Building Procedural Fluency Through Conceptual Understanding

Latrenda Knighten
Elementary Mathematics Instructional Specialist
East Baton Rouge Parish School System
Latrendakpd@gmail.com
@Latrendak
Sum of 1: A Variation of the Game of Nine Cards

- **Materials:** Nine cards numbered $\frac{1}{6}, \frac{5}{24}, \frac{1}{4}, \frac{7}{24}, \frac{1}{3}, \frac{3}{8}, \frac{5}{12}, \frac{11}{12}, \frac{1}{2}$

- **Object:** Be the first person to identify three cards in your hand that add up to 1.

- **Take turns selecting cards.**

- **Note:** You may have more than 3 cards in your hand, but you must use exactly 3 cards to make a sum of 1. (For instance, if you have $\frac{5}{24}, \frac{1}{4}, \frac{7}{24}$ and $\frac{1}{2}$ in your hand, you would win because $\frac{5}{24} + \frac{7}{24} + \frac{1}{2} = 1$. You don’t need to use the $\frac{1}{4}$.)

- **Play the game with a partner. HAVE FUN!!!!**
Think-Pair-Share

• What (if anything) should students know about fractions prior to playing this game?
• How does this game build fluency with adding fractions?
• How would you use this game in your classroom?
• How would you modify this game to fit the needs of your students?
Session Goals:

• Explore strategies for helping students build procedural fluency from conceptual understanding as it relates to fraction operations (primarily addition and subtraction).

• Explore strategies and activities to help students transition from computation with whole numbers to computation with rational numbers.
Building Procedural Fluency Through Conceptual Understanding
Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.

Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems. This sequence is beneficial whether students are building toward fluency with single- and multi-digit computation with whole numbers or fluency with, for example, fraction operations, proportional relationships, measurement formulas, or algebraic procedures.
Conceptual Understanding and Procedural Fluency

When procedures are connected with the underlying concepts, students have better retention of the procedures and are more able to apply them in new situations (Fuson, Kalchman, and Bransford 2005). Martin (2009, p. 165) describes some of the reasons that fluency depends on and extends from conceptual understanding:

*To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.*

### Adding and Subtracting Fractions: A Textbook View

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Sequencing Addition and Subtraction Problems

• Developing a deep understanding of adding and subtracting fractions with unlike denominators likely requires more than a few weeks of instruction.
A variation on the “typical” sequence of fraction problems begins with addition and subtraction of fractions with like denominators, and gradually incorporating whole numbers or mixed numbers with like denominators.

Pose problems in a context

Include fractions that are less than 1 with whole numbers and mixed numbers.
Sequencing Addition and Subtraction Problems

• Provide students with problems involving fractions or mixed numbers with different denominators, beginning with one denominator that is a multiple of the other.

• Use fractions with different denominators that do not share a common factor such as 9 and 4.
Solving Fraction Problems

• Select one of the problem solving situations below:
• Use concrete materials and pictorial models to represent the situation and share your solution strategies with a partner.

- Gerald ate \( \frac{3}{12} \) of a cherry pie, and Donna ate \( \frac{1}{12} \) of the same cherry pie. How much of the cherry pie did Gerald and Donna eat?

- Marty had 4 doughnuts. He ate \( \frac{3}{4} \) of one doughnut. How many doughnuts does Marty have now?

- Thomas had 2 \( \frac{1}{4} \) pounds of meat. He used \( \frac{3}{4} \) of a pound to make a meatloaf. How much meat does Thomas have left?
Scaffolds from Earlier Grades

• Focusing Students on the Unit - Unitizing

• Multiplicative Thinking
  – Partitioning
  – Doubling and Halving
  – Iterating the group
Activity Session

Try at least two of the scaffolding activities with the members of your table group.

• How can these activities facilitate an understanding of fraction concepts?

• How can participating in activities like these help prepare students for work with fraction operations?
Investigating Equivalent Fractions

- [http://illuminations.nctm.org/Lesson.aspx?id=1731](http://illuminations.nctm.org/Lesson.aspx?id=1731)

Use your relationship rods to answer the following questions.

1. What colors can be lined up end-to-end to create the same length as the brown rod? For example, eight white rods can be lined up to create the same length as one brown rod.
Given the whole, find the part

- This dark green rod is one whole.
  - find one-third
  - find two-thirds
  - find six-thirds
- If students successfully solve problems such as this, what can you learn about student understanding?
- How can we support students who struggle with this type of task?
Compare Fractions

• Draw two area models.

• Partition and shade the first model to show \( \frac{1}{2} \). Partition and shade the second model into 5 equal parts. Shade 2 units and label it \( \frac{2}{5} \).

• Partition the area models so that both fractions have common denominators.

• Compare the fractions using >, <, or =

• Select one of the fractions pairs below to compare: \( \frac{1}{5} \) and \( \frac{3}{10} \), \( \frac{1}{4} \) and \( \frac{5}{8} \), or \( \frac{1}{3} \) and \( \frac{3}{4} \).
Importance of the Unit in Addition and Subtraction of Fractions

• Similar to addition with whole numbers, addition with fractions involves combining disjoint sets.

• When adding and subtracting two fractions, students should recognize the following:
  – Both fraction addends need to have the same unit
  – The resulting sum of an addition problem will also have that unit
  – Fractions that are subtracted from one another must refer to the same unit.
  – The context of the problem affects the way you solve the problem or interpret the answer.
Addition and Subtraction Situations

• Similar to when adding whole numbers, students use a variety of strategies such as direct modeling, counting strategies, and the use of derived facts.

• When students begin to add or subtract fractions, they also engage in direct modeling by using physical models or pictorial diagrams.
Addition and Subtraction Situations

• Examine the task below. Solve the task in two different ways. Show how you solved it and explain your thinking.

• Also consider potential misconceptions student might encounter when working on the task.

• How can addition be used to solve the task? How can subtraction be used to solve the task?

Johanna needs $3 \frac{1}{2}$ cups of flour for a cake. She only has $2 \frac{1}{4}$ cups of flour. How much more flour does Johanna need to make the cake?
Race to Zero!

Materials:
– Cuisenaire Rods
– One die

Objective:
– Roll and subtract down to zero

Directions:
1. Form two teams at your table.
2. Start with three dark green rods. Each dark green rod represents one whole.
3. Roll the die and subtract that amount from one of the one whole pieces.
4. The winning team is the first team to make it to zero.
Think-Pair-Share

• How does this game build fluency with subtracting fractions?
• How would you use this game in your classroom?
• How would you modify this game to fit the needs of your students?
Addition and Subtraction Situations

• Examine the tasks on the Fraction Addition and Subtraction page. Use a different diagram or model (fraction strips, number line, grid paper or objects for a set model) to represent each.

• How are your diagrams or models similar? How are they different?

• What type of problem (join, separate, part-part-whole, or comparison) does each task represent? Does the diagram help determine the problem type?
Fraction Addition and Subtraction

- Problem 1: separating action (removing $\frac{3}{4}$ of a page of stickers from a larger group of 2 $\frac{1}{2}$ pages.
- Problem 2: compare (two different groups are compared)
- Problem 3: part-part-whole (one part is $\frac{3}{4}$ of a page, another part is unknown and the whole is 2 $\frac{1}{2}$ pages)
- Problem 4: joining action (combines two separate groups)
Adding Fractions: ½ + 1/3

• Examine the problems on the Adding Fractions handout.
• Which of these problems can be solved by adding ½ + 1/3?
• Show work to support your answer to each problem.
• What misunderstandings or misconceptions do you anticipate your students might experience?
Subtracting Fractions: $\frac{2}{3} - \frac{1}{2}$

- Examine the problems on the Subtracting Fractions handout.
- Which of these problems can be solved by subtracting $\frac{2}{3} - \frac{1}{2}$?
- Show work to support your answer to each problem.
- What misunderstandings or misconceptions do you anticipate your students might experience?
Looking at Student Work

• Examine the student work samples. The student work samples show the reasoning of five students as they used models to show the addition of $\frac{3}{4}$ and $\frac{2}{3}$.

• While looking at the collection of student work, consider the following questions:
  • What representations are evident in the student work?
  • What does the student work tell us about their understanding?
  • How are the representations alike?
  • How do student representations communicate understanding?
Think-Pair-Share

• Which students show errors in their reasoning?
• What questions would you ask these students to help them understand their errors?
• How could you use the students’ diagrams to help them find the sum?
• Does the context influence the type of diagram or model used?
Facilitating Conceptual Understanding

• Students need to build a conceptual understanding of fraction concepts in order to transition from addition and subtraction with whole numbers to addition and subtraction with rational numbers.

• To aid students in making this transition successfully, teachers should provide students with classroom activities that focus on the following:
  – Concept of unit
  – Varied interpretations of fractions
  – and Equivalency and comparison of fractions.
Fluency Interview

How can this type of assessment help assess student understanding of fraction operations?

1. Show how to use a number bond to decompose the difference between \( \frac{16}{9} - \frac{5}{9} \). Record your answer as a mixed number.

2. Jhordan explains that \( \frac{3}{12} + \frac{1}{12} = \frac{1}{3} \). Is he correct? Explain how you know.

3. Solve \( \frac{1}{2} + \frac{1}{3} \). What strategy did you use?

3. Dhruva has \( \frac{3}{8} \) of a medium pepperoni pizza. His dad gives him \( \frac{2}{8} \) more of a medium pepperoni pizza. How much of a medium pepperoni pizza does he have now? Explain how you got your answer.

4. Jasmine solved \( 1 - \frac{1}{3} \) by changing it in her mind to \( \frac{3}{3} - \frac{1}{3} \). Why do you think she did this?
Teacher Actions:

• Provide students with opportunities to use their own reasoning strategies and methods for solving problems.

• Ask students to discuss and explain why the procedures that they are using work to solve particular problems.

• Connect student-generated strategies and methods to more efficient procedures as appropriate.

• Use visual models to support students’ understanding of general methods.

• Provide students with opportunities for distributed practice of procedures.
Student Actions

• Make sure that they understand and can explain the mathematical basis for the procedures that they are using.

• Demonstrate flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.

• Determine whether specific approaches generalize to a broad class of procedures.

• Strive to use procedures appropriately and efficiently.
Reflection

• To reflect on your work in this session, complete a thinking log by responding to one of the sentence stems below.
  
• Right now I am thinking about ...

• I need to rethink ...

• I think in the future I would like to try ...

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