2. Explain your reasoning.
3. Describe the point of view of someone you know who feels the same about the belief statement.
4. How might that belief statement have been developed?
Unproductive Belief Statements

- The primary purpose of assessment is accountability for students through report card marks or grades.
- Assessment in the classroom is an interruption of the instructional process.
- Only multiple-choice and other “objective” paper-and-pencil tests can measure mathematical knowledge reliably and accurately.
- A single assessment can be used to make important decisions about students and teachers.
- Assessment is something that is done to students.
- Stopping teaching to review and take practice tests improves students’ performance on high-stakes tests.
Productive Beliefs Statements

• The primary purpose of assessment is to inform and improve the teaching and learning of mathematics.

• Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.

• Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.

• Multiple data sources are needed to provide an accurate picture of teacher and student performance.

• Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.

• Ongoing review and distributed practice within effective instruction are productive test preparation strategies.
Elicit and Use Evidence of Student Thinking
Session Goals

• Develop the process of eliciting and using student thinking
• Connect the effective teaching practices that have been examined during the institute
Paired Interviews

• Interview the person sitting next to you and ask:
  • What is the most surprising student evidence of learning you have collected?
  • How did you collect this information? When did it happen?
  • Why was it surprising?
  • What did you do differently as a result of learning this information?
Let’s consider...

• What counts as evidence...
  – Prerequisite/progression of concept
  – Representations
  – Concepts/conceptual understanding
  – Possible procedures/symbols

• How you might collect evidence?
Task/Question to investigate:

• Interpreting their thinking
  – Do your students understand?
  – How do you know?

• Making In-The-Moment Decisions
  – How is their work similar/different than what you expected?
  – How do you reflect on what you expected?
  – How might you adapt what you expected?
  – Does anything surprise you?

• What might you change?
What Counts As Evidence
What Might Count As Evidence?

Second grade students were given the task of sorting a variety of problems by grouping together problems that they believed were alike. Students were then asked to explain their sorting choices.

Examine the student work and look for evidence of students attention to underlying structures.
Making Use of Evidence: What Do We Notice?

1. Jimmy found fruit snails counted 3 snails in his bag. He ate 17 snacks. If each fruit snail in his bag, Lana ate 5 candy last six pieces. How many candy has eaten in the days?

2. Keisha had 32 comic books. She gave 14 to her cousin. How many does she have now?

3. Sara ran 31 miles last week. Stephanie ran 42 miles last week. How many more miles did Stephanie run that Sara?

4. Sari can play 37 songs on the piano. Aldo can play 44 songs on the piano. How many more songs can Aldo play than Sari?

5. Raida loved to put together puzzles. On Saturday she put together 9 puzzles and on Sunday she put together 15 puzzles! How many more puzzles did she put together on Sunday than Saturday?

6. Lana ate 9 pieces of candy last night and six pieces today. How many pieces of candy has Lana eaten in the last two days?

Explain why these problems go together.

Problems go together because they all have days of the week.
Making Use of Evidence
What do we notice?

• What reasoning do you think each student used in grouping the problems he or she selected as alike?

• Which students were paying attention to the underlying structures, and which were not?

• What do these samples of students’ groupings reveal that could be used to inform instructional next steps?
Interpreting Student Thinking
Adding Two-Digit Numbers

These four assessment tasks were used with first and second grade students who were beginning to add two-digit numbers.

Task A
Stickers come in packs of 10. A box of stickers holds 10 packs. Susie has 4 packs and 3 more stickers. Jose has 6 packs and 9 more stickers. How many stickers do they have in all?

Task B
Susie has 43 stickers. Jose has 69 stickers. How many stickers do they have in all?

Task C
43 + 69 = ?

Task D
\[
\begin{array}{c}
43 \\
+ 69 \\
\end{array}
\]
Adding Two-Digit Numbers

- How might students approach each of these tasks?
- What tools might you provide to help students solve these problems?
- In what order might you use the tasks?
- How might changing the numbers in the tasks affect their level of difficulty or the strategies students might use?
Amy’s solution
Amy made drawings to show the stickers that Susie and Jose had:

Susie

\\begin{array}{cccc}
10 & 10 & 10 & 10 \\
\end{array}

\[ 10 + 60 = 100 \]

Jose

\\begin{array}{cccc}
10 & 10 & 10 & 10 \\
10 & 10 & 10 & 10 \\
\end{array}

\[ 3 + 9 = 12 \]

\[ \frac{112}{112} \text{ stickers in all} \]
Interpreting Student Thinking:

Ben’s solution
Ben’s teacher distributed smiley-face stickers grouped in tens and ones; Ben arranged them as shown and then counted by tens and ones:

Susie

José

10 20 30 40 50 60 70 80 90 100
1 2 3 4 5 6 7 8 9 10 11 12

112 stickers in all

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Interpreting Student Thinking:

Craig’s solution
Craig solved task C by thinking about place value:

$$43 + 69 = 40 + 60 + 3 + 9 = 100 + 12 = 112$$
Interpreting Student Thinking:

Juan’s and Fran’s solutions
Juan and Fran both approached task D by thinking about place value but arrived at different answers:

Juan
43
+69
100
+12
112

Fran
43
+69
1012

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Reflecting on Evidence of Student Learning to Inform Next Steps
What Happens Next?

First grade students were asked to solve a series of word problems.

• Examine the student work samples and describe possible next steps for each student.
Putting It All Together
(The Institute)

- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Build procedural fluency from conceptual understanding.
- Facilitate meaningful mathematical discourse.
- Elicit and use evidence of student thinking.
- Establish mathematics goals to focus learning.
- Pose purposeful questions.
- Support productive struggle in learning mathematics.
Putting it Together

• Each table will receive one of the Principles to Actions Practices on a slip of paper.
• Your table group will create a poster for your assigned practice. Your job is to design and draw a picture, symbol, or metaphor for that practice and complete the other sections on the chart paper and post for a *Gallery Walk*. 
Design Your Practice!

“In a Nutshell”
3 Big Ideas about the Practice

Design a picture, symbol or metaphor for the practice

What are teacher behaviors of the practice?

What are student behaviors of the practice?

Practice
During the *Gallery Walk*, use:

- a **green** sticky dot to show that you **agree** with the design for the practice.
- a **blue** sticky dot to show that the design prompted you to think about the practice in a new way.
- a **yellow** sticky dot to show that the design helped you remember something about the practice that you learned.
Institute Closing:
Personal reflection on practices

- Which is the easiest for you to implement?

- Which is most challenging?

- How might you use your strength to develop your challenge?
Institute Closing: Content Debrief

What might you **Start** as a result of the Institute?

What might you **Stop** as a result of the Institute?

What might you **Keep** as a result of the Institute?
Disclaimer

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