Quick and Meaningful Tasks to Engage and Assess Mathematical Thinking

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NCTM’s Principles to Actions

“An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.”

NCTM Principles to Actions: Ensuring Mathematical Success for All  NCTM 2014
Mathematics Teaching Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

*Principles to Actions: Ensuring Mathematical Success for All. NCTM 2014*
Students need to “do math”

“In order to develop students’ abilities to ‘do mathematics’, classrooms must become environments in which students are able to engage actively in rich, worthwhile mathematical activity.”

Marjorie Henningsen and Mary Kay Stein

Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning
Choosing a Meaningful Task

• Does the task serve the students’ learning goal?
• Does the task allow students to make connections to content they already know?
• Does the task allow for multiple solution methods or approaches?
• Does it encourage students to reason about mathematics and allow for them to communicate mathematically?
• Does the task engage all learners?

*From Designing and Implementing Worthwhile Tasks, Breyfogle and Williams*
Meaningful Ten Minute Tasks

- Visual / Spatial Tasks
- Mental Math / Silent Math
- Formative Assessment Probes
- Mini- Rich Tasks / Mini-Lessons / Concept Games
- Online Adaptive Technology Tasks
NCTM Process Standards and the CCSSM Mathematical Practices

<table>
<thead>
<tr>
<th>NCTM Process Standards</th>
<th>CCSS Standards for Mathematical Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td></td>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td></td>
<td>3. Critique the reasoning of others.</td>
</tr>
<tr>
<td></td>
<td>4. Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>

| Reasoning and Proof    | 1. Make sense of problems and persevere in solving them. |
|                        | 2. Reason abstractly and quantitatively. |
|                        | 3. Construct viable arguments. |
|                        | 4. Model with mathematics. |
|                        | 5. Use appropriate tools strategically. |

| Communication          | 1. Make sense of problems and persevere in solving them. |
|                        | 2. Reason abstractly and quantitatively. |
|                        | 3. Construct viable arguments. |

| Connections            | 1. Make sense of problems and persevere in solving them. |
|                        | 2. Reason abstractly and quantitatively. |
|                        | 3. Construct viable arguments. |

| Representations        | 1. Make sense of problems and persevere in solving them. |
|                        | 2. Reason abstractly and quantitatively. |
|                        | 3. Construct viable arguments. |

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Standards for Mathematical Practice

1. I make sense of math problems and keep trying even when problems are challenging.

2. I use numbers and symbols to describe math situations.

3. I justify my strategies and listen to see if other students’ ideas make sense.

4. I make models of math ideas.

5. I can decide which math tool to use and I know how to use it correctly.

6. I try to be accurate when I compute and I am specific when I talk about math ideas.

7. I use what I know about numbers, patterns, and properties to help me find answers.

8. I notice when things repeatedly happen and try to figure out rules to explain what is happening.
Quick Images

Students briefly view a geometric figure or design and either draw or build it from the mental image they formed during the brief viewing.

from *Ten Minute Math* by Cornelia Tierney /Susan Jo Russell
Why do Quick Images?

- Goals of Quick Images
  - Organize and analyze visual images
  - Develop concepts and language to communicate about spatial relationships
  - Use geometric vocabulary to describe shapes and patterns
  - Use number relationships to describe patterns
Quick Images Procedure

1. Flash image for three seconds.
2. Students draw or build what they see.
3. Flash image again, for revision.
4. Show image 3rd and final time.
5. Discuss the mental images students formed.
Quick Images – Seeing Numbers
3.G Reason with shapes and their attributes
4.G.3 Recognize a line of symmetry for a 2-d figure
5.G.3/4 Classify 2-d figures into categories based on properties
2- D Quick Image
Use Pattern Blocks for Quick Images
More 2-D Quick Images

[Diagram of geometric shapes]

- Star-like shape
- Cubic structure
- Triangular division
- Rectangular division
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
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3-D Quick Images
3 – D Quick Image
3 – D Geometry Quick Image
5.MD.3-5 Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.
CCSSM Standards for Volume

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
   b. A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.
Park City Mathematics Institute (PCMI) of the Institute for Advanced Study, Princeton, NJ, offers comprehensive professional development for mathematicians and teachers of mathematics, as well as programs for students aspiring to a career in mathematics.

- The Summer School Teachers Program (SSTP) Summer Session is a 3-week residential program in Park City, Utah and is part of the larger PCMI program. The SSTP is for school teachers who specialize in mathematics grades 3-12.

- Go to https://pcmi.ias.edu/ for more information.
Number Talks

• 5-10 minute conversations around purposefully crafted computation problems that are solved mentally
• Help develop essential processes and habits of mind of doing math.
• Designed to elicit specific strategies that focus on number relationships
• Lead to development of more accurate, efficient, and flexible strategies

Number Talks by Sherry Parrish
Let's view a Number Talk

70 - 34

Take a few seconds to solve MENTALLY

Share with others how you solved and listen to what strategies other people used.

CCSSM 3.NBT.2 Fluently add and subtract within 1000.
Third graders solve 70-34

When watching the video, focus on:
1. what the teacher is doing and saying
2. how the students communicate their answers and strategies.

Discussion Questions:
1. What structures are present in the classroom to encourage student thinking / participation?
2. How is the relationship between addition and subtraction used to build efficient strategies?
3. Why did Grant choose to increase both numbers by 6? Other ways he could have altered the problem?
Strategies used for 70 - 34

**Removal**
- 70 - 34
- 70 - 30 = 40
- 40 - 4 = 36

**Adding Up**
- 70 - 34
- 34 + 6 = 40
- 40 + 30 = 70

**Constant Difference**
- 70 - 34
- 70 - 34
- + 6 + 6
- 76 - 40 = 36
What if my students do not come up with a strategy?

Use an Introductory Number Talk that readily encourages a specific strategy

*Removal Strategy*

35 – 10  
73 – 50  
Rick has 70 marbles. He decides to give his friend 34 of his marbles.

35 – 13  
73 – 51  
How many marbles will Rick have left?

35 – 20  
78 – 20  
35 – 33  
78 – 27  
Contextualize with word problems that imply taking or removing an amount from the whole.
What if my students need more practice for a strategy?

Use a Number Talk with specific chosen numbers for students that have been successful with a strategy.

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-89</td>
<td>750-709</td>
</tr>
<tr>
<td>100-69</td>
<td>750-599</td>
</tr>
<tr>
<td>100-49</td>
<td>750-449</td>
</tr>
</tbody>
</table>
What if my students need more challenge with a strategy?

Use a number talk that allows for students to use and extend a targeted strategy.

*Keeping a Constant Difference Strategy*

<table>
<thead>
<tr>
<th>Subtraction 1</th>
<th>Subtraction 2</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 – 19</td>
<td>300 – 214</td>
<td>In this strategy, both the subtrahend and minuend are adjusted by the same amount. Allows students to make friendlier numbers.</td>
</tr>
<tr>
<td>48 – 29</td>
<td>500 – 289</td>
<td></td>
</tr>
<tr>
<td>35 – 18</td>
<td>700 - 477</td>
<td></td>
</tr>
<tr>
<td>41 – 13</td>
<td>1000 – 674</td>
<td></td>
</tr>
</tbody>
</table>
Key Components of Number Talks

• Classroom Environment
• Classroom Discussions
• Teachers Role
• Mental Math
• Purposeful Computation

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to continually adjust instruction in ways that support and extend learning.

From Principles to Actions: Ensuring Mathematical Success for All. NCTM, 2014
## Evidence of Student Thinking

### Teacher and Student Actions: Elicit and Use Evidence of Student Thinking

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Identifying what counts as evidence of student progress toward mathematics learning goals.</td>
<td>• Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse.</td>
</tr>
<tr>
<td>• Eliciting and gathering evidence of student understanding at strategic points during instruction.</td>
<td>• Reflecting on mistakes and misconceptions in order to improve their mathematical understanding.</td>
</tr>
<tr>
<td>• Interpreting student thinking to assess mathematical understanding, reasoning, and methods.</td>
<td>• Asking questions, responding to, and giving suggestions to support the learning of their classmates.</td>
</tr>
<tr>
<td>• Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.</td>
<td>• Assessing and monitoring their own progress toward mathematics learning goals, and identifying areas in which they need to improve.</td>
</tr>
<tr>
<td>• Reflecting on evidence of student learning to inform planning next instructional steps.</td>
<td></td>
</tr>
</tbody>
</table>
Formative Assessment Probes

Short sets of diagnostic questions to elicit students’ prior knowledge and misconceptions

“Diagnostic strategies allow for instruction that builds on individual student existing understanding while addressing their identified difficulties.”

*Uncovering Student Thinking in Mathematics Grades K-5* by Cheryl Rose Tobey and Leslie Minton
What is the Value of the Digits?

5.NBT.3 Read, write, and compare decimals to thousandths...

<table>
<thead>
<tr>
<th>Statement</th>
<th>Explanation (why circled or not circled)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) There is a 3 in the ones place.</td>
<td></td>
</tr>
<tr>
<td>B) There is a 2 in the ones place.</td>
<td></td>
</tr>
<tr>
<td>C) There are 21.3 tenths.</td>
<td></td>
</tr>
<tr>
<td>D) There are 13 tenths.</td>
<td></td>
</tr>
<tr>
<td>E) There is a 1 in the tenths place.</td>
<td></td>
</tr>
<tr>
<td>F) There is a 3 in the tenths place.</td>
<td></td>
</tr>
</tbody>
</table>
Quadrilaterals: Probe 21

Circle the name or names of the people you agree with. Explain your choices.

A square is always a rectangle.
A rectangle is sometimes a rhombus.
A rhombus is always a square.
A rectangle is sometimes a square.
A parallelogram is always a rhombus.
Connection to the CCSS-M

3.G.1 Understand that shapes in different categories may share attributes...

4.G.2 Classify two-dimensional shapes based on presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.

5.G.3 Understand that attributes belonging to a category of 2-D figures also belong to all subcategories of that category.

5.G.4 Classify 2-d figures in a hierarchy based on properties.
<table>
<thead>
<tr>
<th>Always, Sometimes, Never</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiples of 5 end in 5</td>
</tr>
<tr>
<td>Always □ Sometimes □ Never</td>
</tr>
<tr>
<td>Justify your answer</td>
</tr>
<tr>
<td>Multiples of 2 end in an odd number.</td>
</tr>
<tr>
<td>Always □ Sometimes □ Never</td>
</tr>
<tr>
<td>Justify your answer</td>
</tr>
<tr>
<td>If you add the digits together in a number made up of more than two digits it will be a multiple of 3.</td>
</tr>
<tr>
<td>Always □ Sometimes □ Never</td>
</tr>
<tr>
<td>Justify your answer</td>
</tr>
<tr>
<td>Multiples of 7 are odd numbers.</td>
</tr>
<tr>
<td>Always □ Sometimes □ Never</td>
</tr>
<tr>
<td>Justify your answer</td>
</tr>
<tr>
<td>Multiples of 10 end in .0.</td>
</tr>
<tr>
<td>Always □ Sometimes □ Never</td>
</tr>
<tr>
<td>Justify your answer</td>
</tr>
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</table>

From *Mathematics Formative Assessment* by Page Keeley and Cheryl Rose Tobey
Where do you see the SMP in Formative Assessment Probes?

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<table>
<thead>
<tr>
<th>Puzzle 5</th>
<th>Puzzle 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>You say my number when you start at 0 and count by 15’s.</td>
<td>You say my number when you start at 0 and count by 25’s.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Puzzle 5</td>
<td>Puzzle 5</td>
</tr>
<tr>
<td>My number is less than 200.</td>
<td>My number is even.</td>
</tr>
</tbody>
</table>
Number Puzzles and the SMP and CCSSM Content Standards

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Online Ten Minute Tasks

• MANGAHIGH – adaptive technology
  www.mangahigh.com

• SUMDOG
  www.sumdog.com

• NCTM Illuminations
  http://illuminations.nctm.org

• First in Math
  www.firstinmath.com
Fraction of the Week from K-5 Learning Resources

http://www.k-5mathteachingresources.com
Vo-Back-ulary

1. Using an index card on string, write a vocabulary word and put it in on a student’s back.

2. Students ask others a yes or no question and keep track of clues to figure out their word.

3. Or, have three students give the student with the word on their back clues. If the student can use those three clues to solve they win the card. Rotate players.
Mini Rich Tasks

- www.illustrativemathematics.org

- Super site for gaining a better understanding of the mathematics content by common core standard through tasks ..... 

- Sample task
1. Make sense of problems and persevere in solving them.

I am learning to:
- Make sense of problems
- Make a plan for solving problems
- Try different strategies, even when a problem is hard
- Solve a problem in more than one way
- Check whether a solution makes sense
- Find connections between mathematical ideas

Heather Canzaurlo, Parma City School District

2. Reason abstractly and quantitatively.

I am learning to:
- Represent problem situations with objects, pictures, numbers, words, and symbols
- Explain to others the meanings of objects, pictures, numbers, words, and symbols

Heather Canzaurlo, Parma City School District

3. Construct viable arguments and critique the reasoning of others.

I am learning to:
- Explain both what to do and why it works
- Make sense of others’ mathematical thinking

Heather Canzaurlo, Parma City School District

4. Model with mathematics.

I am learning to:
- Solve real-world problems using mathematics
- Use models such as graphs, drawings, tables, symbols, and diagrams to solve problems

Heather Canzaurlo, Parma City School District

https://ccssmresources.wikispaces.co
5. Use appropriate tools strategically.

I am learning to:
- Choose appropriate tools to solve problems
- Use mathematical tools correctly and efficiently
- Estimate an answer before using a tool

6. Attend to precision.

I am learning to:
- Clearly communicate my thinking to others
  ▪ Speak, Read, Write, and Listen mathematically
- Decide whether an estimate or exact answer is needed
- Be accurate when I count, measure, and compute

7. Look for and make use of structure.

I am learning to:
- Notice, continue, and create patterns
- Use patterns to solve problems

8. Look for and express regularity in repeated reasoning.

I am learning to:
- Use patterns to create and explain rules and shortcuts
- Use rules and properties of number to more easily solve problems
- Reflect on my thinking before, during, and after solving a problem
Silent Multiplication

Goals of Silent Multiplication:
Explore patterns in factors and products in order to develop understanding of multi-digit multiplication. Develop strategies to solve complex problems by using what is already known about simpler, related problems. Gain skill with computation, number sense, and problem solving.

from Lessons for Extending Multiplication by Marylin Burns and Mary Ann Wickett
How it works...

• A star is drawn on the board, indicating the beginning of the activity and silence by everyone, including the teacher.

• A problem is written on the board and students signal with a “thumbs up” if they know the answer.

• A student is handed the “chalk” to solve the problem.

• Students signal agreement with a “thumbs up”, disagreement with “thumbs down,” or confusion with “thumbs sideways.”

• A discussion follows a series of Silent Multiplication problems. What patterns did the students notice?
## Silent Multiplication Series

### Doubling one factor
- \( 4 \times 2 = \)
- \( 8 \times 2 = \)
- \( 16 \times 2 = \)
- \( 32 \times 2 = \)
- \( 3 \times 4 = \)
- \( 4 \times 6 = \)
- \( 4 \times 12 = \)
- \( 4 \times 24 = \)
- \( 8 \times 24 = \)

### Multiplying by 10
- \( 4 \times 1 = \)
- \( 4 \times 10 = \)
- \( 40 \times 10 = \)
- \( 41 \times 10 = \)
- \( 45 \times 10 = \)
- \( 451 \times 10 = \)
Fraction Multiplication / Division

\[
8 \div 2 = \quad 8 \times \frac{1}{2} = \\
8 \div \frac{1}{2} = \quad 8 \times 2 =
\]

From Marylin Burns *Lessons for Multiplying and Dividing Fractions*
Patterns in Division and Multiplication

\[
\begin{align*}
8 \div 2 &= 4 & 8 \times \frac{1}{2} &= 4 \\
7 \div 2 &= 3 \frac{1}{2} & 7 \times \frac{1}{2} &= 3 \frac{1}{2} \\
6 \div 2 &= 3 & 6 \times \frac{1}{2} &= 3 \\
5 \div 2 &= 2 \frac{1}{2} & 5 \times \frac{1}{2} &= 2 \frac{1}{2} \\
4 \div 2 &= 2 & 4 \times \frac{1}{2} &= 2 \\
3 \div 2 &= 1 \frac{1}{2} & 3 \times \frac{1}{2} &= 1 \frac{1}{2} \\
2 \div 2 &= 1 & 2 \times \frac{1}{2} &= 1 \\
1 \div 2 &= \frac{1}{2} & 1 \times \frac{1}{2} &= \frac{1}{2}
\end{align*}
\]
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students. NCTM’s Institutes, an official professional development offering of the National Council of Teachers of Mathematics, supports the improvement of pre-K-6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students. It is a forum for the exchange of mathematics ideas, activities, and pedagogical strategies, and for sharing and interpreting research. The Institutes presented by the Council present a variety of viewpoints. The views expressed or implied in the Institutes, unless otherwise noted, should not be interpreted as official positions of the Council.
Thank you for coming!!

Email Annemarie at newhouse@sel.k12.oh.us for any requests, questions, or comments