Capturing Quantities

An Instructional Routine to Foster Math Practice 2

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#NCTMINST
1. Solve both tasks.

2. Anticipate how students might solve the tasks.

A school bought some math books and 4 times as many science books. The cost of a math book was $12 while a science book cost $8. Altogether the school spent $528. How many science books did the school buy?

Juan had 3 red marbles. He found 12 more marbles that were blue. How many more blue marbles than red marbles did Juan have?
Deepen your understanding of reasoning abstractly and quantitatively (MP2). You will know your learning is on track if you can:

– Describe how you and/or a colleague is reasoning abstractly and quantitatively.
– Articulate abstract and quantitative reasoning that will be helpful in future problem solving.
Learn how the *Capturing Quantities* instructional routine embodies NCTM Mathematics Teaching Practices. You will know your learning is on track if you can identify when and how the routine:

- Establishes a Math Goal to Focus Learning
- Uses and Connects Mathematical Representations
- Supports Meaningful Mathematical Discourse
- Poses Purposeful Questions
Opening: Intro, Goals, Framing, and Do Now

Unpack MP2 Reason Abstractly and Quantitatively

Do Math: *Capturing Quantities* Routine

Reflect: NCTM Teaching Principles

Analyze Student Work
Juan had 3 red marbles. He found 12 more marbles that were blue. How many more blue marbles than red marbles did Juan have?

\[3 + 12 = 15 \text{ marbles}\]
A school bought some math books and 4 times as many science books. The cost of a math book was $12 while a science book cost $8. Altogether the school spent $528. How many science books did the school buy?

\[4 \times 8 = 32\]

\[
\begin{array}{c}
32 \mid 528 \\
\underline{32} \\
\hline
16 \\
\end{array}
\]

16 Science Books
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. **Reason abstractly and quantitatively.**

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
MP1 Make Sense of Problems and Persevere in Solving Them

- MP2
  - MP3
  - MP4
  - MP5
  - Quantities & Relationships

- MP7
  - MP3
  - MP4
  - MP5
  - MP6
  - Structure

- MP8
  - MP3
  - MP4
  - MP5
  - MP6
  - Repetition

Routines for Reasoning
Kelemanik, Lucenta, Janssen Creighton
MP2 Reason Abstractly and Quantitatively

Attend to…

Quantities and Relationships

Ask yourself…

• What can I count or measure in this problem situation?
• How do the quantities relate to each other?
• How can I represent this problem?
• What does this (expression, variable, number, shaded region, etc.) represent in the problem context?
Entering the task from a Reasoning Abstractly and Quantitatively Avenue

Paid attention to

• Number of red marbles Juan had
• Number of blue marbles Juan found
• Relationship between the number of red marbles and the number of blue marbles

Asked yourself...

• What can I count in this situation?
• How are the number of red marbles related to the number of blue marbles?
• How can I represent Juan’s marbles so that I can see a relationships between the number of blue and red marbles?
Put into Action...

• Identify quantities explicitly mentioned in the problem situation
• Surface implied quantities
• Abstract problem situations
• Use representations to see quantities and relationships
• Recall and consider referents
Quantitative Reasoning Shifts

The numbers and key words in a problem statement

The quantities and relationships those numbers describe

Routines for Reasoning
Kelemanik, Lucenta, Janssen Creighton
How do we teach students to “reason abstractly and quantitatively?”
Instructional routines designed to develop the habit of attending to quantities and relationships
Capturing Quantities

An instructional routine that develops the quantitative reasoning avenue of thinking
Diagram a Problem Situation
Think like a Mathematician!
To build the habit of looking for quantities and relationships in problem situations
How?

1. Identify Quantities & Relationships
2. Create Diagrams
3. Discuss Diagrams
4. Reflect on Learning
Penny had a bag of candies. She gave $\frac{1}{3}$ of her candies to Rebecca. Then Penny gave $\frac{1}{4}$ of the candies she had left to John. After giving candies to Rebecca and John, Penny had 24 candies left in her bag.
Penny had a bag of candies. She gave \( \frac{1}{3} \) of her candies to Rebecca. Then Penny gave \( \frac{1}{4} \) of the candies she had left to John. After giving candies to Rebecca and John, Penny had 24 candies left in her bag.

A quantity I found was the number of...

A relationship I found was ...
Penny had a bag of candies. She gave \( \frac{1}{3} \) of her candies to Rebecca. Then Penny gave \( \frac{1}{4} \) of the candies she had left to John. After giving candies to Rebecca and John, Penny had 24 candies left in her bag.

How can I show the relationships between the quantities?
1. Share how you started to represent quantities and relationships
2. Together create a diagram

How did you represent...?

I showed... by...
Where are the quantities in the diagram?
Where are the relationships in the diagram?

The relationship is... So I looked for...

I noticed... So I looked for two quantities that...
• When looking for quantities in a word problem, I learned to __________.

• When analyzing a diagram, I learned to pay attention to __________ because __________.
Penny had a bag of candies. She gave \( \frac{1}{3} \) of her candies to Rebecca. Then Penny gave \( \frac{1}{4} \) of the candies she had left to John. After giving candies to Rebecca and John, Penny had 24 candies left in her bag.
CAPTURING QUANTITIES ROUTINE

1. Launch the Routine:
   Introduce thinking goal, review routine’s steps

2. Identify Quantities & Relationships
   - Individual Think Time
   - Pairs
   - Share: Discuss & Annotate

3. Create Diagrams
   - Individual Think Time
   - Pairs

4. Discuss Diagrams
   - Individual Think Time
   - Pairs
   - Share: Discuss & Annotate

5. Reflect on Your Thinking
   - Individual Write Time
   - Pairs
   - Share & Record

Thinking Goal: Building an Avenue of Thinking

Routines for Reasoning
Kelemanik, Lucenta, Janssen Creighton
1. Identify Q & R? (T-P-S)
2. Create Diagram (T-P)
3. Discuss Diagrams (Full group)
4. Reflect on Learning (W-P-S)
Reflecting on the *Capturing Quantities* Routine and the NCTM Mathematics Teaching Practices

AT YOUR TABLES discuss how the *Capturing Quantities* routine:

- Establishes a Math Goal to Focus Learning
- Poses Purposeful Questions
- Supports Meaningful Mathematical Discourse
- Uses and Connects Mathematical Representations
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Think like a Mathematician!
To build the habit of looking for quantities and relationships in problem situations
## Capturing Quantities

### Mathematical Goals to Focus Learning

<table>
<thead>
<tr>
<th>Sample Thinking Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can identify quantities and relationships in problem situations.</td>
</tr>
<tr>
<td>Look for quantities and relationships that are implied in problem situations.</td>
</tr>
<tr>
<td>Learn to ask yourself, <em>What can I count?</em> and <em>What can I measure?</em> to identify quantities in a problem.</td>
</tr>
<tr>
<td>I can create a diagram that shows how quantities are related.</td>
</tr>
<tr>
<td>I can use a diagram to surface hidden relationships in the problem.</td>
</tr>
<tr>
<td>I can describe relationships between quantities in multiple ways.</td>
</tr>
</tbody>
</table>

*Kelemanik, Lucenta, Janssen Creighto*n

*Heinemann*
• When looking for quantities in a word problem, I learned to ______________.

• When analyzing a diagram, I learned to pay attention to __________ because __________.
### Capturing Quantities
Mathematical Goals to Focus Learning

<table>
<thead>
<tr>
<th>When looking for quantities in a word problem, I learned to look for _____</th>
<th>When capturing quantities, I look for _____</th>
</tr>
</thead>
<tbody>
<tr>
<td>When looking for relationships in a word problem, I learned to look for _____</td>
<td>When capturing quantities, I ask myself _____</td>
</tr>
<tr>
<td>When finding quantities/relationships in a diagram, I learned to look for _____</td>
<td>When capturing quantities, next time I will _____</td>
</tr>
<tr>
<td>When identifying quantities, I learned to ask myself _____</td>
<td>When capturing quantities, I learned _____</td>
</tr>
<tr>
<td>When identifying relationships, I learned to ask myself _____</td>
<td></td>
</tr>
<tr>
<td>When analyzing a diagram, I learned to pay attention to _____ because _____</td>
<td></td>
</tr>
<tr>
<td>The secret to identifying quantities/relationships in a word problem is _____</td>
<td></td>
</tr>
</tbody>
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**Reflecting on Learning Prompts**
AT YOUR TABLES discuss how the *Capturing Quantities* routine:

- Establishes a Math Goal to Focus Learning
- **Poses Purposeful Questions**
- Supports Meaningful Mathematical Discourse
- Uses and Connects Mathematical Representations
Penny had a bag of candies. She gave $\frac{1}{3}$ of her candies to Rebecca. Then Penny gave $\frac{1}{4}$ of the candies she had left to John. After giving candies to Rebecca and John, Penny had 24 candies left in her bag.
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Capturing Quantities
Pose Purposeful “ask yourself” Questions

- What can I count or measure?
- What quantity or relationship does this number describe?
- How do the quantities relate to each other?
- How much bigger/smaller (how many times bigger/smaller) is one quantity than another?
- How can I represent this situation so that I can see the quantities and relationships?
- How can I show how much bigger/smaller (how many times bigger/smaller) one quantity is in my diagram?
- What quantities or relationships do I see in the diagram?
- Is there a “hidden” quantity or relationship that I can now see in the representation?
- What does this (expression, variable, number, shaded region, etc.) represent in the problem context?
Reflecting on the *Capturing Quantities* Routine and the NCTM Mathematics Teaching Practices

AT YOUR TABLES discuss how the *Capturing Quantities* routine:

- Establishes a Math Goal to Focus Learning
- Poses Purposeful Questions
- **Supports Meaningful Mathematical Discourse**
- Uses and Connects Mathematical Representations
Capturing Quantities Supports Meaningful Math Discourse

<table>
<thead>
<tr>
<th>All Purpose</th>
<th>For Use During Partner Work</th>
<th>For Use During Full-Group Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capturing Quantities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The number of _____</td>
<td>• How did you represent _____?</td>
<td>• They noticed _____.</td>
</tr>
<tr>
<td>• The amount of _____</td>
<td>• I showed _____ by _____.</td>
<td>• A relationship that I see in their diagram is _____.</td>
</tr>
<tr>
<td>• Quantity A is _____ (relationship) _____ Quantity B</td>
<td>• The relationship is _____ so I looked for _____.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• I noticed _____ so I looked for two quantities that _____.</td>
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Reflecting on the *Capturing Quantities* Routine and the NCTM Mathematics Teaching Practices

AT YOUR TABLES discuss how the *Capturing Quantities* routine:

- Establishes a Math Goal to Focus Learning
- Poses Purposeful Questions
- Supports Meaningful Mathematical Discourse
- **Uses and Connects Mathematical Representations**
Student Work Analysis

Look for evidence that students are attending to quantities and relationships between quantities.
**Quantitative Reasoning “Look-Fors”**

<table>
<thead>
<tr>
<th>Your students know that:</th>
<th>Your students regularly take the following actions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A quantity has three parts—a value, unit, and sign.</td>
<td>• Say “the number of ____” and “the amount of ____” when talking about quantities.</td>
</tr>
<tr>
<td>• The values of some quantities are given in a problem and the value of at least one quantity is not, and it’s the quantities with the “unknown” values that are typically the one(s) the problem is asking you to find.</td>
<td>• Ask themselves (and answer!) the question, <em>What can I count or measure?</em></td>
</tr>
<tr>
<td>• You use the relationships between the quantities to solve the problem.</td>
<td>• Ask themselves (and answer!) <em>How are these quantities related?</em></td>
</tr>
<tr>
<td>• Representing quantities and the relationships between them visually can help you make sense of the problem and often see a solution path.</td>
<td>• Look for implied or “hidden” quantities in problem statements and visuals.</td>
</tr>
<tr>
<td></td>
<td>• Create diagrams (and other visuals) that capture the quantities and relationships in a problem.</td>
</tr>
<tr>
<td></td>
<td>• Describe quantities and relationships in more than one way.</td>
</tr>
</tbody>
</table>
**Quantities.** In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours .... They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.
Routines for Reasoning: Fostering the Mathematical Practices in All Students

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