Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

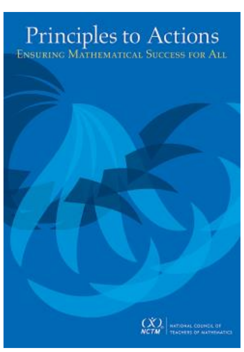
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*



National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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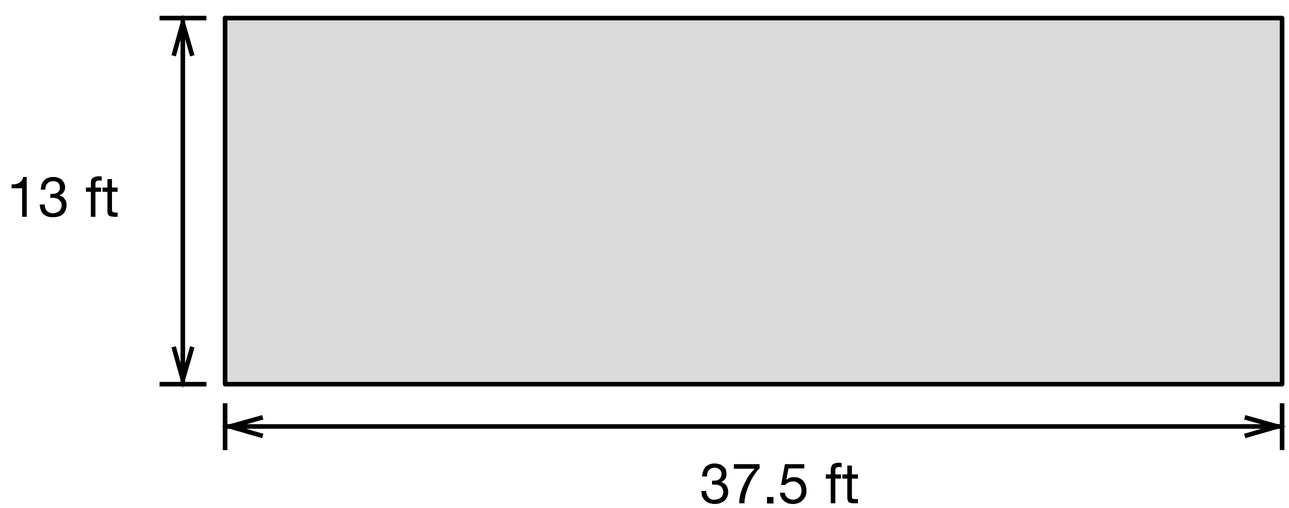
The Arena Logo Task

A sports and entertainment group is proposing a new arena for New York City to compete with Madison Square Garden. The Wilson Parallelogram Garden is seeking city approval for arena construction to begin. They have used the logo below in their promotional materials.

The WPG group is designing the arena façade and has a rectangular area for the exterior lighted arena logo shown below (area to scale).



1. Using any tools available to you, scale the WPG logo to fit the façade area. Explain how you produced your dilation.

2. The WPG group ordered the lighted logo sign. They received a sign like the one below (picture to scale). Is this sign correct? Explain how you know using mathematical language.

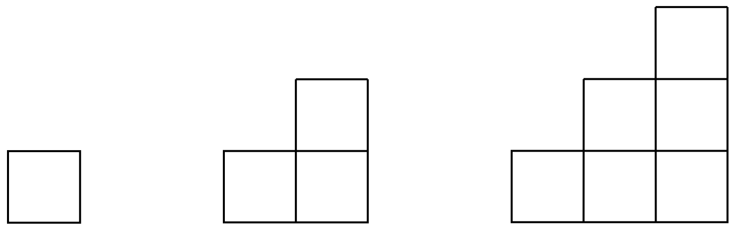
3. Write a rule or process for producing a scale copy of the WPG logo for the exterior arena sign.

Building Rabbit Pens

You are going to build a rectangular pen for your rabbit, Euclid. You have decided to build the pen using some portion of the back of your house as one side of the pen and enclosing the other three sides with the fencing that was left over from another project. If you want Euclid to have as much room as possible (after all he spends most of the day in his pen), what would the length and width of the pen be?

The Staircase Task

Below are the first three instances of a pattern of staircases built from square blocks.



The first staircase is 1 step high and is made with 1 block.
The second staircase is 2 steps high and is made with 3 blocks.
The third staircase is 3 steps high and is made with 6 blocks.

Consider the following questions:

1. How many blocks would you need to construct the 50th staircase in this pattern, that is, a staircase 50 steps high?

2. How many blocks would you need to construct the staircase that is 100 steps high?

3. How many blocks would you need to construct a staircase that is *n* steps high?

Epidemic! When Viruses Attack (MS edition)

Considering the Context

1. What do you already know about plagues and epidemics? What have you seen in a movie, read in a book, or learned in class related to this topic?
2. In general, comparing life in the 1600s to life today, what might increase or decrease the spread of epidemics in today's world?

Simulating the Spread

3. After the simulation is conducted, try to represent the pattern between each round and the number of infected students by creating x-y data tables. Continue each data table beyond the number of rounds completed in class for each simulation.
4. What patterns do you notice?
5. Write a general description for how the pattern is changing.
6. Suppose a simulation was conducted with an entire school of 2000 students starting with 4 infected students. How many students would be infected after the eighth round? Use drawings, models, or words to explain your reasoning.
7. How would you describe the number of infected students for any stage of the epidemic?
8. Consider the representations of other groups working on the task. What do each of these different representations allow you to see about the epidemic situation?

Culminating Questions

9. Predict how the situation would change if every person infected 2 new people (rather than just 1).
10. In real life, infected people might come in contact with other infected people (rather than infecting someone new). How would this change the outcome of the simulation?
11. In real life, some people might be immune. Predict how this could change the outcome of the simulation.
12. Name other real-world phenomena that could result in exponential growth or decay.

Routines for Mathematical Discourse

Protocol for Workshopping a Task/Lesson:

1. Determine order for sharing & assign a time-keeper.
2. Have the first person decide if they want a recorder. (The recorder takes notes on the discussion for the person who is sharing the lesson). Now, share lesson seed/ lesson idea including materials, etc (This does not need to be in full lesson plan format.)
3. **Notice**: Participants make observations. These are not judgmental in a positive or negative way. Just noticing. Some
4. Question: Participants ask clarifying questions.
5. What would happen if...Participants ask questions:
    - I wonder what would happen if…
        - you moved that question later on in the lesson
        - you had the students try the task first

- What if? Teacher asks clarifying questions of the participants.

Teacher summarizes

The Wet Phones Task

Part 1

A shipper is shipping large boxes of iPhones from Shanghai. Each large box measures 36"x18"x16". It is filled completely with iPhones, which are in boxes that are 3"x4"x6".

One box falls off the boat and submerges completely, but is pulled out of the water almost immediately.

Upon talking with Apple, the shipper concludes that all iPhones that were touching the outside of the wet box will have to be returned to Apple to check if they still work. How many iPhones will have to go back? Be sure to explain how you arrived at your answer, and use words, symbols, and/or diagrams to support your explanation.

Part 2

Upon hearing the bad news, Apple decides that they need to ship their iPhones in a larger box. They want to design a box that has twice the volume of the original and that minimizes the number of iPhones that would be damaged in a similar accident.

What are the dimensions of a box that serves this purpose?

If Apple's box supplier charges by the square inch of surface area, how much more will the new box cost?

Task Analysis Guide

Lower-level demands (memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas

that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1986) and Resnick on high-level thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and H