Gus and Ike Running Task*

Gus and Ike both went running this week.

Adding the distance Gus ran this week and the distance Ike ran this week gives a total of 48 miles.

Gus ran three times as far as Ike this week.

How many miles did Ike run this week?

Diagrams as Problem-Solving and Communication Tools for English Learners

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Today we will explore...

• How diagrams support you.
• How diagrams support English learners.
• Supporting language access and production.

All in the context of thinking about ratios.
Mathematical Visual Representations

What is a mathematical visual representation?
How is it used?
Mathematical Visual Representations

Includes:

• *Drawing* (e.g., in enhancing figures in geometry tasks)

• *Diagramming* (e.g., in thinking about word problems and other quantitative tasks)
  – including tape diagrams and number lines
Why Visual Representations (VRs)?

- Competent mathematical thinkers use VRs flexibly in problem solving (Stylianou, 2002; Stylianou & Silver, 2004)

- Learning to use VRs can enhance mathematical problem solving

- Students should learn to use a variety of VRs appropriate to each task (2012 IES Practice Guide on Problem Solving in Grades 4-8, Woodward et al., 2012)

- VRs reinforce students’ conceptual understanding of rational number (Gersten et al., 2009; Siegler et al., 2010)
Why VRs for ELs?

- VRs are recommended for use with ELs (Baker et al., 2014), particularly because they can serve as an intermediate step between the textual and symbolic phases of solving a word problem, revealing to the solver the mathematical structure of the problem (Ng & Lee, 2009).

- VRs support both engagement with the mathematics of a task and language development.
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Turn to a different partner:

List three ways a diagram supported you with Gus and Ike (or could support someone).
Purposes of Diagramming

Using diagrams as a thinking tool

• Start work on a task by representing important information.
• Work on answering (or generating) mathematical questions.
• Share mathematical thinking.

Using diagrams to support productive and receptive language

• Produce mathematical language.
• Develop understanding of language involved in a task.
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Framework shown on the following slides from:

### The Five Dimensions of Mathematically Powerful Classrooms

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The Mathematics</strong></td>
<td>How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections?</td>
</tr>
<tr>
<td><strong>Cognitive Demand</strong></td>
<td>What opportunities do students have to make their own sense of mathematical ideas? How can we create more opportunities?</td>
</tr>
<tr>
<td><strong>Access to Mathematical Content</strong></td>
<td>Who does and does not participate in the mathematical work in the class, and how? How can we create opportunities for each student to participate meaningfully?</td>
</tr>
<tr>
<td><strong>Agency, Authority, and Identity</strong></td>
<td>What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?</td>
</tr>
<tr>
<td><strong>Formative Assessment</strong></td>
<td>What do we know about each students current mathematical thinking? How can we build on it?</td>
</tr>
</tbody>
</table>
### Observe as if you were a student

| **The Mathematics**          | • What’s the big mathematical idea in this lesson?  
|                             | • How does it connect to what I already know? |
| **Cognitive Demand**         | • How long am I given to think, and to make sense of things?  
|                             | • What happens when I get stuck?  
|                             | • Am I invited to explain things, or just give answers? |
| **Access to Mathematical Content** | • Do I get to participate in meaningful math learning?  
|                             | • Can I hide or be ignored? |
| **Agency, Authority, and Identity** | • Do I get to explain, to present my ideas? Are they built on?  
|                             | • Am I recognized as being capable and able to contribute in meaningful ways? |
| **Formative Assessment**      | • Do classroom discussions include my thinking?  
|                             | • Does instruction respond to my thinking and help me think more deeply? |
### Observe as a teacher

| The Mathematics | **Are students learning important mathematics?**  
<table>
<thead>
<tr>
<th></th>
<th><strong>Are opportunities made for meaningful connections?</strong></th>
</tr>
</thead>
</table>
| Cognitive Demand| **How long do students spend on each prompt?**  
|                 | **Do they engage in productive struggle?**  
|                 | **Do teacher questions invite explanations or answers?** |
| Access to Mathematical Content | **Are there multiple ways to get involved productively?**  
|                             | **Does the teacher ask a range of students to respond?** |
| Agency, Authority, and Identity | **Who explains most: the teacher or the students?**  
|                                 | **Do the students give extended explanations?** |
| Formative Assessment | **Does the teacher follow up on student responses?**  
|                     | **Does the teacher vary the lesson in the light of student responses?** |
Agency, Authority, and Identity

How did this work with visual representations support this student’s agency, authority, and identity?

Share evidence you saw of the student’s mathematical thinking or communication.
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Mathematical Visual Representations

Exploring the Mystery Ribbons Task:

Supporting Access and Production for students who are English learners.
Mathematical Visual Representations

Exploring the Mystery Ribbons Task:

Supporting Access and Production for students who are English learners.
Who are students who are English learners?
Mathematical Visual Representations

Exploring the Mystery Ribbons Task:

Supporting **Access and Production** for students who are English learners.
Access and Production

Language access

• Receptive language
• Listening and reading
• Access to opportunities to engage with the math

Language production

• Productive language
• Speaking and writing
• Participate in the mathematical discourse
Mystery Ribbons Task

Sam cut an orange ribbon into two pieces.

After Sam cut the ribbon, the longer piece of ribbon is 24 inches in length.

The ratio of the lengths of the two pieces is 1:3.
Sam cut an orange ribbon into two pieces.

After Sam cut the ribbon, the longer piece of ribbon is 24 inches in length.

The ratio of the lengths of the two pieces is 1:3.

Jo started a diagram to find the length, in inches, of the whole ribbon before Sam cut it. Complete Jo’s diagram to find the length of the ribbon before Sam cut it.
Sam cut an orange ribbon into two pieces. After Sam cut the ribbon, the longer piece of ribbon is 24 inches in length. The ratio of the lengths of the two pieces is 1:3.

Jo started a diagram to find the length, in inches, of the whole ribbon before Sam cut it. Complete Jo’s diagram to find the length of the ribbon before Sam cut it.

Share with a partner:

Jo’s diagram shows ...

Jo can use this diagram to find out... by...
Sam cut an orange ribbon into two pieces. After Sam cut the ribbon, the longer piece of ribbon is 24 inches in length. The ratio of the lengths of the two pieces is 1:3.

Jo started a diagram to find the length, in inches, of the whole ribbon before Sam cut it. Complete Jo’s diagram to find the length of the ribbon before Sam cut it.

Full Group:
What words or phrases were useful or confusing when talking about the diagram and problem?
Mystery Ribbons Task – #2

Sam cut a green ribbon into two pieces in the ratio of 2:3. After the cut, the longer piece of ribbon is 30 inches.

Create a diagram like Jo’s to find the length, in inches, of the ribbon before Sam cut it:

Create a different diagram that you could use to solve this problem.

1) Individual work
2) Work in pairs
3) Share in full group

**Use words or phrases from our word bank to share your diagram**

This diagram shows the ratio 2:3 ...
Agency, Authority, and Identity

What opportunities did you have during Mystery Ribbons to see yourself and others as powerful doers of mathematics? What supported that?

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Language Access & Production

- Three reads
- Acting out and realia
- Clarifying vocabulary
- Differentiated teacher questions
- Frayer model
- Teacher revoicing
- Sentence starters and frames
- Co-constructed word bank
- Pairs work

Plan with a partner how to use one of today’s tasks with students with one language strategy integrated.
Planning to Support Students

- Choose one student to focus on

- How will you support that student’s opportunities for access and production of mathematical thinking and communication for today’s task?

- Integrate at least one strategy.
Questions?

To reach me:
Johannah: jnikula@edc.org

A couple resources:
Mathematical Thinking and Communication: Access for English Learners (book available through Heinemann)

Supporting English Learners: Lessons from Research (article available through NCTM Mathematics Teaching in the Middle School journal)

Mathandlanguage.edc.org (a work in progress)
Implications for Your Work

- What ideas or questions do you have about how visual representations can support *mathematical thinking*?

- What ideas or questions do you have about supporting *mathematical communication (access and production)* for ELs?
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