Thinking through a Lesson Protocol (TTLP) Planning Template

Learning Goals What understandings will students take away from this lesson?	**Evidence** *What will students say, do, or produce that will give evidence of their understandings*?
Task *What is the main activity that students will be working on in this lesson?*	**Instructional Support—Tools, Resources, Materials** *What tools or resources will be available to give students entry to—and help them reason through—the activity?*
Prior Knowledge *What prior knowledge and experience will students draw on in their work on this task?* **Essential Questions** *What are the essential questions that I want students to be able to answer over the course of the lesson?*	**Task Launch** *How will you introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?*

Smith, Margaret, Victoria Bill, and Elizabeth Hughes. "Thinking through a Lesson Protocol: A Key for Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School* 14 no. 3 (2008): 132–38.

Thinking through a Lesson Protocol (TTLP) Planning Template

<table>
<tr><td colspan="2">Anticipated Likely Solutions and Instructional Supports

What are the various ways that students might complete the activity? Be sure to include correct, incorrect and incomplete solutions.

What questions might you ask students that will support their exploration of the activity and bridge between what they did and what you want them to learn? These questions should assess what a student currently knows and advance her or him toward the goals of the lesson. Be sure to consider questions that you will ask students who cannot begin as well as students who finish quickly.</td></tr>
<tr><td>Correct and Incomplete Solutions</td><td>Instruction Supports (Assessing and Advancing questions)</td></tr>
<tr><td>Possible Errors and Misconceptions</td><td>Instruction Supports (Assessing and Advancing questions)</td></tr>
<tr><td colspan="2" align="center">Sharing and Discussing the Task</td></tr>
<tr><td>Selecting and Sequencing
Which solutions do you want students to share during the lesson?
In what order? Why?</td><td>Connecting Responses
What specific questions will you ask so that students—
– make sense of the mathematical ideas that you want them to learn
– make connections among the different strategies and solutions that are presented?</td></tr>
<tr><td colspan="2">Homework/Assessment</td></tr>
</table>

Smith, Margaret, Victoria Bill, and Elizabeth Hughes. "Thinking through a Lesson Protocol: A Key for Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School* 14 no. 3 (2008): 132–38.

Monitoring Tool

(green- complete prior to lesson; yellow-complete during the lesson)

Anticipated Solutions	Instructional Support		Who/What	Order
	Assessing Questions	Advancing Questions		
Unanticipated Solutions				

The Caterpillar Task

On Monday, the hungry caterpillar ate through one apple, but he was still hungry. On Tuesday he ate through two pears, but he was still hungry. On Wednesday he ate through three plums. On Thursday he ate through four strawberries. On Friday he ate through five oranges. How many pieces of fruit did the hungry caterpillar eat during the week?

From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

Multiplication Check-In

1. Find the answer to 4×7.
2. Explain how you figured out the answer.
3. Draw a picture that shows what 4×7 means.
4. Write a story problem for 4×7.

From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

The Swings Task

At recess, 13 children were playing on the swings. Then some more children joined them. Now there are 21 children on the swings. How many children joined the first group?

Make sure you show your thinking, write an equation, and record your answer.

From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

Walking from School Task

Five classmates walk home from school each day. Suzy walks $^1/_2$ mile to get home from school. Harold walks $^2/_3$ mile from school, Derrick walks $^3/_4$ mile, Maria walks $^2/_6$ mile, and Sheila walks $^4/_6$ mile from school.

Shelia says she walked the farthest because the numbers in her fractions are the biggest. Did Shelia walk the farthest?

Draw diagrams to show and use words to explain whether or not Shelia walked the farthest. (Note: You can use as many number lines or other visual models as you need.)

Adapted from Institute for Learning (2015b). Lesson guides and student workbooks available at ifl.pitt.edu

From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

The Odd and Even Task

- Is the sum of two even numbers, even or odd, and how do you know?
- Is the sum of two odd numbers, even or odd, and how do you know?
- Is the sum of an odd number and an even number, even or odd, and how do you know?

Describe your observations and justify your claims with several examples.

From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

The Pencil Task

1. Find the solution to the Pencil task, make a diagram, and write an equation.

   Mrs. Washington cleans her classroom on the last day of school and finds some pencils left over from the year. She knows there are 24 pencils in a full box. Mrs. Washington finds $^3/_4$ of a box in her desk. How many pencils does she find in her desk?

2. Now use what you know about the first box of pencils to help you think about and solve this problem.

   Mrs. Washington finds 1 $^1/_8$ boxes of pencils in the closet of her classroom. How many pencils does she find in the closet?

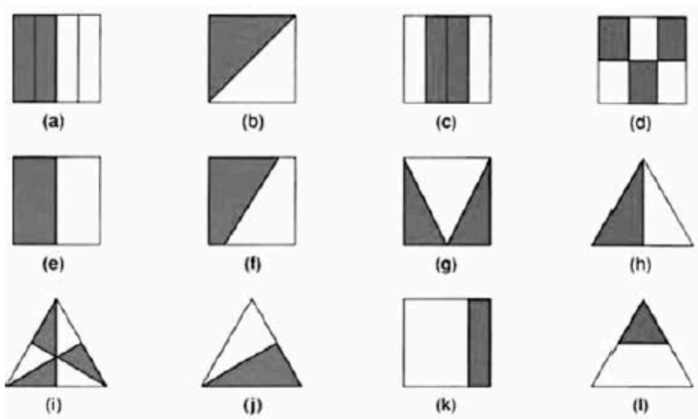
3. What patterns do you notice when solving the set of pencil problems?

Adapted from Institute for Learning (2015c). Lesson guides and student workbooks available at ifl.pitt.edu

Finding One-Half

Task

Identify all of the figures that have one-half of it shaded and be prepared to explain and justify how you know that one-half of the figure is or is not shaded. Write a description giving your reasons why each figure is or is not showing halves.



Adapted from: Watanabe, Tad. "Ben's Understanding of One-half." *Teaching Children Mathematics 2*, no. 8 (1996): 460–464.

From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

Seth's Birthday Cake Task

After his birthday party, Seth had one-fifth of his cake leftover. Eight of his friends spent the night. His friends plan to eat the remaining cake as a snack later that night, and they each expect to receive an equal share of the remaining cake. They want to know what fractional part of the original cake will each friend get to eat for a snack.

1. What fractional part of the cake did each friend eat for a snack? Draw and clearly label a diagram that shows how much of the cake is eaten by one person for the snack. Use words to explain your reasoning and decisions for how you drew the diagram.
2. Seth claims that to find the answer you can use the expression $8 \div {}^1\!/_5$. His friend Frankie says that the expression would be ${}^1\!/_5 \div 8$. Who is correct? Why?

Adapted from Institute for Learning (2015a). Lesson guides and student workbooks available at ifl.pitt.edu

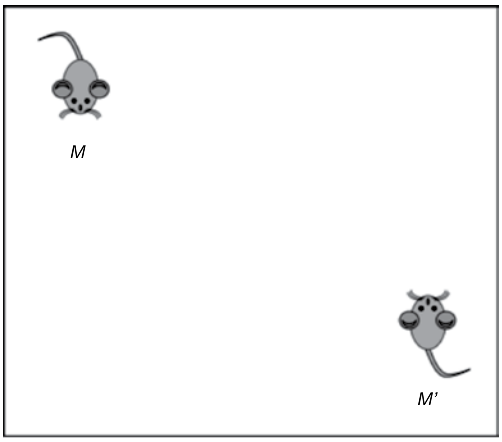
From Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.

Task 4: Robot Mouse

Below is a diagram of a robot mouse in the original position M and the location of its final image M′. Assume that the robot is flat so that you can reflect it. Create different sequences of motions (translations, rotations, and reflections) that will move the robot mouse from position M to M′ on the field.

To get started:

- Make copies of the robot mouse on at least four different transparencies.
- Write sequences of steps to move the robot mouse from position *M* to position *M′*. Keep in mind that:
  - — each step is a single transformation—either a reflection, translation, or rotation;
  - — you will need to draw and label the vector, line of reflection, or center and angle of rotation for each step in your diagram; and
  - — you will need to tape down a copy of the robot mouse to show the image created by each step.



1. Describe and show a sequence of *at least three* steps that will move the robot mouse from position *M* to *M′*.
2. Describe and show a sequence of *exactly two* steps that will move the robot mouse from position *M* to *M′*.
3. Describe and show a sequence of steps that will move the robot mouse from position *M* to *M′* that includes *at least one* reflection.

Make a conjecture:

A. Following a sequence of translations, reflections, and/or rotations, what will an original figure and its image *always* have in common?

B. Explain why your conjecture makes sense.

Institute for Learning. *Eighth Grade Lesson Set: Understanding Congruence and Similarity*. Pittsburgh, Pa.: University of Pittsburgh, 2015.

The Mixing Juice Task

Arvind and Mariah attend summer camp. Everyone at the camp helps with cooking and cleanup at meal times. One morning, Arvind and Mariah are in charge of making orange juice for all the campers. They plan to make the juice by mixing water and frozen orange juice concentrate. To find the mix that tastes best, they decide to test some recipes.

Mix A	**Mix B**
2 cups concentrate **3 cups cold water**	**1 cup concentrate** **4 cups cold water**
Mix C	**Mix D**
4 cups concentrate **8 cups cold water**	**3 cups concentrate** **5 cups cold water**

1. Which recipe will make juice that is the most "orangey"? Explain.
2. Which recipe will make juice that is the least "orangey"? Explain.

Taken from Lappan et al. (1997).

Lappan, Glenda, James T. Fey, J., William M. Fitzgerald, Susan N. Friel, and Elizabeth D. Phillips. *Connected Mathematics Project: Comparing and Scaling*. Upper Saddle River, N.J.: Pearson (1997).

The Building a Pizza Task

You and your friends are going to buy pizza from Domino's. From previous orders, you know that a medium pizza with two toppings costs $14.00 and that a medium pizza with five toppings costs $20.00.

a) If Domino's charges the same amount for each topping added to a plain cheese pizza, what is the cost per topping?

b) If you wanted to order a medium cheese pizza with no additional toppings, how much would you expect to pay?

c) Write a general rule that you can use to determine the price of any medium Domino's pizza.

For each part of the task, be sure to explain how you got your answer and why it makes sense.

Adapted from Mathalicious. http://www.mathalicious.com/lessons/domino-effect

Mathalicious. "Domino Effect." http://www.mathalicious.com/lessons/domino-effect (accessed December 12, 2016).

Supreme Court Handshake

When the nine justices of the Supreme Court meet each day, each justice shakes the hand of every other justice, to show harmony of aims, if not views.

1. If each justice shakes hands exactly once with each of the other justices, how many handshakes take place?
2. How can you determine the number of handshakes for a group too large to model?

Illuminations. *Supreme Court Handshake*. http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Supreme-AS-Handshakes.pdf. National Council of Teachers of Mathematics, 2005.

Pick Your Cable Provider

In the past, cable television companies charged a flat rate for a cable television package. Recently, some cable companies are starting to offer packages where you pay a flat rate for access plus a fee per channel.

- TV Party charges a $40.00 flat rate plus $1.50 per channel.
- Cable Club charges a flat rate of $20.00 plus $3.00 per channel.

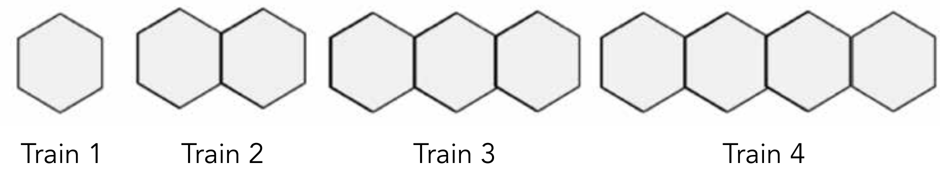
Your friend Felicia argues that because TV Party's flat rate is two times as much Cable Club and their price per channel is half of Cable Club's price per channel, the cost will be the same for any number of channels. Explain why you agree or disagree with Felicia.

Adapted from Institute for Learning (2013). Lesson guides and student workbooks are available at ifl.pitt.edu.

Institute for Learning. *Eighth Grade Lesson Set: Understanding and Solving Systems of Linear Equations.* Pittsburgh, Pa.: University of Pittsburgh, 2013.

The Hexagon Task

Trains 1, 2, 3, and 4 (shown below) are the first four trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



1. Compute the perimeter for each of the first four trains.
2. Draw the fifth train, and compute the perimeter of that train.
3. Determine the perimeter of the tenth train without constructing it.
4. Write a description that can be used to compute the perimeter of any train in the pattern.
5. Determine which train has a perimeter of 110.

Adapted from Foreman and Bennett (1995).

Foreman, Linda C., and Albert B. Bennett. *Visual Mathematics Course 1*. Salem, Ore.: Math Learning Center, 1995.

The Bag of Marbles Task

Ms. Rhee's math class was studying statistics. She brought in three bags containing red and blue marbles. The three bags were labeled as shown below:

Bag X

Bag Y

Bag Z

Ms. Rhee shook each bag. She asked the class, "If you close your eyes, reach into a bag, and remove one marble, which bag would give you the best change of picking a blue marble?"

Which bag would you choose?

Explain why that bag gives you the best choice of picking a blue marble. You may use the diagrams above in your explanation.

This task was adapted from the QUASAR Cognitive Assessment Instrument (Lane 1993).

Lane, Suzanne. "The Conceptual Framework for the Development of a Mathematics Performance Assessment Instrument." *Educational Measurement: Issues and Practice* 12, no. 2 (1993): 16–23.

From Smith, Margaret, Michael Steele and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics, 2017.

The Ducklings Task

The local nature club is carrying out a survey of the number of ducklings in each family of ducks in the lake. Here are the results of the survey:

4, 7, 6, 5, 8, 7, 5, 4, 10,
4, 9, 6, 5, 4, 4, 5, 9, 8, 4

How many ducks are in a typical family? Use tables, graphs, or arithmetic to justify your answer.

Adapted from http://www.insidemathematics.org/assets/common-core-math-tasks/ducklings.pdf (Noyce Foundation, 2012).

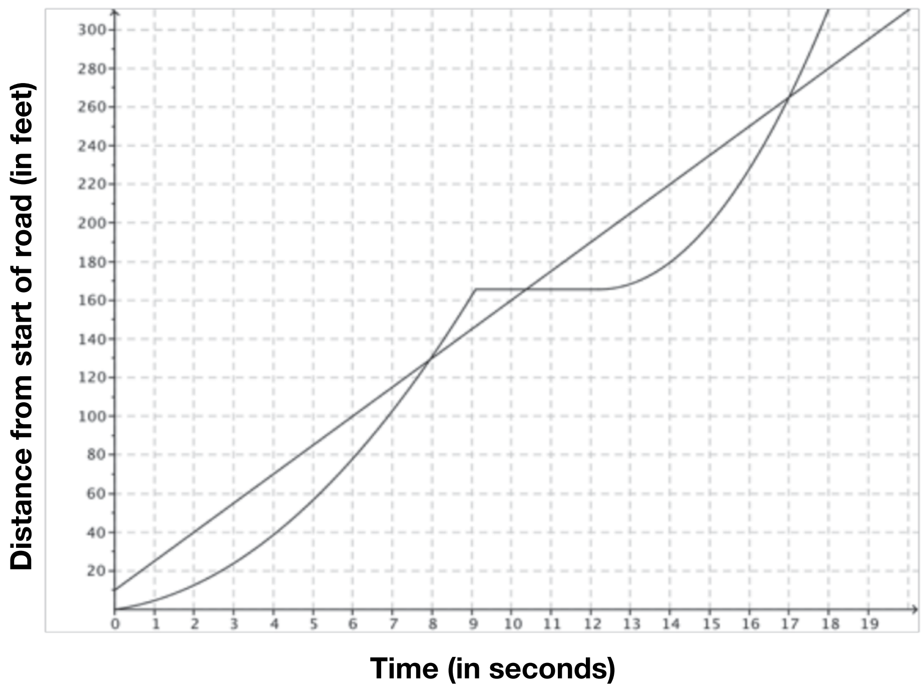
Noyce Foundation. Performance Assessment Task—Ducklings Grade 7. http://www.insidemathematics.org/assets/common-core-math-tasks/ducklings.pdf. 2012.

The Pay It Forward Task

In the movie *Pay It Forward*, a student, Trevor, comes up with an idea that he thinks could change the world. He decides to do a good deed for three people, and then each of the three people would do a good deed for three more people and so on. He believes that before long there would be good things happening to billions of people. At stage 1 of the process, Trevor completes three good deeds. How does the number of good deeds grow from stage to stage? How many good deeds would be completed at stage 5? Describe a function that would model the Pay It Forward process at *any* stage.

The Bicycle and Truck Task

A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.



1. Label the graphs appropriately with B(t) and K(t). Explain how you made your decision.
2. Describe the movement of the truck. Explain how you used the values of B(t) and K(t) to make decisions about your description.
3. Which vehicle was first to reach 300 feet from the start of the road? How can you use the domain and/or range to determine which vehicle was the first to reach 300 feet? Explain your reasoning in words.
4. Jack claims that the average rate of change for both the bicycle and the truck was the same in the first 17 seconds of travel. Explain why you agree or disagree with Jack.

From *Algebra 1 Creating and Interpreting Functions: A Set of Related Lessons* (2015). Pittsburgh, PA: Institute for Learning, University of Pittsburgh. Lesson guides and student workbooks are available at ifl.pitt.edu.

Institute for Learning. *Algebra 1 Creating and Interpreting Functions: A Set of Related Lessons.* Pittsburgh, Pa.: Institute for Learning, 2015.

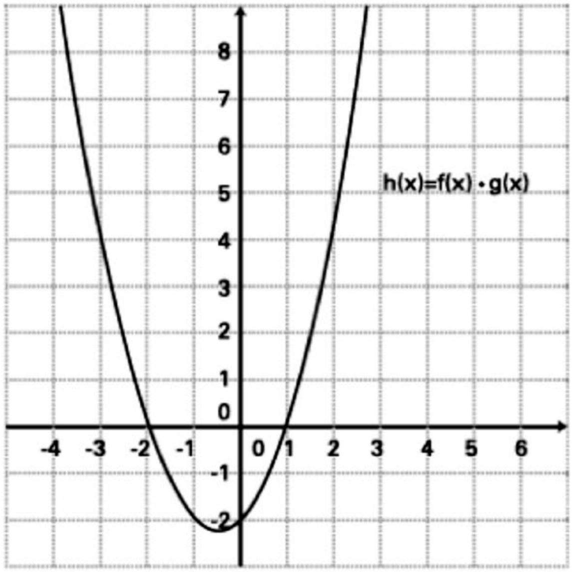
From Boston, Melissa, Fredrick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12.* Reston, VA: National Council of Teachers of Mathematics, 2017.

The Missing Function Task

Missing Function

If h(x) = f(x) · g(x), what can you determine about g(x) from the given table and graph? Explain your reasoning.

x	*f(x)*
-2	0
-1	1
0	2
1	3
2	4



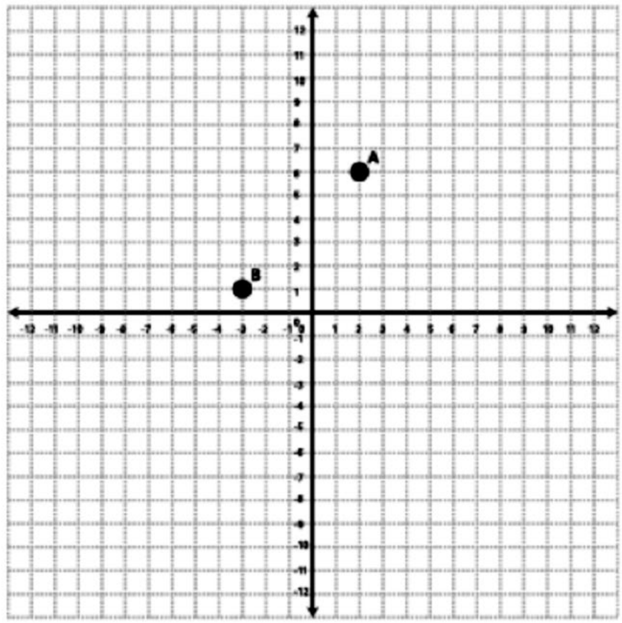
Taken from *Algebra 2 Set of Related Lessons: Building Polynomial Functions* (2013a). Pittsburgh, PA: Institute for Learning, University of Pittsburgh. Lesson guides and student workbooks are available at ifl.pitt.edu.

Institute for Learning, *University of Pittsburgh. Algebra 2 Set of Related Lessons: Building Polynomial Functions.* Pittsburgh, Pa.: Institute for Learning, 2013a.

The Playground Task

Building a New Playground

The City Planning Commission is considering building a new playground. They would like the playground to be equidistant from the two elementary schools, represented by points A and B in the coordinate grid that is shown.



PART A

1. Determine at least three possible locations for the park that are equidistant from points A and B. Explain how you know that all three possible locations are equidistant from the elementary schools.

2. Make a conjecture about the location of all points that are equidistant from A and B. Prove this conjecture.

Taken from *Geometry Set of Related Lessons: Investigating Coordinate Geometry and Its Use to Solve Problems* (2013b). Pittsburgh, PA: Institute for Learning, University of Pittsburgh. Lesson guides and student workbooks are available at ifl.pitt.edu.

Institute for Learning, University of Pittsburgh. *Geometry Set of Related Lessons: Building Polynomial Functions.* Pittsburgh, Pa.: Institute for Learning, 2013b.

The Student Homework Problem

Math teachers frequently claim that doing homework helps students get better grades in their math classes. To test this claim, a survey of high school math students was conducted from which the following results were obtained:

- 48% completed math homework regularly.
- 55% have a B average or better in math class.
- 40% do not complete math homework regularly AND have less than a B average in math class.

Do these data support the claim that students who complete math homework regularly are more likely to get a B or better in math class? Justify your answer using mathematics.

Oregon Department of Education (2016).

Oregon Department of Education ODE), "Mathematics Work Sample Assessment: Student Homework Problem," 2016. http://www.ode.state.or.us/wma/teachlearn/subjects/mathematics/assessment/worksamples/statistics_homeworkandgrades_task.pdf.

Four Situations

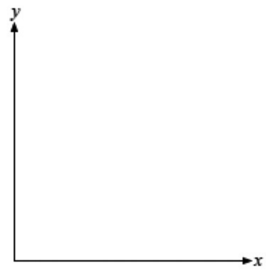
1. Sketch a graph to model each of the following situations.
   Think about the shape of the graph and whether it should be a continuous line or not.

A: Candle

Each hour a candle burns down the same amount.

x = the number of hours that have elapsed.

y = the height of the candle in inches.

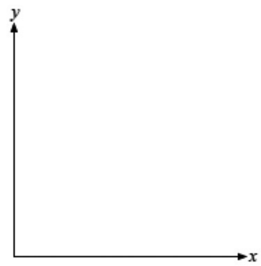


B: Letter

When sending a letter, you pay quite a lot for letters weighing up to an ounce. You then pay a smaller, fixed amount for each additional ounce (or part of an ounce.)

x = the weight of the letter in ounces.

y = the cost of sending the letter in cents.

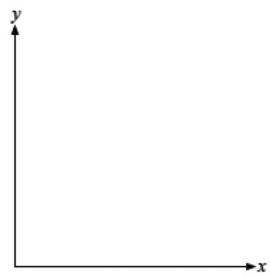


C: Bus

A group of people rent a bus for a day. The total cost of the bus is shared equally among the passengers.

x = the number of passengers.

y = the cost for each passenger in dollars.

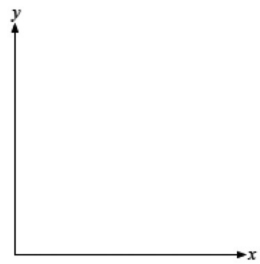


D: Car value

My car loses about half of its value each year.

x = the time that has elapsed in years.

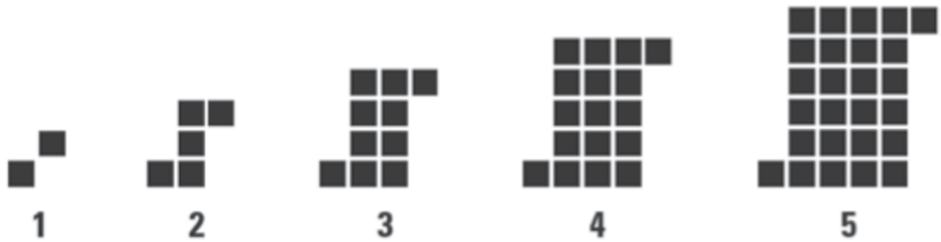
y = the value of my car in dollars.



Mathematics Assessment Resource Service. "Four Situations Task." University of Nottingham and UC Berkeley, 2015. http://map.mathshell.org/lessons.php?unit=9260&collection=8.

From Boston, Melissa, Fredrick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. Reston, VA: National Council of Teachers of Mathematics, 2017.

The S-pattern Task



1. What patterns do you notice in the set of figures?
2. Sketch the next two figures in the sequence.
3. Describe a figure in the sequence that is larger than the 20th figure without drawing it.
4. Determine an equation for the total number of tiles in any figure in the sequence. Explain your equation, and show how it relates to the visual diagram of the figures.
5. If you knew that a figure had 9,802 tiles in it, how could you determine the figure number? Explain.
6. Is there a linear relationship between the figure number and the total number of tiles? Why or why not?

Adapted from Foreman and Bennett (1995).

Foreman, Linda C., and Albert B. Bennett. *Visual Mathematics Course 2*. Salem, Oreg.: Math Learning Center, 1995.

The Amazing Amanda Task

Amanda claims to have an amazing talent. "Draw any polygon. Don't show it to me. Just tell me the number of sides it has, and I can tell you the sum of its interior angles."

Is Amanda's claim legitimate? Does she really have an amazing gift, or is it possible for anyone to do the same thing?

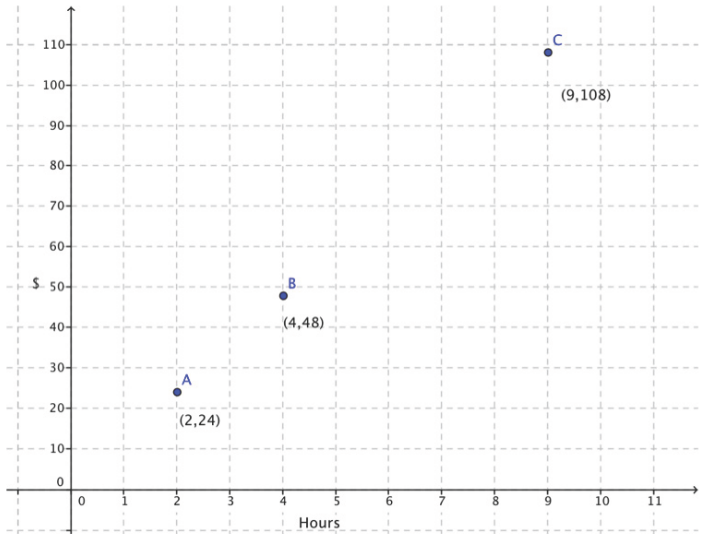
1. Working individually, investigate the sum of the interior angles of at least two polygons with 4, 5, 6, 7, or 8 sides. Use a straight-edge to draw several polygons. Make sure that some are irregular polygons. Subdivide each polygon into triangles so you can use what you already know about angle measures to determine the sum of the interior angles of your polygon. Organize and record your results.
2. As a group, combine your results on a single recording sheet and answer these questions:
   - How did group members subdivide their polygons into triangles? Did everyone do it in the same way? If different, how did that affect your calculations?
   - Does whether the polygon is regular or irregular affect the sum of the angle measures? Why or why not?
   - What patterns did you notice as you explored this problem?
   - What is the relationship between the number of sides of the polygon and the sum of the measures of the interior angles of the polygon? Express this relationship algebraically and explain how you know that your expression will work for *any* convex polygon.

Name____

TASK 2

Lifeguard

Benjamin's friend, Maleeka, is taking a job as a lifeguard at the city pool. She will make a constant rate per hour. The graph below represents Maleeka's potential earnings for three different numbers of hours of summer work.



1. Maleeka's short-term goal is to earn more than $80 for new games. Describe the number of hours that she must work in order to make enough money for the games. Show all work and represent your answer using words, inequality notation, and a number line.

2. Maleeka's long-term summer goal is to make more than $750. Determine the number of hours that she must work in order to meet this goal. Show all work and represent your answer using words, inequality notation, and a number line.