2017 NCTM Summer Interactive Institutes
Baltimore, MD
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SUPPORTING STUDENTS' PRODUCTIVE STRUGGLE

Peg Smith
University of Pittsburgh

EFFECTIVE MATHEMATICS TEACHING PRACTICES

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

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6. Build procedural fluency from conceptual understanding.
7. **Support productive struggle in learning mathematics.**
8. Elicit and use evidence of student thinking.

CONSIDER SOMETHING THAT YOU LEARNED TO DO, BUT WEREN'T VERY GOOD AT INITIALLY….

- The video I showed here can be found at

https://www.youtube.com/watch?v=7EZlEtZagCc

The first 34 seconds only.

TAKE THE CHILD DIVING INTO THE POOL…

IT WASN'T FROM LISTENING TO A LECTURE….

OR SITTING ON THE SIDELINES….

OR WATCHING AN EXPERT DIVER….

Greg Louganis

HOW DOES THE CHILD DIVING INTO THE POOL GET BETTER?

He will get better from opportunities to work hard while getting guidance from an experienced other

By engaging in productive struggle……

SO WHAT IS PRODUCTIVE STRUGGLE AND HOW CAN WE SUPPORT IT?

WHAT IS PRODUCTIVE STRUGGLE?

From Merriam-Webster:

Productive -- *adjective* pro·duc·tive \prə-ˈdək-tiv, prō-\
doing or achieving a lot; working hard and getting good results

Struggle - *intransitive verb* strug·gle \ˈstrə-gəl\
to try very hard to do, achieve, or deal with something that is difficult or that causes problems

Productive Struggle – working hard at something that is difficult to do, persevering when the going gets tough, and making progress.

SUPPORT PRODUCTIVE STRUGGLE IN LEARNING MATHEMATICS

Productive struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

PRODUCTIVE STRUGGLE

"…students expend effort to make sense of mathematics, to figure something out that is not immediately apparent…The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed (Hiebert et al., 1996)."

Hiebert, James, and Douglas A. Grouws. "The Effects of Classroom Mathematics Teaching on Students' Learning." In *Second Handbook of Research on Mathematics Teaching and Learning*, edited by Frank K. Lester, Jr., pp. 371–404. Charlotte, N.C.: Information Age; Reston, Va.: National Council of Teachers of Mathematics, 2007. [p. 387]

PRODUCTIVE STRUGGLE

"…productive struggle comprises the work that students do to make sense of a situation and determine a course of action when a solution strategy is not stated, implied, or immediately obvious. From an equity perspective, this implies that each and every student must have the opportunity to struggle with challenging mathematics and to receive support that encourages their persistence without removing the challenge."

Boston, Melissa D., Fredrick Dillon, Margaret S. Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12.* Reston, VA: National Council of Teachers of Mathematics, 2017. [p.208]

WHEN IS STRUGGLE PRODUCTIVE?

The struggle is productive if:

- The intended goals and the cognitive demand of the task are maintained;
- Students' thinking is supported by acknowledging effort and mathematical understanding; and
- Students are able to move forward in the task execution through student actions.

Warshauer, Hiroko Kawaguchi. "Productive Struggle in Middle School Mathematics Classrooms." *Journal of Mathematics Teacher Education* 17, no.4 (2015): 375-399. [p. 390]

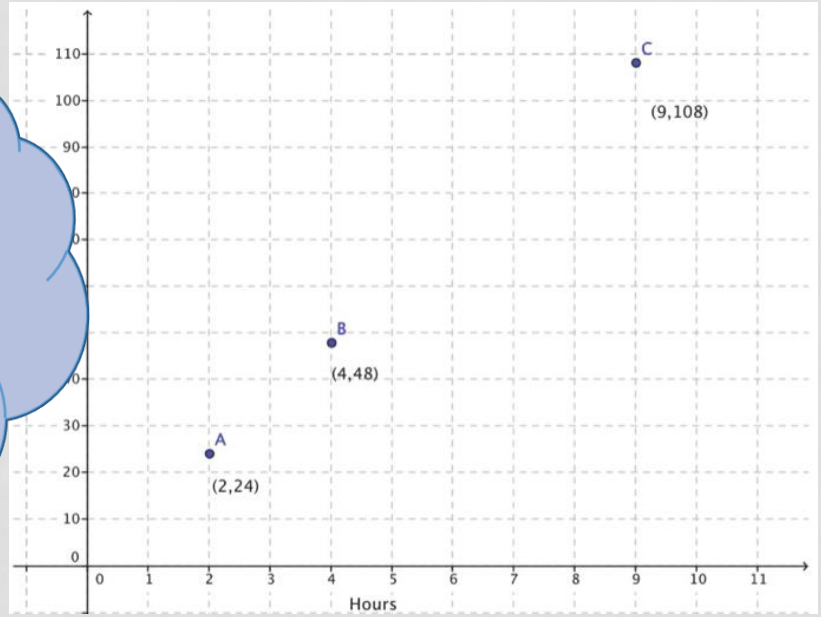
ACTIVITY 1

WHAT **IS** THE TEACHER DOING TO SUPPORT STUDENTS PRODUCTIVE STRUGGLE?

THE LIFEGUARD TASK

Benjamin's friend, Maleeka, is taking a job as a lifeguard at the city pool. She will be paid a constant rate per hour. The graph below represents Maleeka's potential earnings for three different numbers of hours of summer work.

- How could you solve this task?
- What might students struggle with?

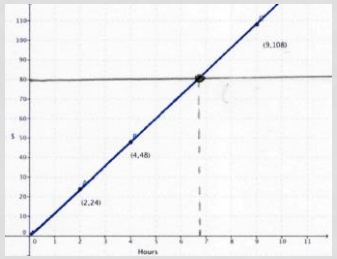


1. Maleeka's short-term goal is to earn more than $80 for new Xbox One™ games. Describe the number of hours that she must work in order to make enough money for the games. Show all work and represent your answer using words, inequality notation, and a number line.
2. Maleeka's long-term summer goal is to make more than $750. Determine the number of hours that she must work in order to meet this goal. Show all work and represent your answer using words, inequality notation, and a number line.

Adapted from Seventh Grade Set of Related Lessons:

WAYS TO SOLVE THE LIFEGUARD TASK

- Conclude that the Maleeka makes $12 per hour -- the relationship between the hours worked and the money earned – then divide $80 by 12 and get 6 2/3. So she needs to work **more than** 6 hours and 40 minutes.
- Draw a line connecting the three points on the graph; Draw a horizontal line through $80 dollars; then draw a vertical line from the point of intersection to the x-axis. This will be the number of hours she needs to work to make $80. So it is anything greater than this.



- Make a table with the given points and fill in the missing points by seeing the pattern – the money earned is 12 x the hours worked. Then determining the she works between 6 and 7.

Hours	Earnings
1	12
2	**24**
3	36
4	**48**
5	60
6	72
7	84
8	96
9	**108**

POSSIBLE CHALLENGES

- Recognizing that the rate per hour is $12.
- Solving an equation rather than an inequality
- Making a table but not being sure how to find the missing values
- Narrowing it down to between 6 and 7 hours and not sure how to get more precise
- Not sure what to do

LIFEGUARD MINI-DIALOGUES

Read the mini-dialogues then:

- Discuss the nature of each students' struggle.
- Identify what the teacher does to help students move beyond the impasse they had reached.
- Determine whether or not the teacher supported students' productive struggle.

DISCUSSION OF MINI-CASES OF PRODUCTIVE STRUGGLE

- In which of the mini-dialogues do you think the student was supported in productive struggle? Why?
- What was it about the interaction between the teacher and the student that supported (or did not support) productive struggle?

DIALOGUE 1

Not Productive Struggle

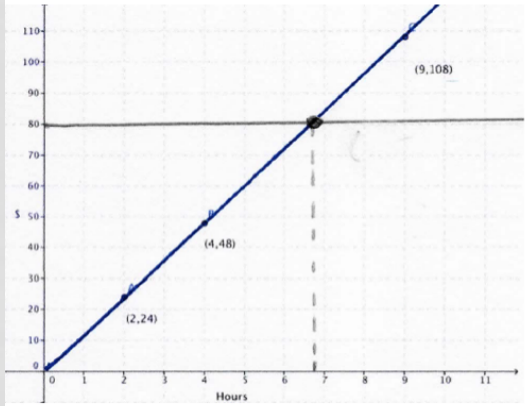
A student lists the ordered pairs shown below.
(2, 24) (4, 48) (9, 108)

Teacher: Where did these numbers come from?
Student: These are the labels for the points on the graph.
Teacher: So how do they help you answer the question?
Student: Maleeka wants to make $80, and I know that's between four hours and nine hours, but I am not sure how to get closer than that.
Teacher: How much money did Maleek make in two hours?
Student: $24.
Teacher: So, how much money would she make in one hour? *[Student divides 24 by 2 on her paper.]*
Student: $12?
Teacher: That's right. So now you know that she makes $12 in one hour, and we want to find out how many hours she needs to work to make more than $80. Do you remember how we used equations to solve problems like this?
Student: 80 ÷ 12?
Teacher: Yes! Solve 12x = $80 by dividing 80 by 12 see what you get.

DIALOGUE 2

Productive Struggle

A student made the table shown below.



Teacher: Tell me what you have here.

Student: I drew a line connected the three points that were given. Then I drew a horizontal line through $80 because that is the least amount of money she wants to make.

Teacher: So what did this tell you?

Student: The point where the two lines intersect is the number of hours where she makes $80. So it is (x, 80). The value of x is somewhere between 6 and 7 hours, but closer to 7.

Teacher: So how can you figure it out more precisely?

Student: I am not sure.

Teacher: If you look at the three points that are given, how are hours worked related to the amount earned?

Student: The hours worked times 12 gives you the amount she earned.

Teacher: So can this help you figure out how to find the x-coordinate of your point of intersection?

Student: I can set up an equation $x\,(12) = 80$ and solve for x.

Teacher: That sounds like a good plan. I will check back in with you later.

DIALOGUE 3

Productive Struggle

A student makes the table shown below.

Hours	Earnings
2	24
4	48
9	108

Teacher: Tell me how you got the numbers in your table?
Student: I read the points in the graph.
Teacher: Does that help you answer the question?
Student: Not really. She had to work more than four hours but less than eight. I am not sure where to go from here.
Teacher: I am wondering if you could expand your table to include other values that were not on the graph. How could you get started?
Student: I could put in hours 1 through 9 and then see if I could figure out how much she would make for that number.
Teacher: Okay. So how will you figure out how much she earns for any number of hours?
Student: Well, if I look at the three values in the table, it seems that each of the hours was multiplied by 12 to get the earnings. So that means she must make $12 an hour. So, I will just multiply all the hours in my new table by 12 and see if I can find what number of hours get her more than $80.
Teacher: Sounds good!

DIALOGUE 4

Not Productive Struggle

A student cannot get started.

Teacher: What have you figured out so far?
Student: Nothing. I am not sure what to do.
Teacher: The first thing you need to do is to figure out how much money she makes every hour. Do you remember what operation you need to use to do that?
Student: Divide?
Teacher: What are you going to divide?
Student: 2 ÷ 24?
Teacher: No, 24 ÷ 2.
Student: So, it's 12.
Teacher: So this is the amount she makes per hour. Now you need to divide 80 by 12.

DIALOGUE 5

Not Productive Struggle

A student can't get started.

Teacher: What have you figured out so far?
Student: Nothing. I am not sure what to do.
Teacher: Why don't you start by reading the problem again and then looking really closely at the graph. I will be back in a few minutes.

How would you categorize the teacher's response in each of the five dialogues?

TEACHER STRUGGLE

1. Telling

Telling involves supplying students with information (e.g., a strategy to use) that removes the struggle, thereby allowing them to make progress on the task by following the prescribed strategy.

2. Directed Guidance

Directed guidance involves redirecting students to a different strategy, one that is consistent with the teacher's way of thinking rather than the student's way of thinking, thereby enabling the student to move beyond the impasse.

3. Probing Guidance

Probing guidance involves determining what a student is currently thinking, encouraging a student's self-reflection, and offering ideas that are based on the student's thinking.

4. Affordance

Affordance involves asking students to articulate what they have done, encouraging their continued efforts with limited intervention, and allowing them time to continue their work.

5. Unfocused or Vague

Unfocused or vague responses do not direct students to a particular strategy or build on students' thinking but instead provides a suggestion that is often too general to be helpful.

Warshauer, Hiroko Kawaguchi. "Productive Struggle in Middle School Mathematics Classrooms." *Journal of Mathematics Teacher Education* 17, no.4 (2015): 375-399. [p. 387]

DIALOGUE 1

Directed Guidance

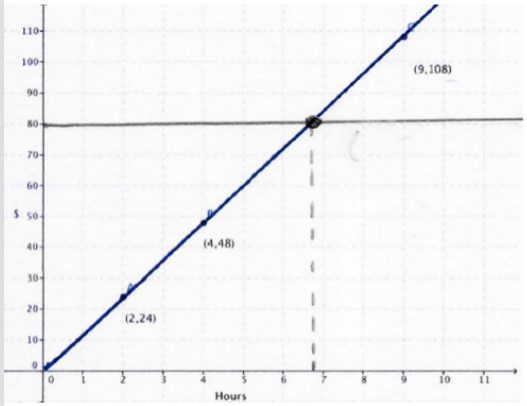
A student lists the ordered pairs shown below.
(2, 24) (4, 48) (9, 108)

Teacher: Where did these numbers come from?
Student: These are the labels for the points on the graph.
Teacher: So how do they help you answer the question?
Student: Maleeka wants to make $80, and I know that's between four hours and nine hours, but I am not sure how to get closer than that.
Teacher: How much money did Maleek make in two hours?
Student: $24.
Teacher: So, how much money would she make in one hour? *[Student divides 24 by 2 on her paper.]*
Student: $12?
Teacher: That's right. So now you know that she makes $12 in one hour, and we want to find out how many hours she needs to work to make more than $80. Do you remember how we used equations to solve problems like this?
Student: 80 ÷ 12?
Teacher: Yes! Solve 12x = $80 by dividing 80 by 12 see what you get.

DIALOGUE 2

Affordance

A student made the table shown below.



Teacher: Tell me what you have here.

Student: I drew a line connected the three points that were given. Then I drew a horizontal line through $80 because that is the least amount of money she wants to make.

Teacher: So what did this tell you?

Student: The point where the two lines intersect is the number of hours where she makes $80. So it is (x, 80). The value of x is somewhere between 6 and 7 hours, but closer to 7.

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Teacher: If you look at the three points that are given, how are hours worked related to the amount earned?

Student: The hours worked times 12 gives you the amount she earned.

Teacher: So can this help you figure out how to find the x-coordinate of your point of intersection?

Student: I can set up an equation $x\,(12) = 80$ and solve for x.

Teacher: That sounds like a good plan. I will check back in with you later.

DIALOGUE 3

Probing Guidance

A student makes the table shown below.

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Teacher: Tell me how you got the numbers in your table?
Student: I read the points in the graph.
Teacher: Does that help you answer the question?
Student: Not really. She had to work more than four hours but less than eight. I am not sure where to go from here.
Teacher: I am wondering if you could expand your table to include other values that were not on the graph. How could you get started?
Student: I could put in hours 1 through 9 and then see if I could figure out how much she would make for that number.
Teacher: Okay. So how will you figure out how much she earns for any number of hours?
Student: Well, if I look at the three values in the table, it seems that each of the hours was multiplied by 12 to get the earnings. So that means she must make $12 an hour. So, I will just multiply all the hours in my new table by 12 and see if I can find what number of hours get her more than $80.
Teacher: Sounds good!

DIALOGUE 4

Telling

A student cannot get started.

Teacher: What have you figured out so far?
Student: Nothing. I am not sure what to do.
Teacher: The first thing you need to do is to figure out how much money she makes every hour. Do you remember what operation you need to use to do that?
Student: Divide?
Teacher: What are you going to divide?
Student: 2 ÷ 24?
Teacher: No, 24 ÷ 2.
Student: So, it's 12.
Teacher: So this is the amount she makes per hour. Now you need to divide 80 by 12.

DIALOGUE 5

Vague or Unfocused

A student can't get started.

Teacher: What have you figured out so far?
Student: Nothing. I am not sure what to do.
Teacher: Why don't you start by reading the problem again and then looking really closely at the graph. I will be back in a few minutes.

RESEARCH FINDINGS

Five Types of Teacher Reponses to Struggle:

1. Telling
2. Directed Guidance

More likely to lower the demands of the task and remove struggle

3. Probing Guidance
4. Affordance

More likely to maintain the demands of the task and support productive struggle

5. Unfocused or Vague

Warshauer, Hiroko Kawaguchi. "Productive Struggle in Middle School Mathematics Classrooms." *Journal of Mathematics Teacher Education* 17, no.4 (2015): 375-399. [p. 387]

SO WHAT DOES IT TAKE?

EFFECTIVE MATHEMATICS TEACHING PRACTICES

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
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7. *Support productive struggle in learning mathematics.*
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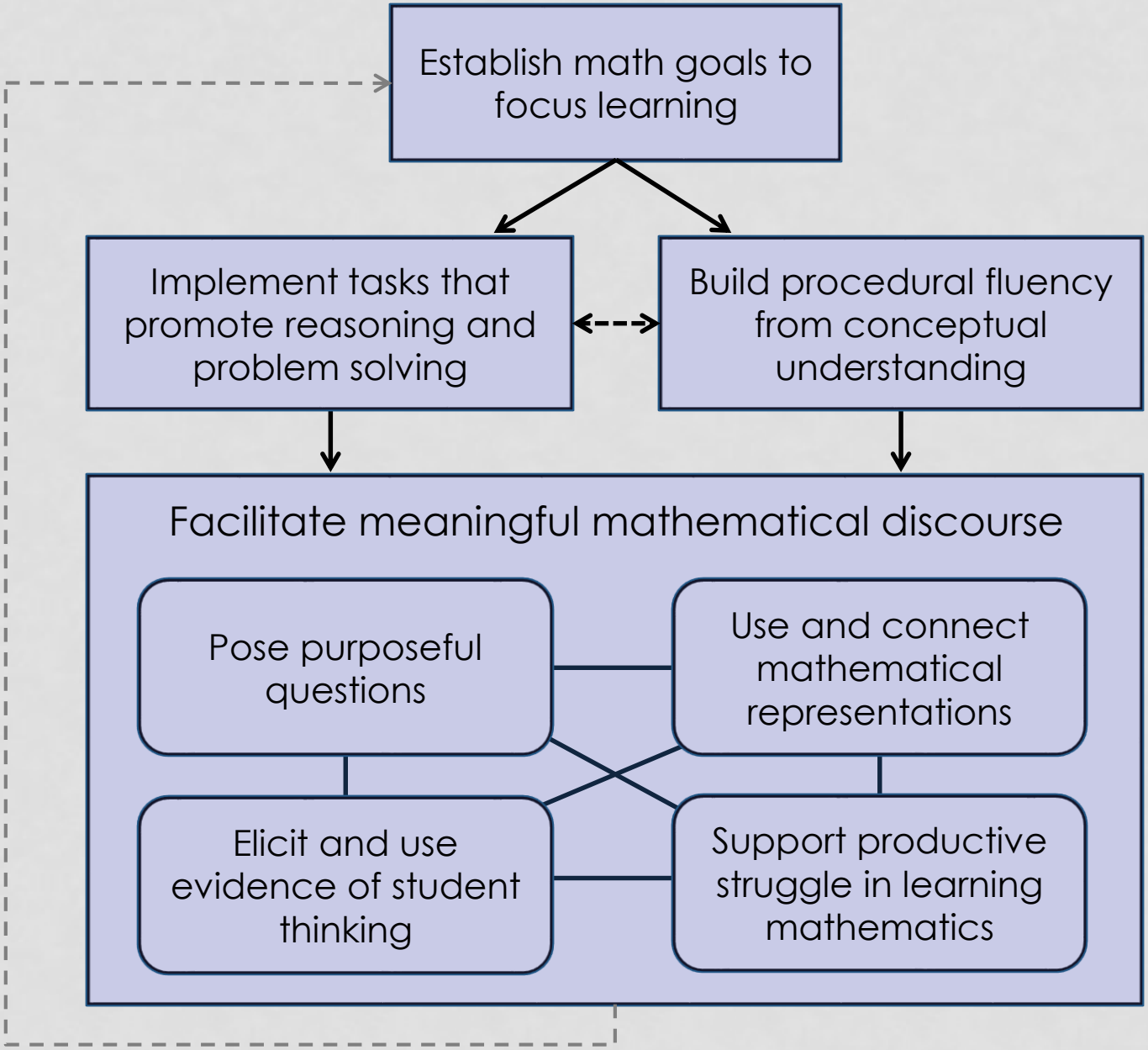
NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Effective Mathematics Teaching Practices
"Building a Teaching Framework"



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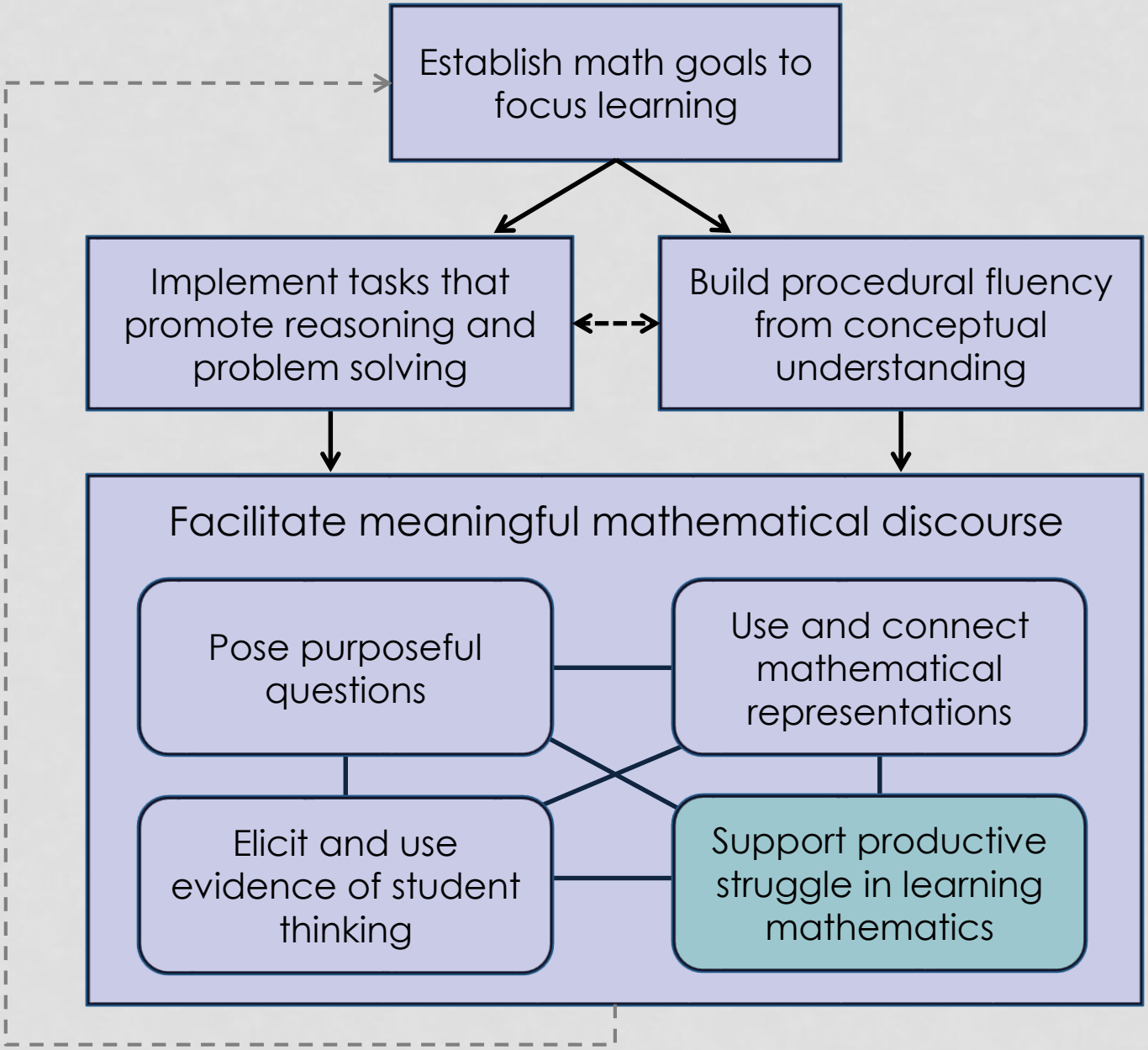

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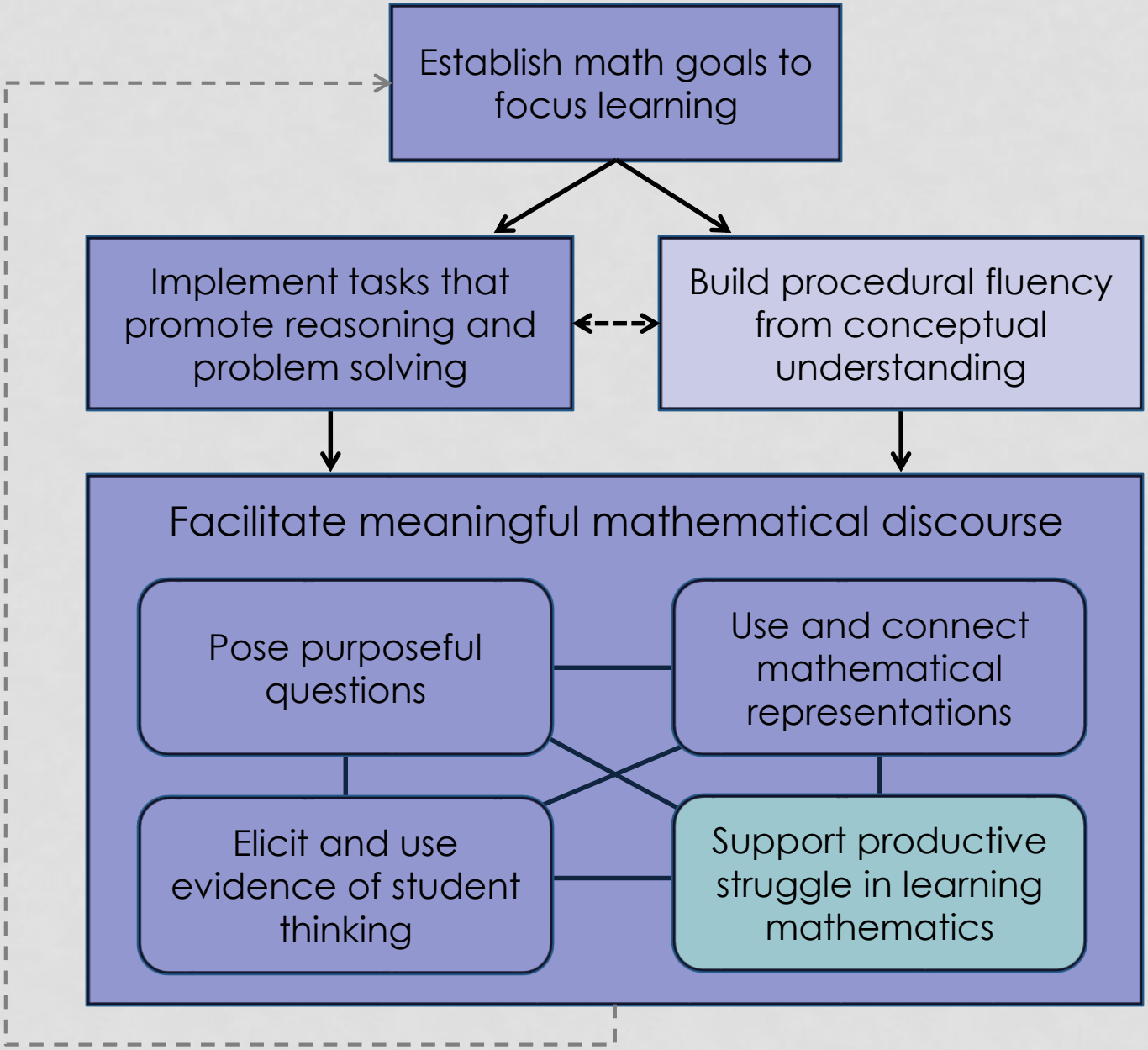
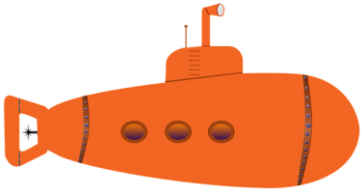


ACTIVITY 2

WHAT ARE THE CHARACTERISTICS OF A TEACHER'S INTERACTIONS THAT SUPPORT PRODUCTIVE STRUGGLE?

DEEP DARK SECRET



The deep sea is the lowest level of the ocean floor. Sunlight does not reach the deep sea, and scientists are discovering that it is home to amazing creatures that live in total darkness. Deep-sea creatures live more than 1000 feet below the water's surface. Marine biologists in an underwater vessel are descending to study a new species of fish that was discovered in the deep sea. Their vessel is currently located 100 feet below the surface (-100 feet). From this location, the biologists start their timer at t = 0 and begin their descent. They descend at a rate of 25 feet per minute. The vessel continues descending at this constant rate until it has reached a depth where they can study the deep sea creatures.

1. After how many minutes will the vessel be positioned to observe deep-sea creatures? Explain how you know.
2. Write an inequality to represent the depth of the vessel at any point in time, *t*, after the vessel has reached a depth at which the marine biologists can observe deep sea creatures, where *t* represents the number of minutes since the vessel has left its original position of -100 feet.

Adapted from Seventh Grade Set of Related Lessons: Investigating Inequalities, Institute for Learning, University of Pittsburgh, 2015.

SUPPORTING PRODUCTIVE STRUGGLE

Ms. Kaufmann's seventh-grade class is working on the Deep Dark Secret task, which presents students with a situation that they have not yet encountered in their work in the unit—a negative rate of change.

In the dialogue, Ms. Kaufmann is interacting with a group of students (Kaila, Jake, Jerome, and Chris), who have arrived at an incorrect solution to the first part of the Deep Dark Secret task.

What did the teacher do to support the group's productive struggle?

WHAT DID MS. KAUFMANN DO?

She supported productive struggle by--

- selecting a **task** that had the potential to engage students in reasoning and problem solving
- asking **questions** to determine what her students understood and to advance them beyond where they currently were
- connecting **representations** – the problem context and the drawing of the situation represented
- **eliciting and using evidence** of student thinking to advance their understanding of the situation
- facilitating meaning **discourse** within the small group

STRATEGIES FOR SUPPORTING PRODUCTIVE STRUGGLE

- Question
- Encourage
- Give Time
- Acknowledge

Warshauer, Hiroko Kawaguchi. Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20 no. 7 (2015a): 390-393. (p.392)

QUESTION

"Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem."

Ms. Kaufmann asked *questions* that made the students' thinking public (lines 1–9). The answers to this series of questions made clear to the teacher the source of the students' error.

Warshauer, Hiroko Kawaguchi. Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20 no. 7 (2015a): 390-393. (p.392)

ENCOURAGE

"Teachers encourage students to reflect on their work and support student struggle in their effort and not just in getting the correct answers."

Ms. Kaufmann asked students to draw and explain a picture that represented the situations (lines 10–11; 19) and to reread the problem aloud (lines 23-24). This series of interactions *encouraged students to reflect on and reconsider* what they had done and ultimately to revise their drawing (line 33) and their solution (lines 43-52).

Warshauer, Hiroko Kawaguchi. Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20 no. 7 (2015a): 390-393. (p.392)

GIVE TIME

"Teachers give time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students."

Ms. Kaufmann also gave the group *time to work* to create the picture (lines 12-13) without hovering over them, sending the clear message that she had confidence that they could do the work without her assistance.

Warshauer, Hiroko Kawaguchi. Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20 no. 7 (2015a): 390-393. (p.392)

ACKNOWLEDGE

"Teachers acknowledge that struggle is an important part of learning and doing mathematics."

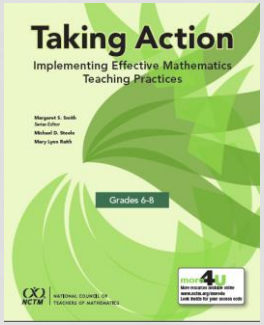
In the end, Ms. Kaufmann *acknowledged* the group's effort and the students' ability to work through the problem (lines 55-57).

Warshauer, Hiroko Kawaguchi. Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20 no. 7 (2015a): 390-393. (p.392)

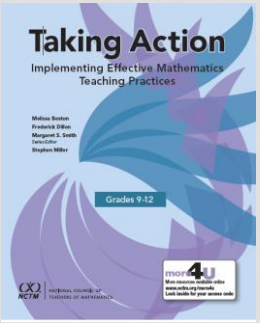
TAKE AWAY

- Productive struggle is central to student learning.
- Students need something with which to struggle – **the task matters**!
- All students know something relevant whether they recognize it or not.
- Teachers need to find ways to support students learning without taking over the thinking for them. The four strategies (question, encourage, time, acknowledge) and the eight effective teaching practices can help.
- Advance planning is key.

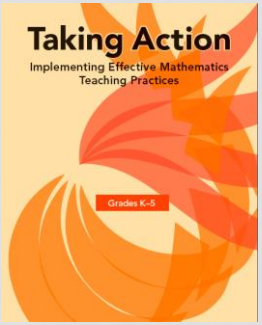
MORE ABOUT THE EFFECTIVE TEACHING PRACTICES



Smith, Margaret, Michael Steele and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics, 2017.



Boston, Melissa, Frederick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. Reston, VA: National Council of Teachers of Mathematics, 2017.



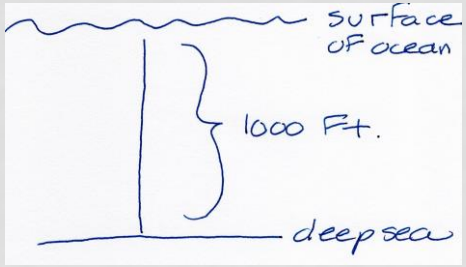
Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.


If there is no struggle, there is no progress.
Frederick Douglass
A-Z QUOTES


ngiyabonga
danke
謝謝
teşekkür ederim
спасибо
dank je
thank you
gracias
tapadh leat
hvala
maururu
bedankt
mochchakkeram
dziękuję
go raibh maith agat
sagolun
sukriya
kop khun krap
obrigado
arigatō
takk
dakujem
grazie
мерси
terima kasih
감사합니다
ευχαριστώ
merci

MS. KAUFMANN'S CLASS

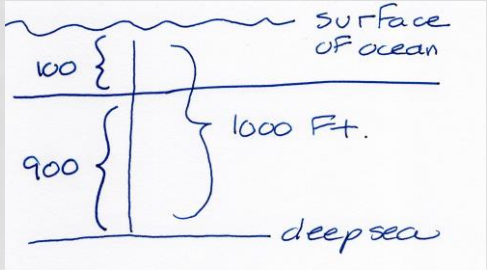
1. Ms. K: So how long do you think it will take before they will see the creatures? Kaila?
2. Kaila: We think it would take 40 minutes.
3. Ms. K: It will take 40 minutes to do what? Jake?
4. Jake: To get to the deep sea.
5. Ms. K: How did you get 40? Chris?
6. Chris: We divided 1000 by 25.
7. Ms. K: Why did you do that? Jerome?
8. Jerome: It says that deep-sea creatures live more than 1000 feet below the surface. So if the
9. vessel goes down 25 feet per minute, you just divide 1000 by 25.
10. Ms. K: Can you draw a picture of the situation to help explain what you did? I will be back
11. in a few minutes to see what you came up with.
12. *[Teacher leaves to respond to other students and returns five minutes later. Members of the*
13. *group work discuss what the picture should look like. Kaila offers to draw.]*
14. Ms. K: So, tell me about your picture.
15. Kaila: Okay, see *[referring to the drawing below]*, we started with the surface of the
16. ocean and then showed where the deep sea started, which is 1000 feet below the
17. surface.



18.

MS. KAUFMANN'S CLASS

19. Ms. K: So, how does this picture explain what you did? Jerome?
20. Jerome: The vessel kept going down 25 feet every minute. So, you need to
21. find out how many 25's it takes to get to 1000. That will tell you how many
22. minutes it would take to get to 1000. So, it takes 40 minutes, just like Kaila said.
23. Ms. K: Okay. So, what else do we know about the situation? Jake, can you reread
24. the problem?
25. *[Jake reads the problem aloud and stops when he reaches the part that says "The vessels*
26. *starts 100 feet below the surface."]*
27. Chris: Oh wait…our drawing isn't right…the vessel starts 100 feet below the surface.
28. We need to fix our picture.
29. Ms. K: Does everyone see what Chris is saying?
30. Jake: Yeah. The vessel was already 100 feet below the surface when they decided
31. to go see the sea creatures.
32. Jerome: Okay, so that means that they only went down 900 feet, not 1000.
33. Kaila: *[Quickly making a change, as in the sketch shown below.]* Now this shows
34. what Jerome just said.



36.
37.

MS. KAUFMANN'S CLASS

38. Ms. K: So does the starting point make a difference? Chris?
39. Chris: Yeah, because like Jerome said, they only went down 900 feet from where
40. they started, not 1000 feet.
41. Ms. K: So, what does the drawing suggest to you about your initial approach to
42. the problem?
43. Jake: So, it should be 900 divided by 25. *[Jerome quickly divides 900 by 25]*.
44. Jerome: Which is 36.
45. Ms. K: Okay, so what does 36 mean in this problem? Kaila?
46. Kaila: It is the number of minutes it took to get to deep sea.
47. Ms. K: Does everyone agree?
48. Chis: I think it takes 36 to get to 1000 feet but you need to get deeper than that
49. to actually see sea creatures because the creatures are more than 1000
50. feet below the surface.
51. Ms. K: So how long will that take?
52. Kaila: More than 36 minutes.
53. Ms. K: Does everyone agree?
54. Students: Yeah!
55. Ms. K: I liked the way you all worked together on this. I knew you could figure it
56. out! Now you are ready to take on the second part of the task. I will check
57. in with you a little later.