Multi-Tiered Systems of Support: What are Effective Interventions and Assessments?

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Reflecting on Your Mathematical Experience

When you think about learners who struggle, what are. . .

- Your biggest challenges?
- The best instructional strategies for them?

Warm up for today's topic

- Using a number family like 9, 6, 15, create an addition or subtraction story problem that you would have students in your classroom solve.

Focus of Session

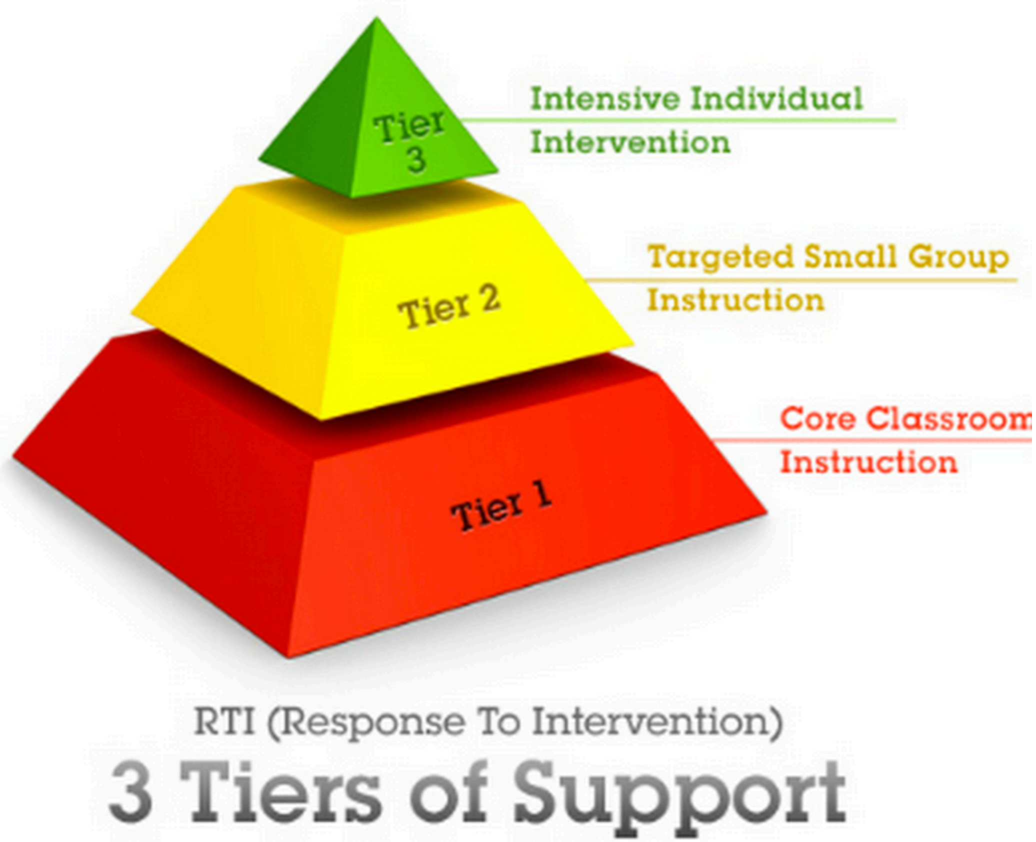
This session focuses on ways to support students who struggle in Tiers 1 and 2 to become:

- Confident in mathematics
- Successful with <u>rigorous</u> mathematics
- Capable of working with complex mathematical ideas

Overview

- Systems of Support
- Important mathematical understandings
- Number understandings
- Problem solving and critical thinking
- Curriculum issues
- General classroom management strategies
- Assessment

3-Tiered Support Model



Models within Tier 2

- Problem-solving model: School-based team uses multiple sources of data to determine interventions, often individualized
- Standard protocol model: Intervention is a research-based, well-formulated curriculum

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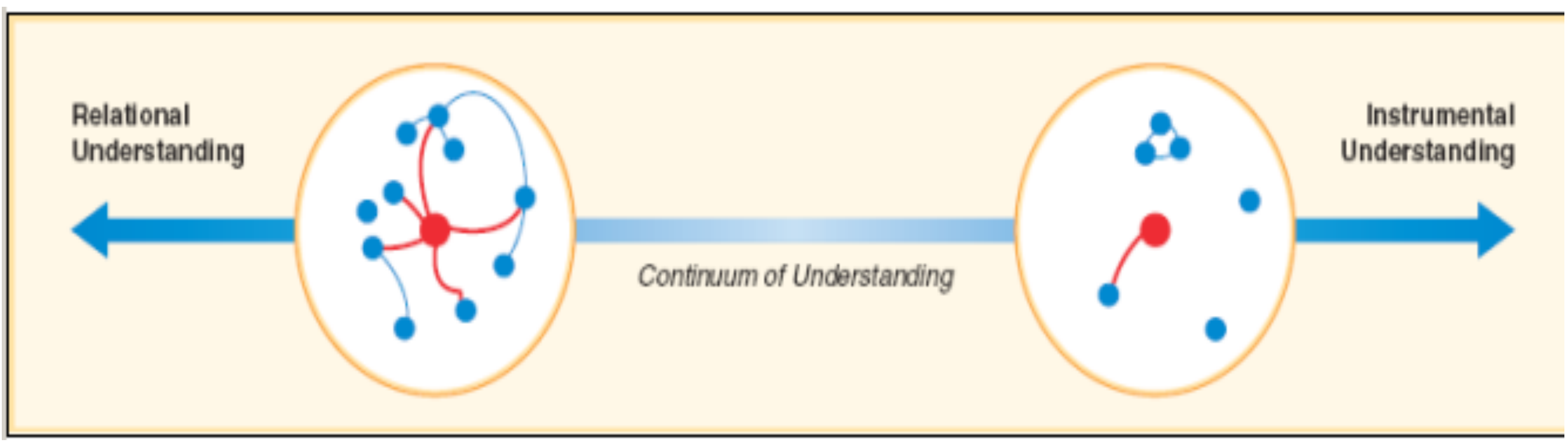
What are characteristics of students in Tier 2?

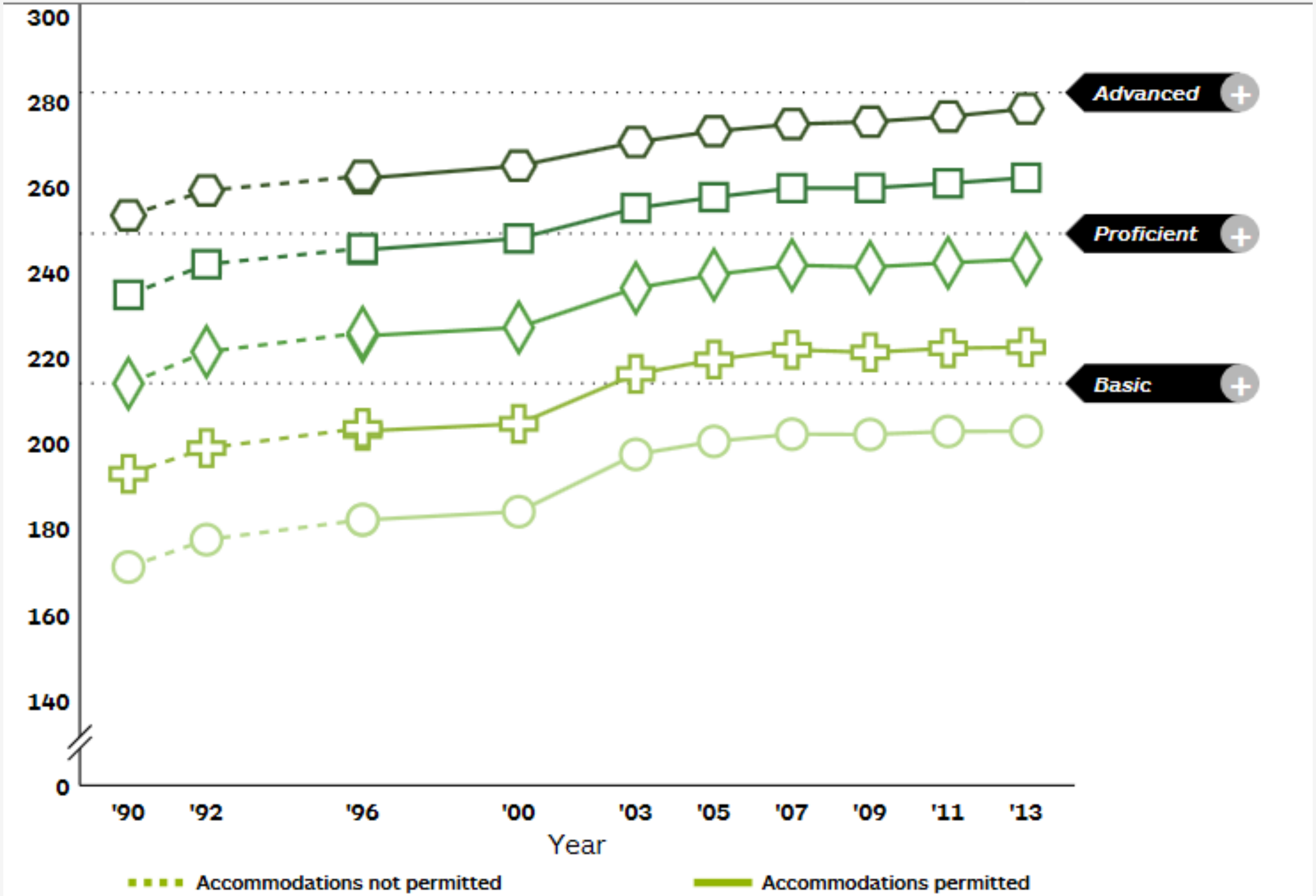
How are students' understandings structured in their brain?

Compared to students who are not struggling, their brains might look very different!

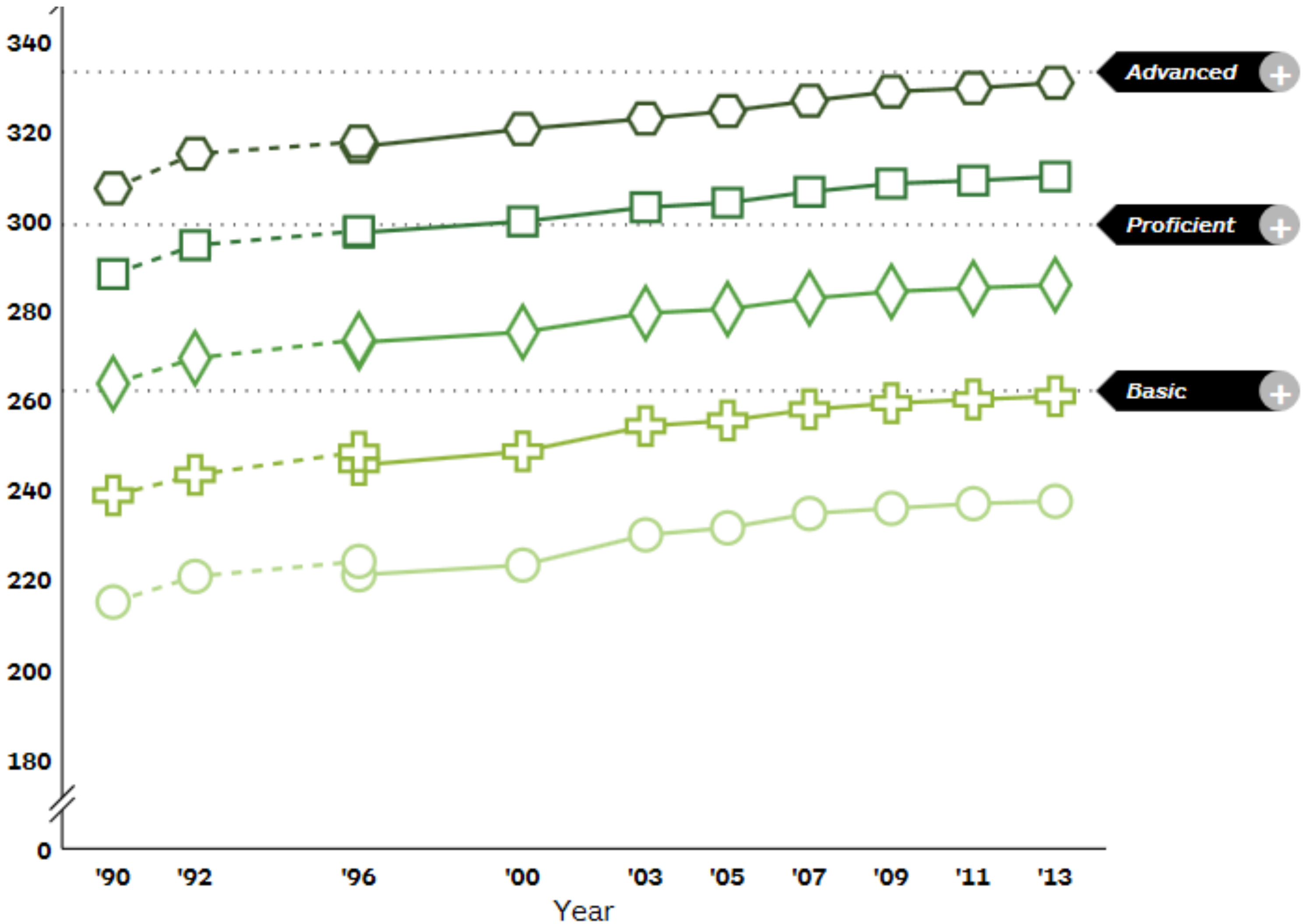


Skemp, R. (1987). *The psychology of learning mathematics,* The Penguin Press.
, Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Has RtI implementation closed the gap? (4th grade)



Are we doing any better at 8th grade?



Mathematics Performance on NAEP 2013

4th grade

At or Above Proficient: 45% of all students and **18% of students with disabilities**

Below Basic: 18% of all students and **45% of students with disabilities**

8th grade

- At or Above Proficient: 39% of all students and **9% of students with disabilities**
- Below Basic: 21% of all students and **65% of students with disabilities**

National Center for Education Statistics, 2013

Components of A Strong Multi-Tiered System of Support Model

Uses a co-teaching approach as a collaboration between general education and special education

Includes research based teaching practices

Uses screening and progress monitoring to Instruct with a preventative approach

Builds from students' strengths

Uses diagnostic assessments to align intervention

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies:Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.

Making Cents

- Take out some coins.
- Multiply the value of the coins in cents by 4.
- Add 10 to the product.
- Multiply your answer by 25.
- Add 115 to your answer.
- Add your age in years.
- Subtract the number of days in a normal year.

Making Cents

- What do you notice about your answers?
- How can you describe what you notice to someone who is not present?

How do you describe what you see in terms of student learning?

Types of Understandings

Procedural - Student can perform a computation or algorithm by following a series of prescribed steps

Conceptual - Student understands the basis of why a computation or algorithm works. They can apply it later without reteaching. Student can identify, describe, and explain a big idea related to a topic or a class of problems

Problem solving - Student can solve a problem when there is no specific solution pathway or algorithm

What should students learn first: concepts or skills?

You decide

Create Mental Residues

- Establishes foundational understanding
- Models the physical action is the important
- Does not fade away or disappear
- Supports their thinking about the operation

Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahweh, NJ: Erlbaum.

Focus on Skills

$$\frac{1}{12} + \frac{7}{8}$$

Focus on Skill

$$\frac{1}{12} + \frac{7}{8} = \frac{23}{24}$$

Focus on Sense Making

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. $\frac{1}{2}$
D. 1

The Two Worlds Collide: Sense Making Meets Skill

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. ½
D. 1

Explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

Petit, Laird,& Marsden, 2010

A Number Problem with Hudson

Me: Okay, then which is larger, 38 or 32?

Hudson: Mmmmmm, 38.

[Starts to walk away quickly. Nana follows him.]

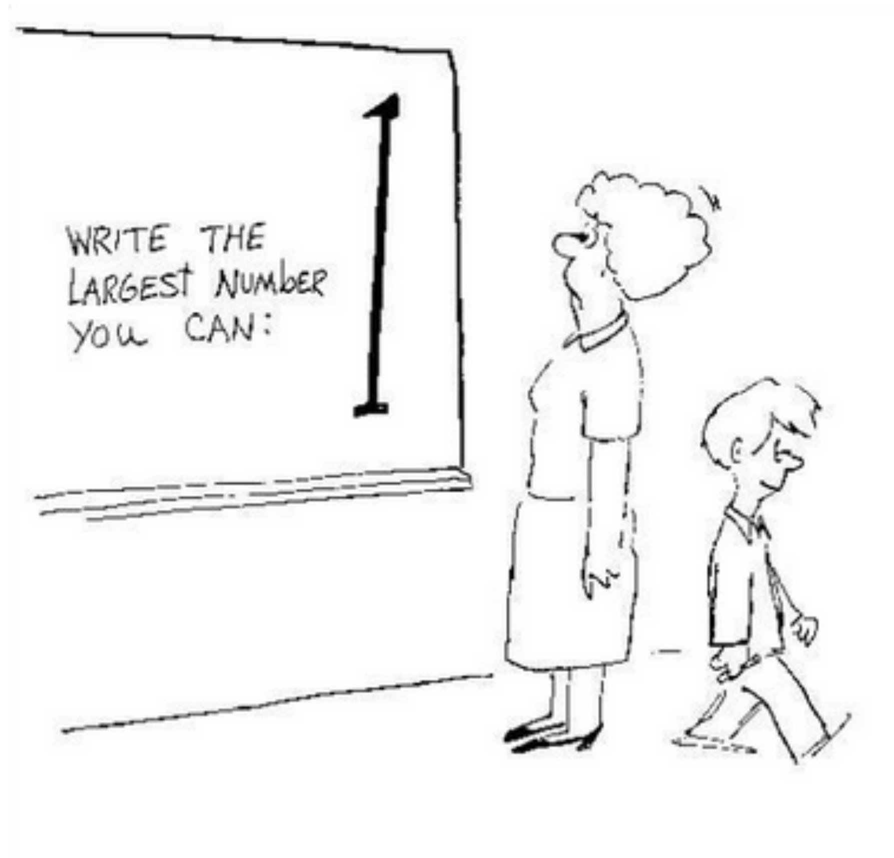
Me: How did you decide that?

A Number Problem with Hudson

Hudson: Well, math is a like a guessing game. I had to pick one so I picked 38 and I got it right. It's like a magic thing. You never know in math if you are getting a problem right until the teacher grades it and you get a smiley face.

Beginning number ideas

- Counting
  - Rote
  - Cardinality
  - Ordinality



Sammi-Jo said, “I counted 4 things.” “No,” said Henna. “There are 8 things.”

Is it possible that Sammi-Jo and Henna counted the same group of objects? why or why not?

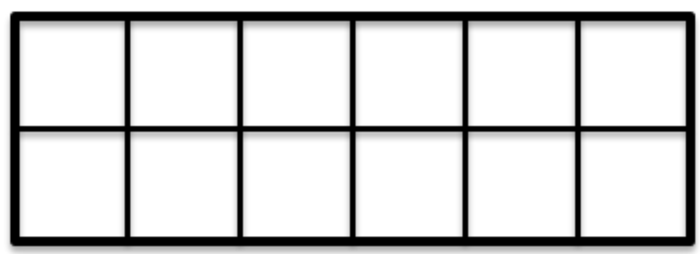
Dougherty, B. J., Flores, A., Sophian, C., & Everett, L. (2010). *Essential understandings: Engaging teachers with mathematics, PreK–2 number concepts*. Reston, VA: National Council of Teachers of Mathematics.

Counting Task

1. How many ways can you find to measure the area in the rectangle below?

2. What would the units look like?

3. How would the units compare?



Dougherty, Zenigami, & Okazaki (2007). *Measure Up*, grade 1. University of Hawaii.

Units are important!

- Instruction often does not explicitly provide opportunities to discuss the importance of unit.
- Unit is a fundamental concept in mathematics.

What do we assume?

$3 < 8$

What first graders had to say. . .

3 < 8

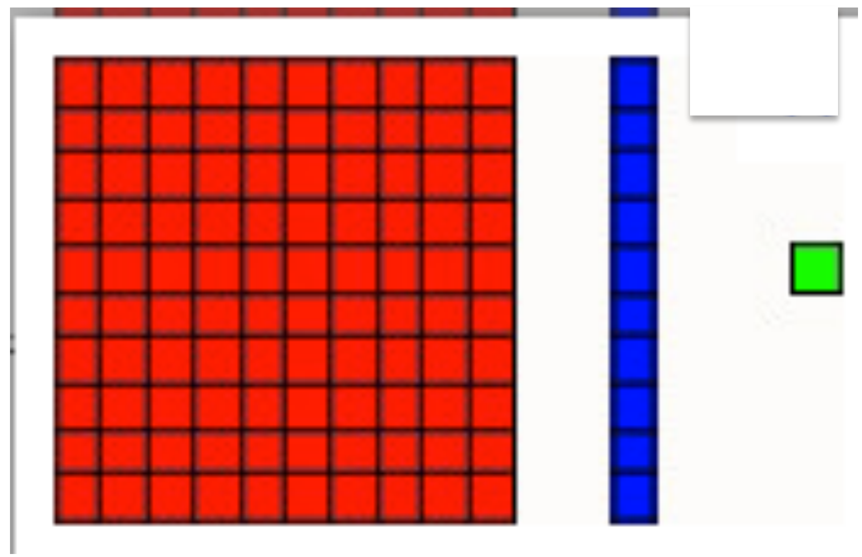
Richard: You can't really tell. 'Cause you could have 3 really really really big units and 8 really really really small units. Then 8 would be less than 3.

Reed: Yeah, but if it's on a number line then it's true 'cause all the units are the same size.

Whole Numbers

If you wanted to use base ten blocks, how could you model 125?

Work with a partner to think about how many different ways you could model 125.



Whole Numbers

- Decomposition
  - Writing equations to symbolize the representations

$125 = 100 + 10 + 10 + 5$

$125 = 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1$

$125 = 60 + 60 + 5$

Comparing Whole Numbers

- What can you say about the relationship between 125 and 152?

- What statements could you write or say?

Comparing Whole Numbers

- What can you say about the relationship between 125 and 152?
- What statements could you write or say?

$125 \neq 152$

$152 \neq 125$

$125 < 152$

$152 > 125$

Comparing Whole Numbers

$125 \neq 152$

$152 \neq 125$

$125 < 152$

$152 > 125$

$125 < 152$ by 27

$152 > 125$ by 27

Comparing Whole Numbers

$125 \neq 152$

$152 \neq 125$

$125 < 152$

$152 > 125$

$125 < 152$ by 27

$152 > 125$ by 27

$152 = 125 + 27$

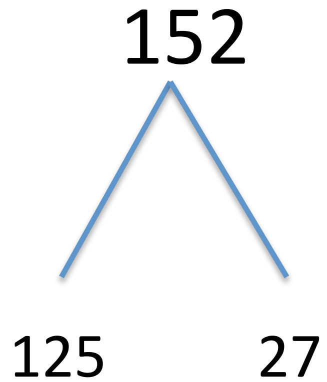
$152 = 27 + 125$

$152 - 27 = 125$

$152 - 125 = 27$

Part-Whole Diagrams

125 < 152 by 27



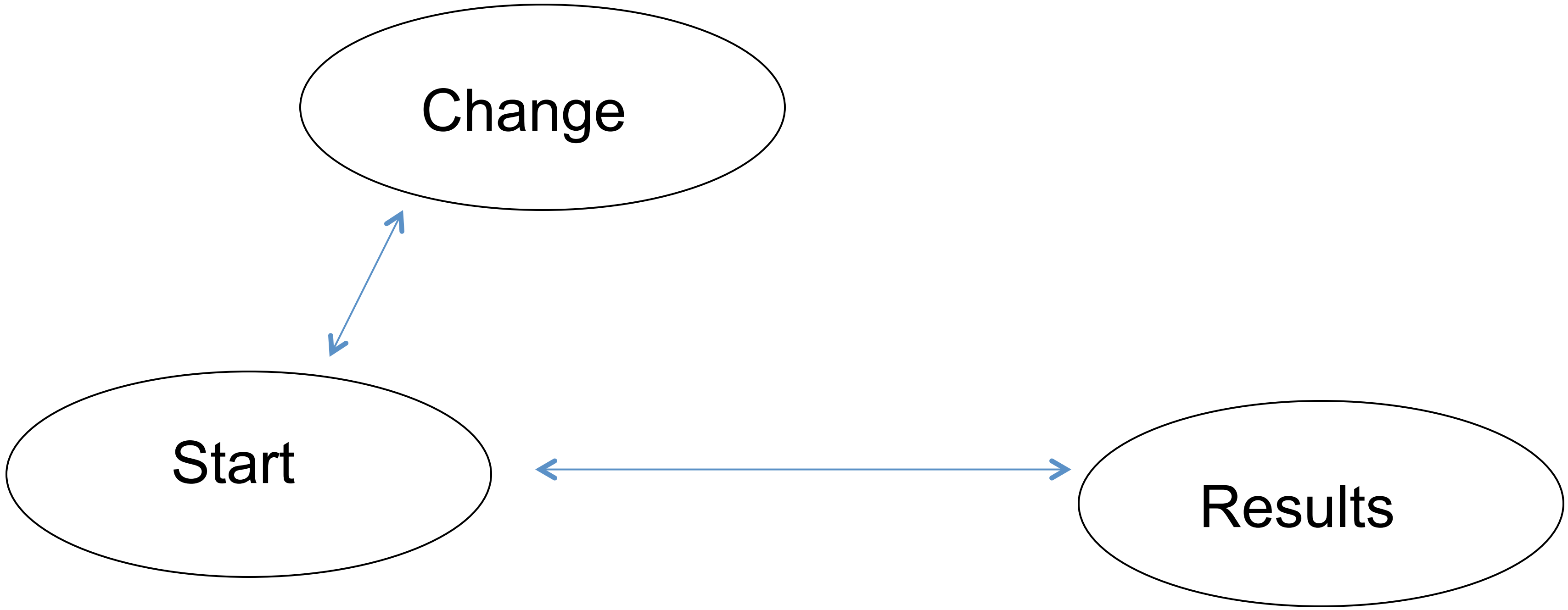
Part-Part-Whole Problems

Lynnette has 14 fiction and 23 nonfiction books. How many books does she have?

Lynnette and her friend Victoria put 37 books into a backpack. Lynnette put in 14 books. How many books did Victoria put in the backpack?

If there are 37 books in the backpack. What are the different combinations that both girls could have placed in the backpack?

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.



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Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S.. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

What's the Action in the Problem?

Join Problems: Use two quantities to find the third

Louise had 11 baseball cards. Elliott gave her 6 more. How many baseball cards does Louise have now?

Connect the action to the equation: $11 + 6 = \boxed{}$

Louise had 11 baseball cards. Elliott gave her some more. Louise now has 17 cards. How many did Elliott give her?

$11 + \boxed{} = 17$ or an equivalent equation $17 - 11 = \boxed{}$

Louise had some baseball cards. Elliott gave her 6 more. Now she has 17. How many baseball cards did Louise have to begin with?

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.
Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S.. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

What's the Action in the Problem?

Separate Problems: Start is the whole or the largest amount (not the result)

Mikal has 12 T-shirts. He gives 3 shirts to his brother Elron. How many T-shirts does Mikal have now?

Mikal has 12 T-shirts. He gives some shirts to his brother Elron. Now he has 9 left. How many T-shirts did Mikal give Elron?

Mikal has some T-shirts. He gives 3 shirts to his brother Elron. Now he has 9. How many T-shirts did Mikal have to begin with?

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Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S.. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

What models of subtraction do these problems represent?

- There were 17 boys playing tag. Twelve boys went home. How many boys are still playing tag?
- Tom has 5 brothers. Juan has 3 brothers. How many more brothers does Tom have than Juan?
- Jesse has 14 golf balls. Teva has 23 golf balls. How many more golf balls does Jesse need to have as many as Teva?

Some people will say…

:

I think this approach will confuse my students!

The Myth of Keywords

- Keywords do not—
  - Develop of sense making or support making meaning
  - Build structures for more advanced learning
  - Appear in many problems

- Students use key words inappropriately

- Multi-step problems are impossible to solve with key words

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Danger: Key Words Ahead

Mark has 33 packages of pencils. There are 6 pencils in each package. How many pencils does he have <u>in all</u>?

39 because it says in all.

The Infamous Shepherd Problem

:

There are 25 sheep and 5 dogs in a flock. How old is the shepherd?

How do you think students solved the problem?

Caldwell, Kobett & Karp (2014) Putting Essential Understandings into Practice: Addition and Subtraction , Grade PreK–2. NCTM.

Results from 214 Students

	Added the numbers	Subtracted the numbers	Multiplied the numbers	Created a ratio	Other Incorrect procedure	Suggested no solution is possible
Third grade (n = 58)	76%	8%	0%	0%	14%	2 %
Sixth grade (n = 71)	48%	9%	21%	8%	6%	8 %
Seventh grade (n = 85)	48%	2%	17%	14%	9%	10 %

Caldwell, Kobett & Karp (2014) *Putting essential understandings into practice in grades preK–2: Addition and subtraction.* A volume in the series edited by B. J. Dougherty, National Council of Teachers of Mathematics..

Really?

Caldwell, Kobett & Karp (forthcoming) *Putting essential understandings into practice in grades preK–2: Addition and subtraction.* A volume in the series edited by B. J. Dougherty, National Council of Teachers of Mathematics..

Another option

Would your students be able to discern which of the following three options would be the correct answer?

- The shepherd is 30 years old
- The shepherd is 125 years old; and
- It is not possible to tell the shepherd's age from the information in the problem.

Caldwell, Kobett & Karp (2014) *Essential understanding of addition and subtraction in practice, grades K-2*. NCTM.

Not All Problem Solving is Created Equal

- Application/routine problems: solved by an algorithm
- Non-routine problems: a creative solution approach is needed

Do you think differently when you solve these problems?

Brett wanted to put carpet on his bedroom floor. His room measures 10’ X 15’. How many square feet of carpet does he need?

You have 8 coins and a balance scale. The coins look alike, but one is counterfeit and lighter than the other 7. Find the counterfeit coin using 2 weighings on the balance scale.

Do you think differently when you solve these problems?

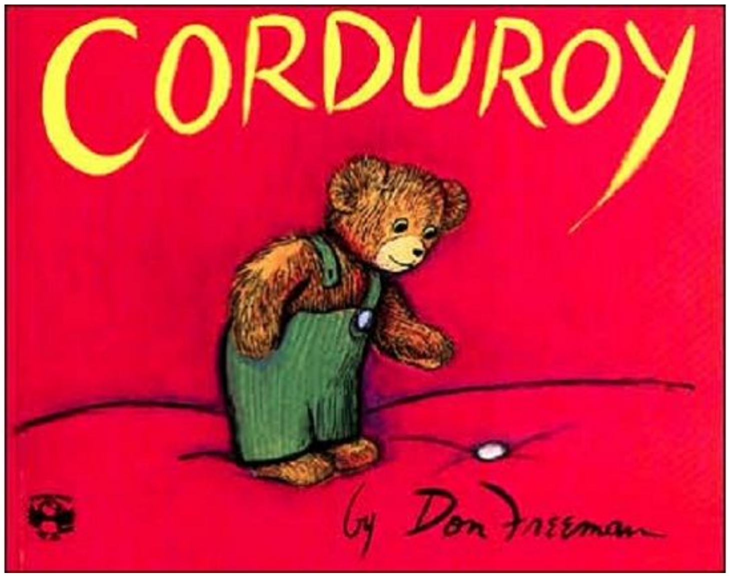
Word/Application

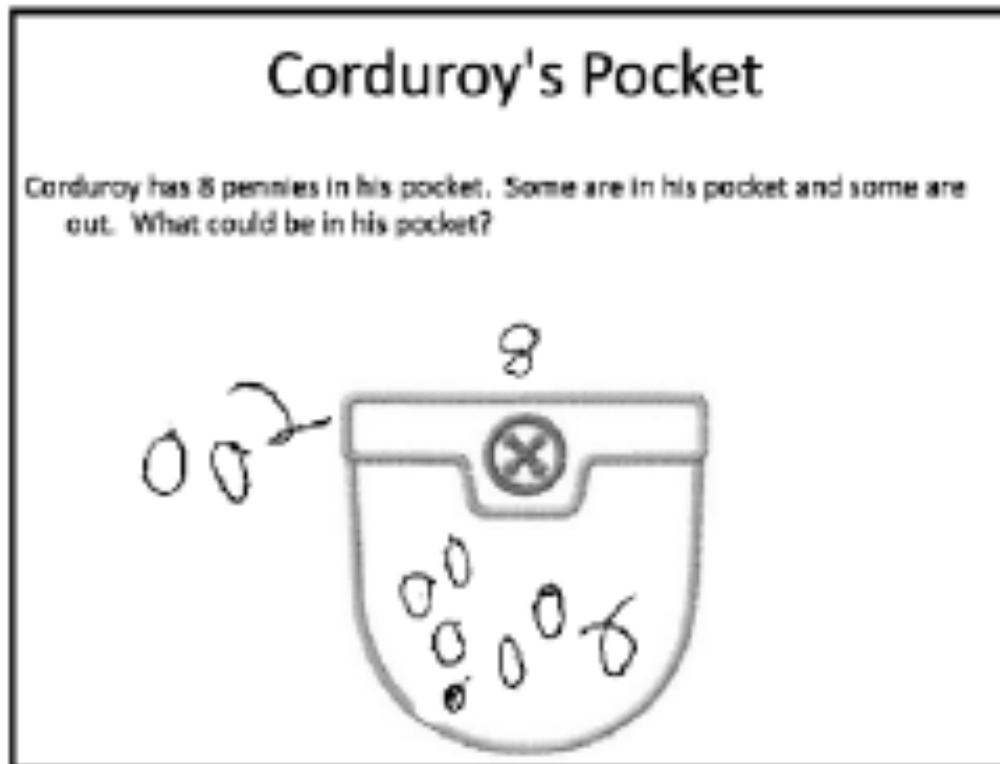
Brett wanted to put carpet on his bedroom floor. His room measures 10’ X 15’. How many square feet of carpet does he need?

Non-routine

You have 8 coins and a balance scale. The coins look alike, but one is counterfeit and lighter than the other 7. Find the counterfeit coin using 2 weighings on the balance scale.

Two parts (addends) unknown



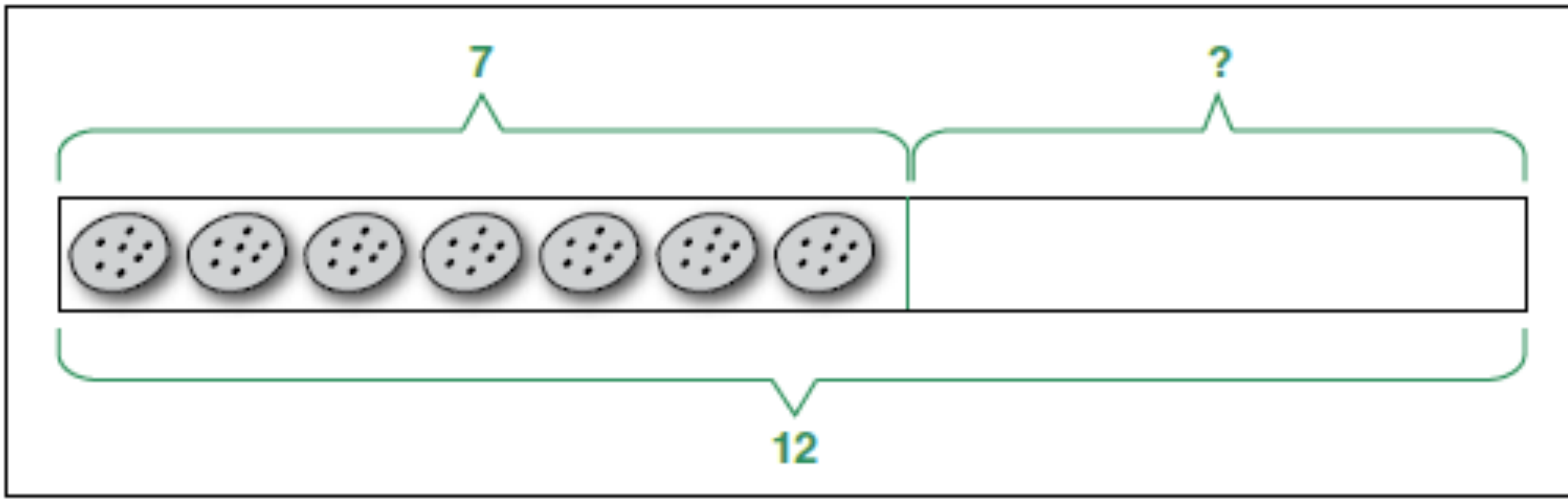


Caldwell, Kobett & Karp (2014) *Essential understanding of addition and subtraction in practice, grades K-2*. NCTM.

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$

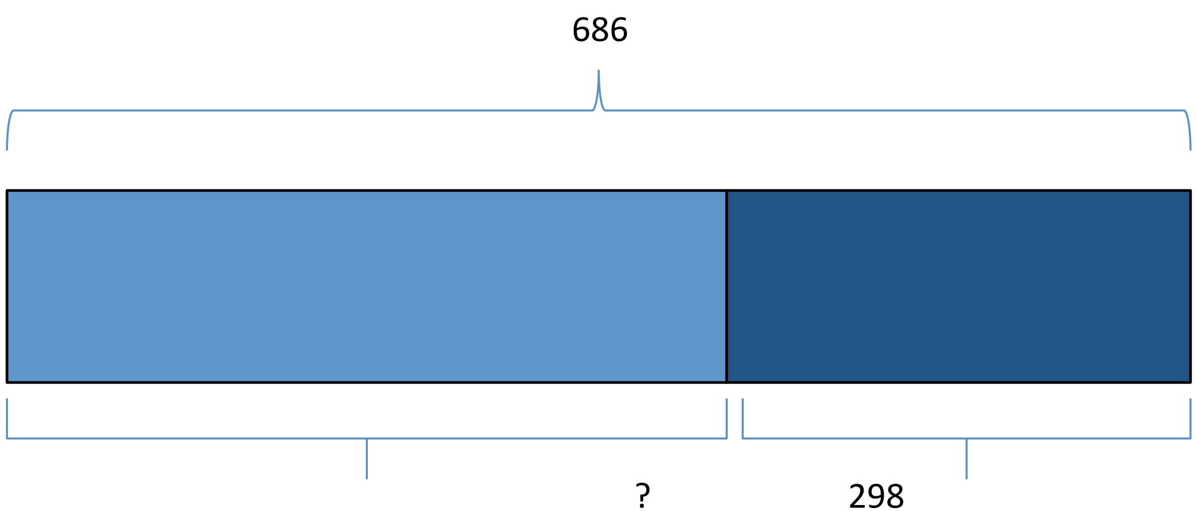
Part-Part-Whole Diagrams – Where does this lead?



Caldwell, Kobett & Karp (2014) *Essential understanding of addition and subtraction in practice, grades K-2*. NCTM.

Keeps on working!!!

- Mary made 686 biscuits. She sold some of them. If 298 were left over, how many biscuits did she sell?



$686 - x = 298$

Beckmann, S. (2004). Solving Algebra and Other Study Problems with Simple diagrams:. *The Mathematics Educator*, 14(1), 42.-46.

Common Multiplication and Division Situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Find A Place

Find-A-Place

- You will need to work in pairs.

Find-A-Place

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A							**Player B**
Hundreds	Tens	Units	Score		Score	Hundreds	Tens	Units
				0				
		7		10				
				50				

Find-A-Place

FIND A PLACE

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A						**Player B**		
Hundreds	Tens	Units	Score		Score	Hundreds	Tens	Units
				0				
		7		10				
				50			4	
				100				

Find-A-Place

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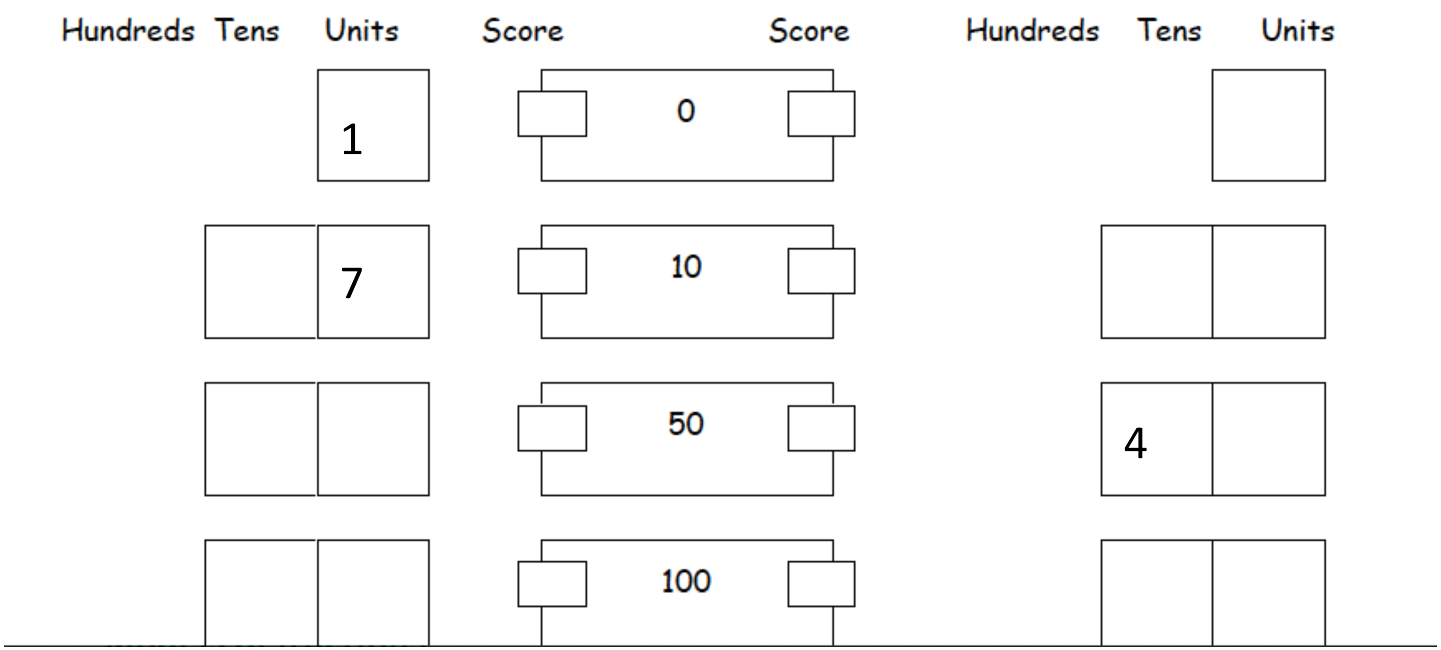

Find-A-Place

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Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A						Player B		
Hundreds	Tens	Units	Score		Score	Hundreds	Tens	Units
		1		0				
		7		10				
				50			4	
				100			8	

Find-A-Place

FIND A PLACE

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A | **Player B**

Hundreds	Tens	Units	Score		Score	Hundreds	Tens	Units
		1		0				0
	1	7		10			0	2
	5	5		50			4	8
	7	8		100			8	1

Equal Sign

Are students acquiring an appropriate understanding of the equal sign when you ask them to explain their thinking?

Are they comfortable using operations on both sides of the equal sign and can use the meaning of equal as "is the same as?"

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Equal Sign–Two Levels of Understanding

Operational: Students see the equal sign as signaling something they must "do" with the numbers such as "give me the answer."

Relational: Students see the equal sign as indicating two quantities are equivalent, they represent the same amount. More advanced relational thinking will lead to students generalizing rather than actually computing the individual amounts. They see the equal sign as relating to "greater than," "less than," and "not equal to."

Knuth, E. et. al (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, *13*, 514–519.

Why is understanding the equal sign important?

Table 1 Percent of students at each grade level who provided each type of equal sign definition as their best definition (n = 375)

Best Definition	Grade 6	Grade 7	Grade 8
Relational	29	36	46
Operational	58	52	45
Other	7	9	8
No response/ don't know	6	3	1

Knuth, E. et. al (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, *13*, 514–519.

Which number sentences would students say are True? False?

$27 = 27$

$22 + 5 = 4 + 23$

$25 + 1 = 27$

$27 = 22 + 5$

- Why?
- What would confuse them?

How is the equal sign interpreted?

Given the task:

$$8 + 4 = \square + 5$$

Hudson's Work

$$8 + 4 = \overset{17}{\cancel{12}} \quad + 5$$

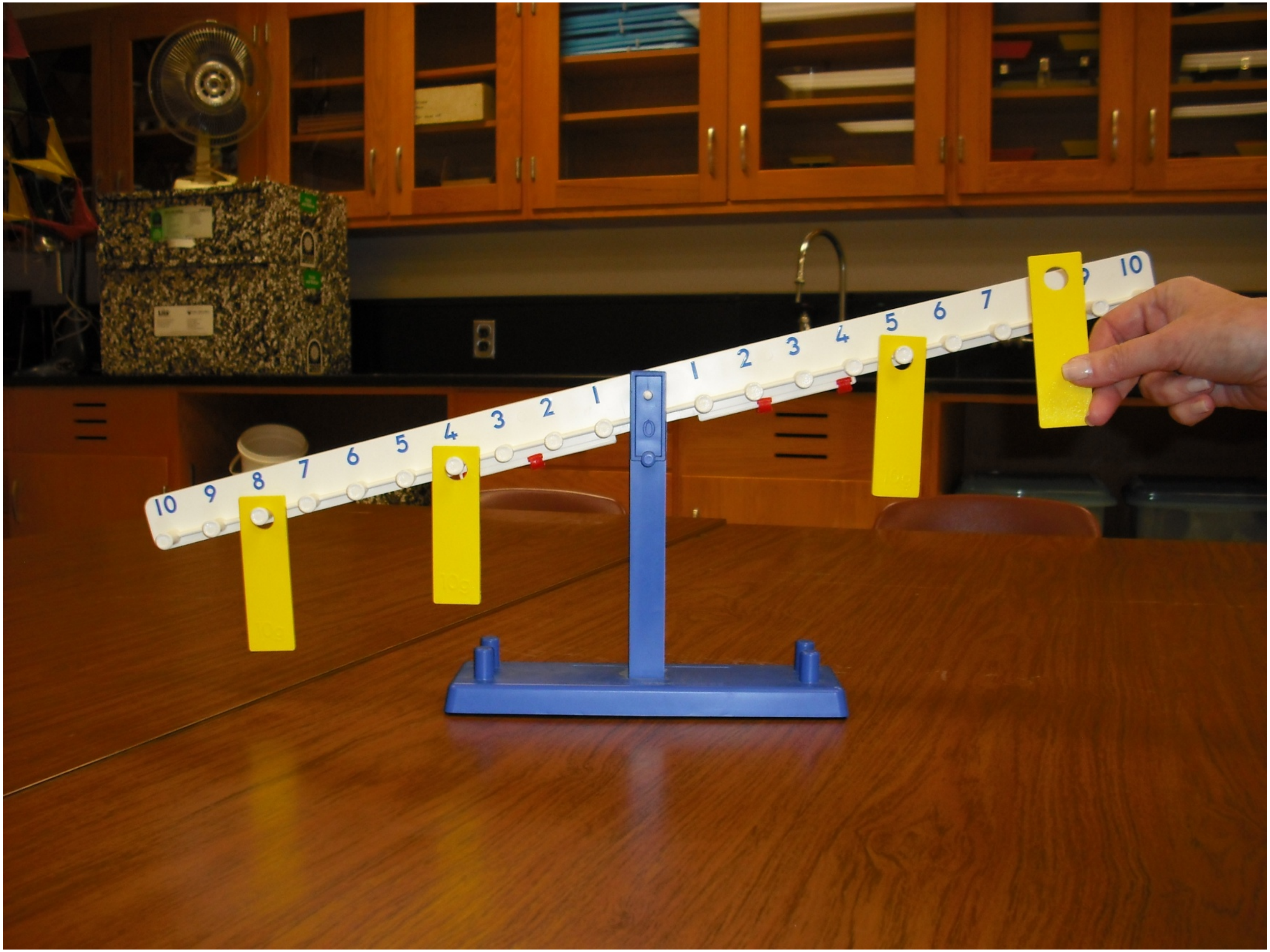
The new 2nd grade outlook

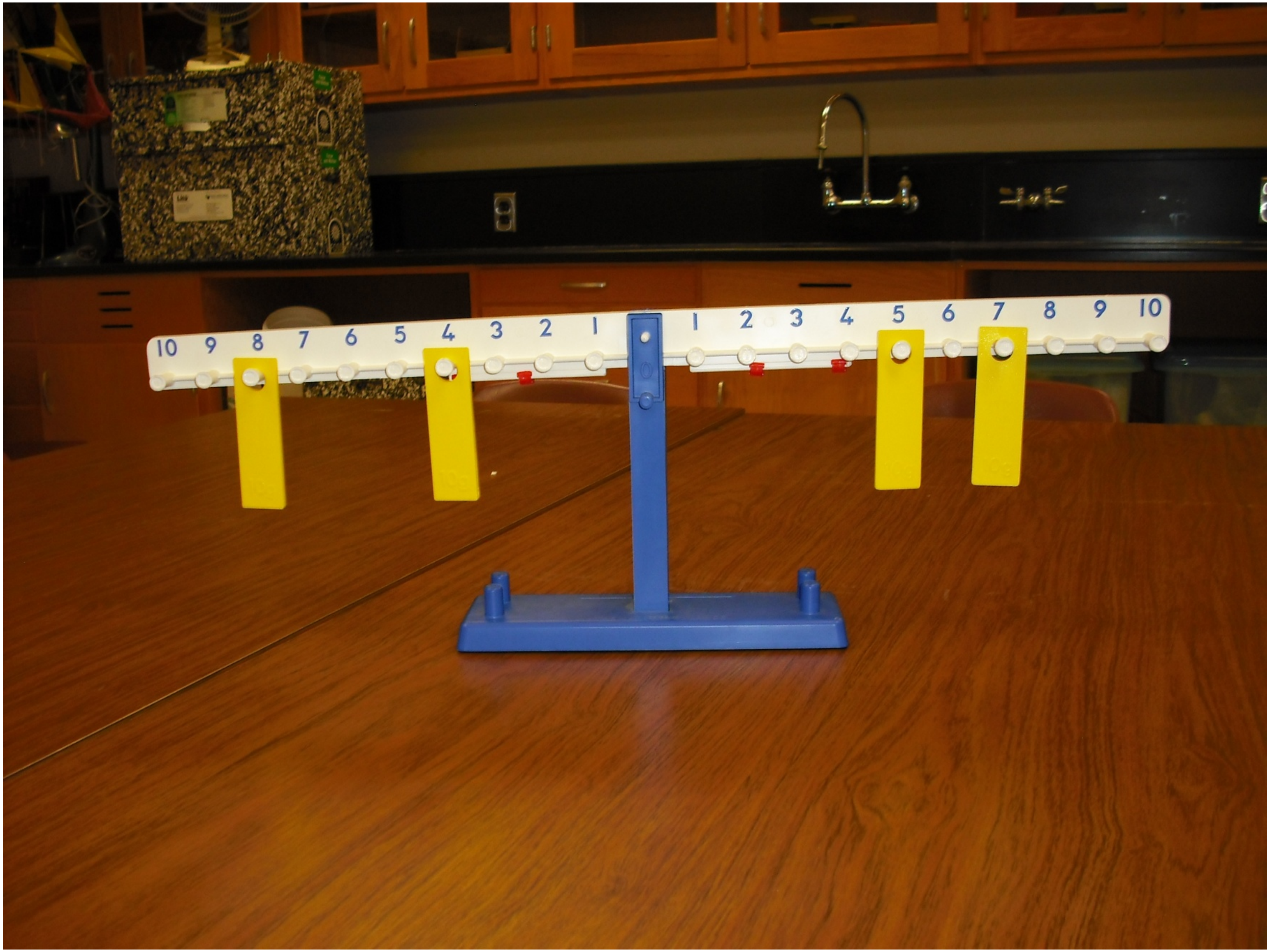
Hudson: Well, it's really kind of easy 'cause the answer comes after the equal sign, then you add the 5 later. This is easy math 'cause when you have numbers you just do whatever to them. Like add, subtract. And I think next year we learn how to multiply.

Nana: How do you know if your answer is correct?

Hudson (looking like why are you asking me that?): Oh, it's like every math thing. Your teacher tells you. As long as you show your work, she won't get mad.

Math is not magic.


10 9 8 7 6 5 4 3 2 1
0
1 2 3 4 5 6 7 9 10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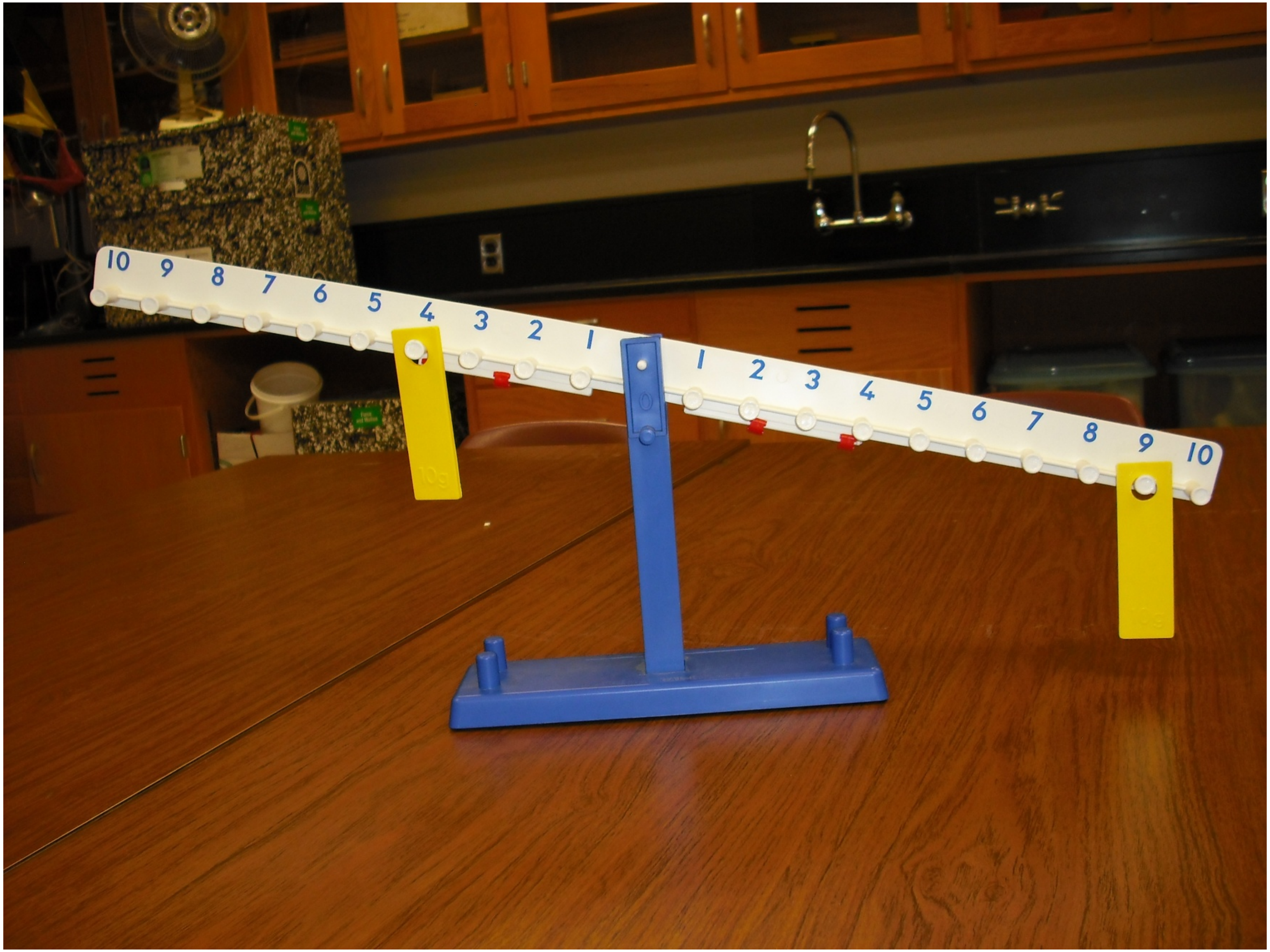
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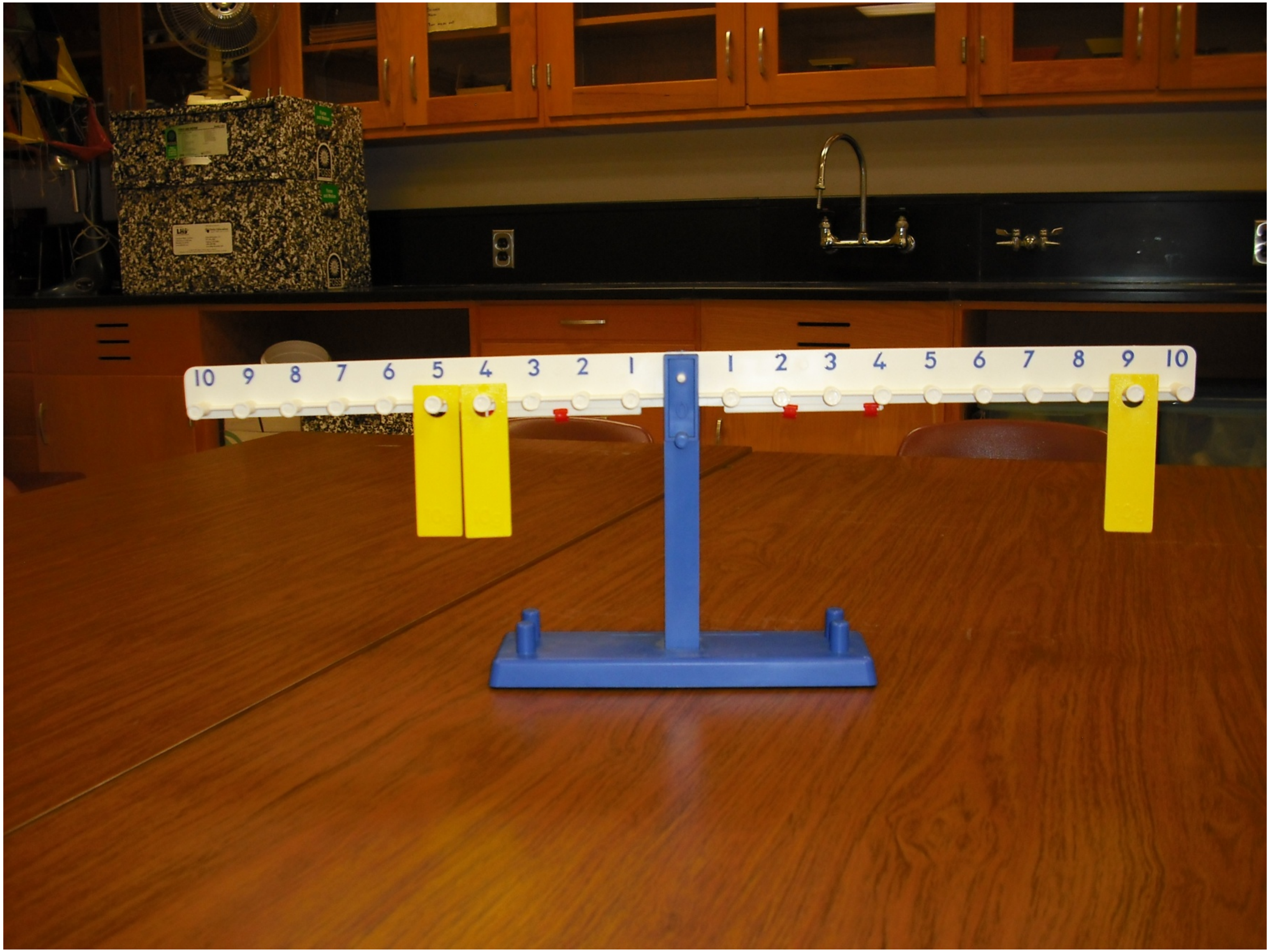

10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10

Talk about this with a partner

Place a weight on 10 on one side of the balance.

Make every combination using two weights on the other side.

How do you know you have all possible combinations?

How does this work?

How can you move one weight from the amount on each side of the balance to a new peg and maintain equality?

How does this work?

$3 \times 4 = 12$?

$2x + 3 = 17$?

Problem Solving: Application/Routine Problems

At all grades, students who struggle see each problem as a separate endeavor.

They focus on steps to follow rather than the behavior of the operations associated with the problem.

They tend to use guess and test – (disconnected thinking).

They need to focus on actions, representations, and general properties of the operations.

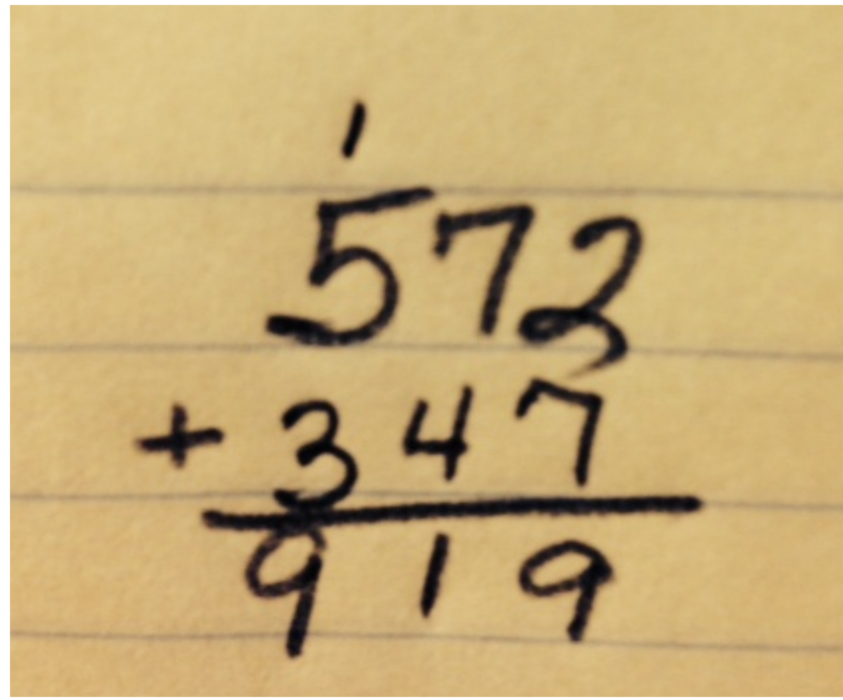
Number Development

Solve in at least three ways:

$$\begin{array}{r} 572 \\ +\ 347 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 862 \\ -\quad 295 \\ \hline \end{array}$$

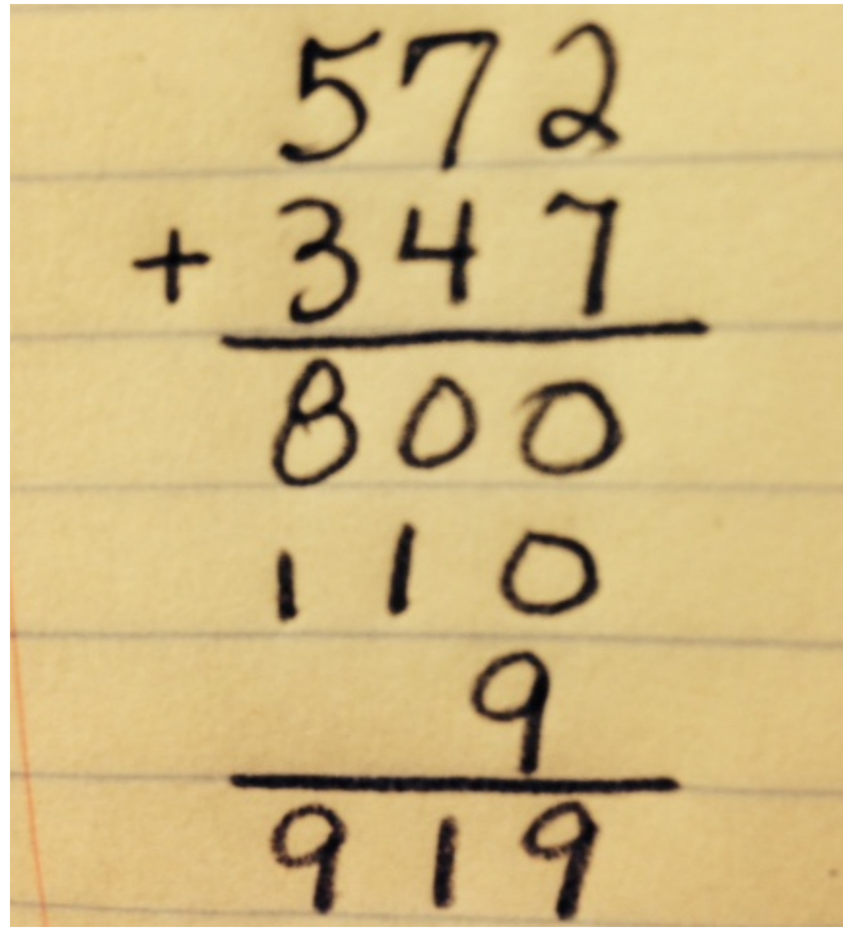
Algorithms

- Traditional
  - Does not preserve placement
  - Does not use number sense or intuition
  - Directionality issues



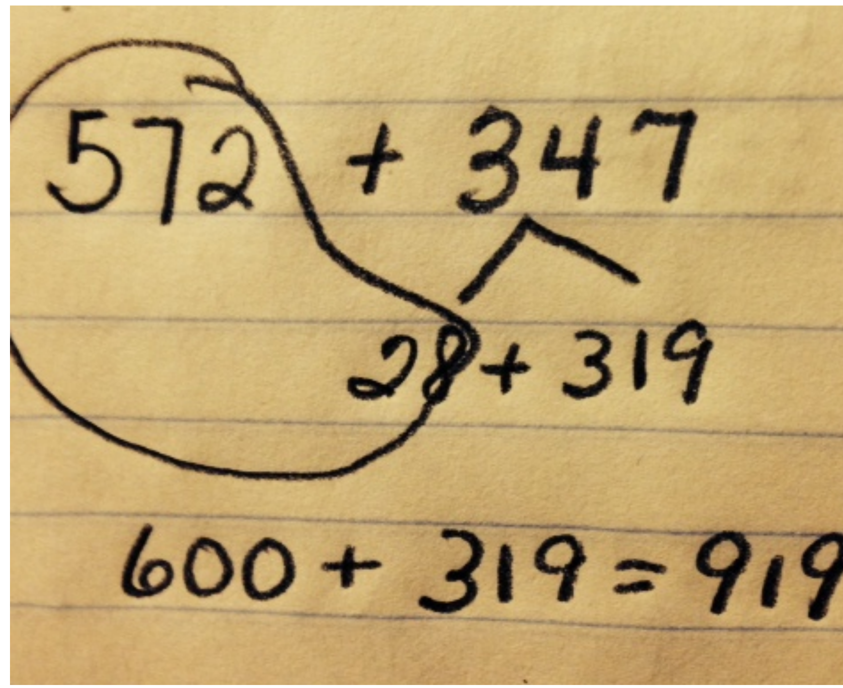
Algorithms

- Partial Sums
  - Preserves place value
  - Left-to-right orientation
  - Supports number sense



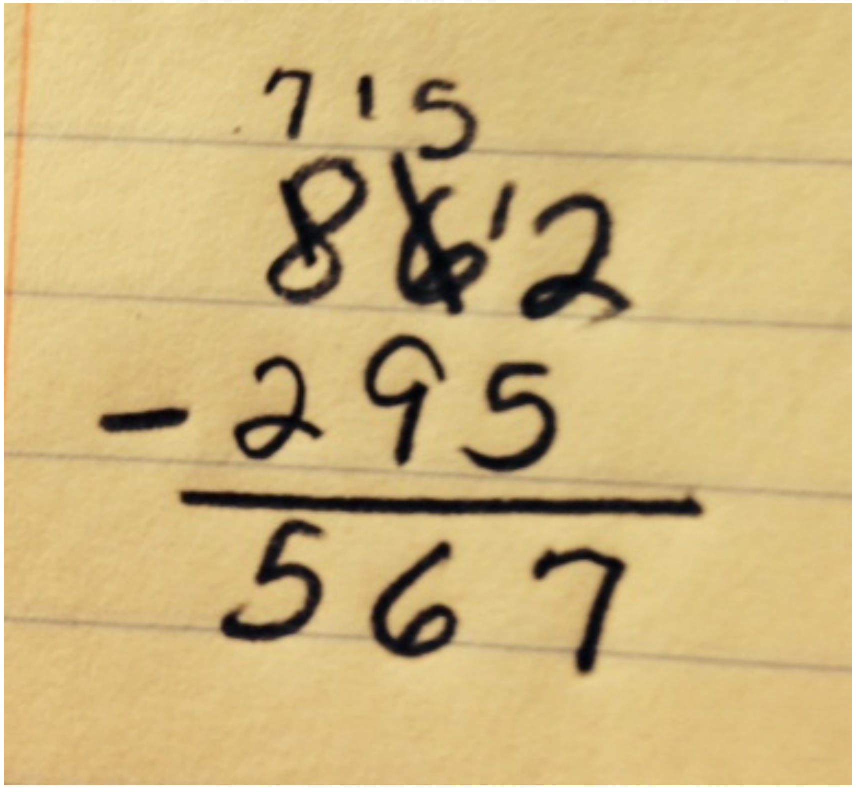
Algorithms

- Round-number strategy
  - Supports number flexibility
  - Uses decomposition
  - Allows for multiple representations



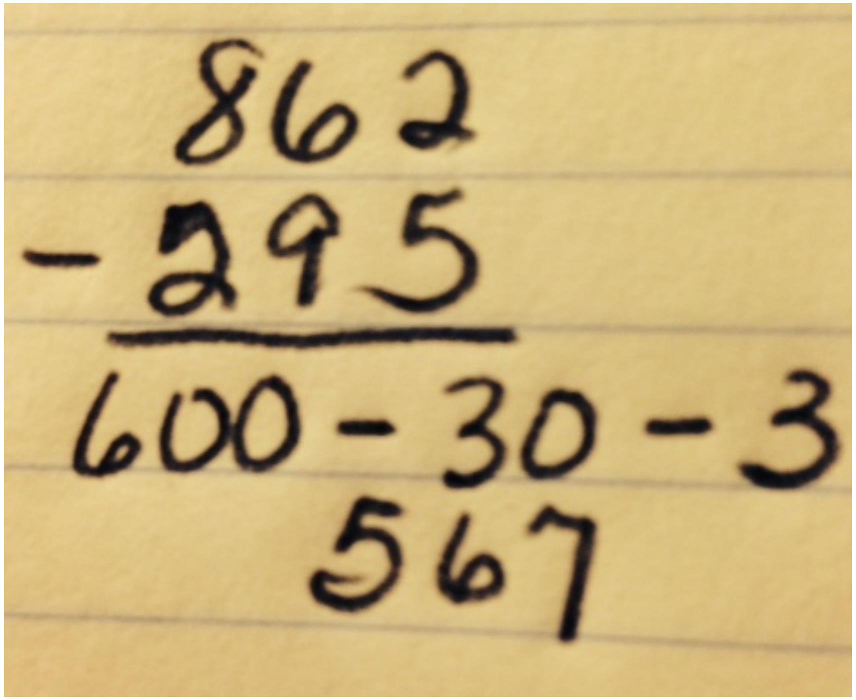
Algorithms

- Traditional
  - Same issues as with traditional algorithm for addition



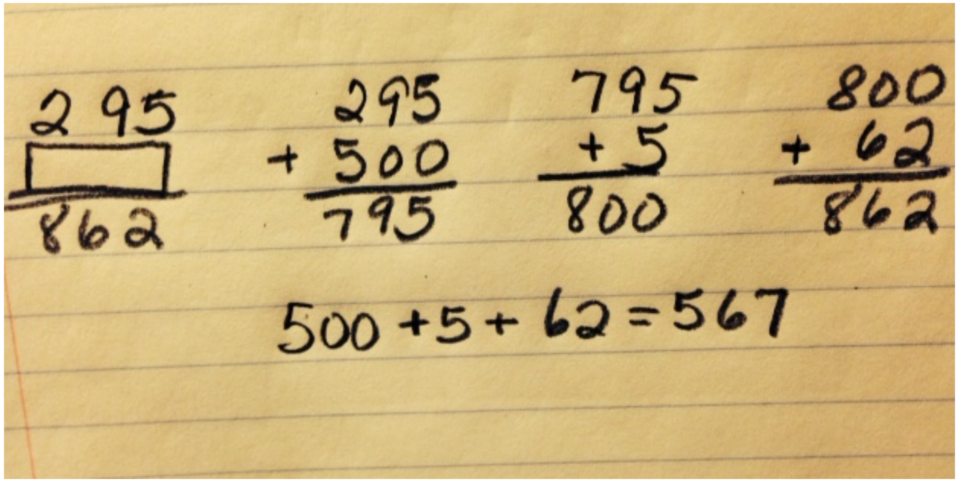
Algorithms

- Partial differences
  - Allows left-to-right direction
  - Supports the idea that you can subtract a smaller number from a larger number



Algorithms

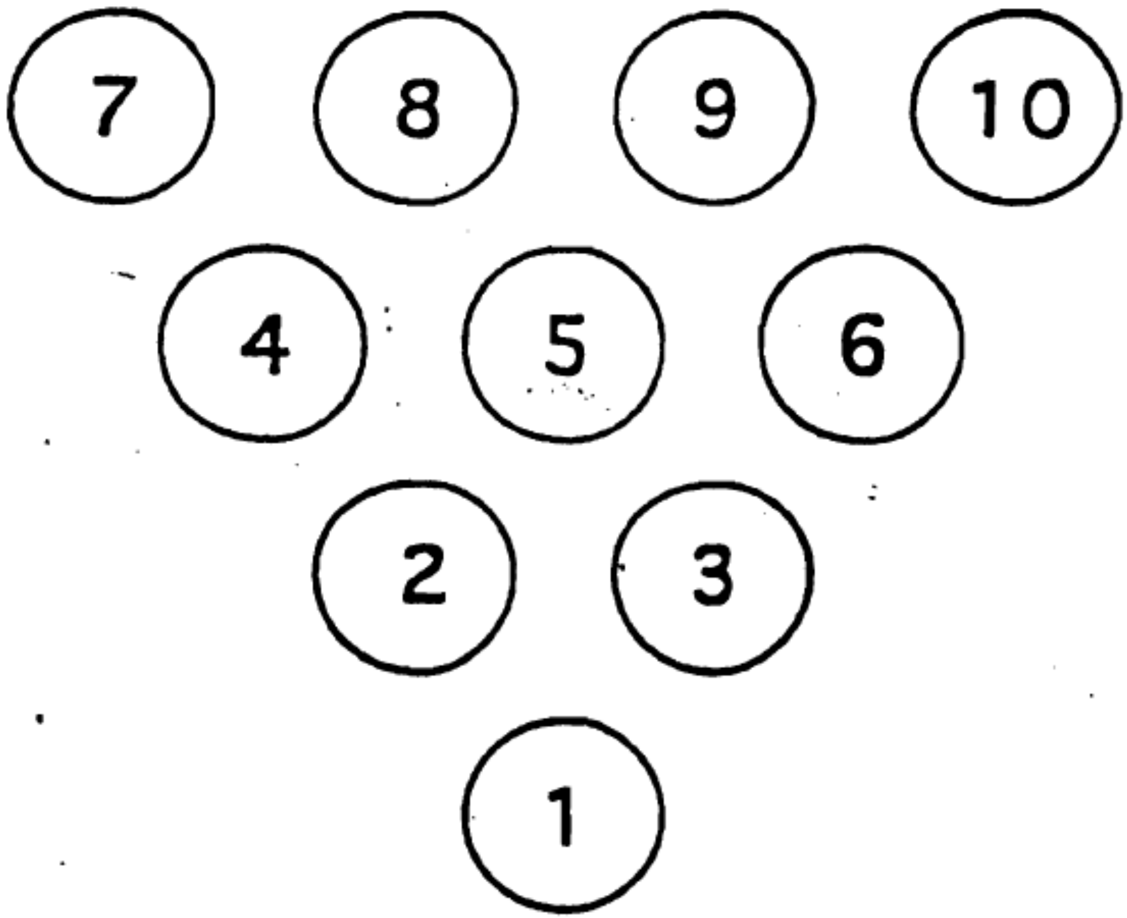
- Missing addend
  - Lessens likelihood of mistake
  - Supports mental computation, mental strategies, and number flexibility



Games to Support Number and Computations

- Make Mine 100 or Make Mine 0
- Bowl-A-Fact
- Salute

Bowl A Fact



Problem of Nine

Example based teaching

- Here's how you. . . . . . .
- Now you solve these.

- I do
- We do
- You do

Shifts in teaching and learning

Moving away from . . .	To. . .
Telling/showing how to do something	Building from concept to skill
Teacher-centric instruction	Student-centered instruction
Problem solving intermittently	Problem solving every day
A focus on only the answer	A focus on justifying and explaining
Showing the steps	Explaining the reasoning
Problems that require only fast calculations	Problems that require thinking

Rules

- With the person sitting next to or around you, decide if the rules shown are always true.
- If it is not always true, find a counterexample.
- Addition and multiplication make larger.
- When you multiply by 10, add 0 to the end of the number.
- Two negatives make a positive.
- The longer the number, the larger the number.

Addition and multiplication make larger.

$32 + 67 = 99$

$-3 + (-14) = -17$

–17 is not larger than
–3 or –14.

$15 \times 10 = 150$

$$\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$$

$0.25 \times .16 = 0.04$

Neither rational number
product is larger than
the factors.

When you multiply by 10, add a 0 at the end of the number.

$15 \times 10 = 150$

$4.5 \times 10 = 45.0$

$4.5 \times 10 \neq 4.50$

Two negatives make a positive.

$$-8 \times (-3) = 24$$

$$-8 + (-3) = -11$$

Addition of negative numbers depends on the absolute value of the integers to determine the sign of the sum.

The longer the number, the larger the number.

$1,278,931 > 1,469$

$1.3 > 1.0118743$

$1.02 < 1.2$

Goal – Try to AVOID DEAD ENDS

- "13 Rules that Expire" (Karp, Bush & Dougherty August 2014 in *Teaching Children Mathematics*) (blog too!)

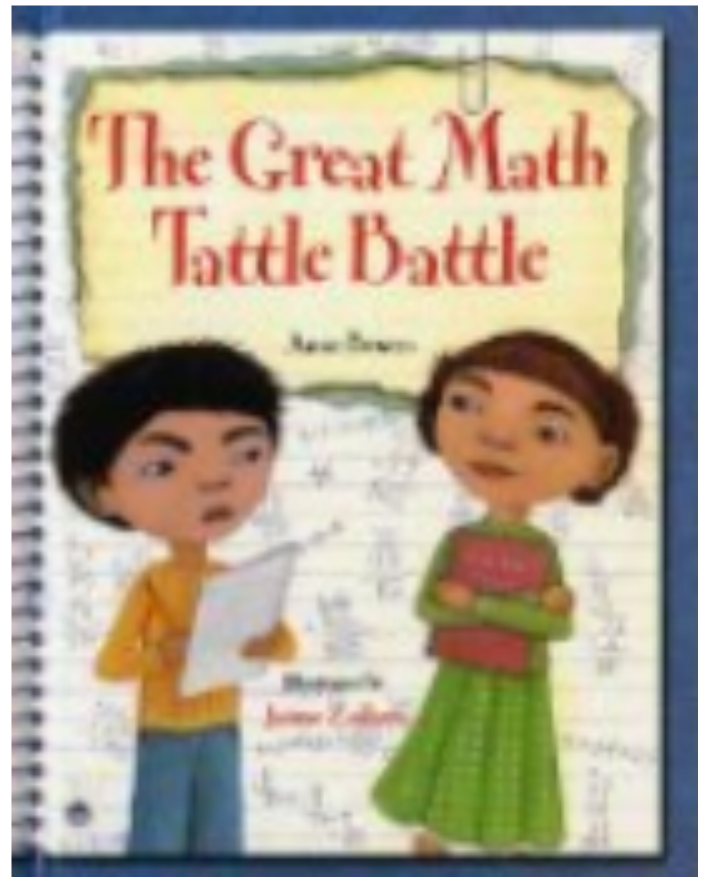
Impact of Rules

- Students use rules as they have interpreted them.
- They often do not think about the rule beyond its application.
- When even the best students find that a rule doesn’t work, it is unnerving and scary.

Infuse Literacy

Great Math Tattle Battle

by Anne Bowen



Harley Harrison

$$\begin{array}{r} 35 \\ +64 \\ \hline 98 \end{array} \qquad \begin{array}{r} 18 \\ +18 \\ \hline 216 \end{array} \qquad \begin{array}{r} 50 \\ -31 \\ \hline 21 \end{array}$$

$$\begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array} \qquad \begin{array}{r} 44 \\ -28 \\ \hline 24 \end{array} \qquad \begin{array}{r} 99 \\ +99 \\ \hline 188 \end{array}$$

Harley Harrison

$$\begin{array}{r} 35 \\ +64 \\ \hline \cancel{9}8 \end{array}$$

$$\begin{array}{r} 35 \\ +64 \\ \hline 99 \end{array}$$

he added this part wrong

$$\begin{array}{r} ^{1}\,18 \\ +18 \\ \hline \cancel{2}\cancel{16} \end{array}$$

$$\begin{array}{r} 18 \\ +18 \\ \hline 36 \end{array}$$

he forgot to regroup. The one from the 16 should go at the top.

$$\begin{array}{r} 4\,\cancel{5}\,\overset{10}{\cancel{0}} \\ -31 \\ \hline \cancel{21} \end{array}$$

he switched those around

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{10}{\cancel{0}} \\ -31 \\ \hline 19 \end{array}$$

he forgot to regroup.

$$\begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array}$$

$$\begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array}$$

he got the write answer, but forgot to regroup

$$\begin{array}{r} 3\,\cancel{4}\,14 \\ -28 \\ \hline \cancel{2}4 \end{array}$$

he forgot to regroup

$$\begin{array}{r} \overset{3}{\cancel{4}}\,{}^{1}4 \\ -28 \\ \hline 16 \end{array}$$

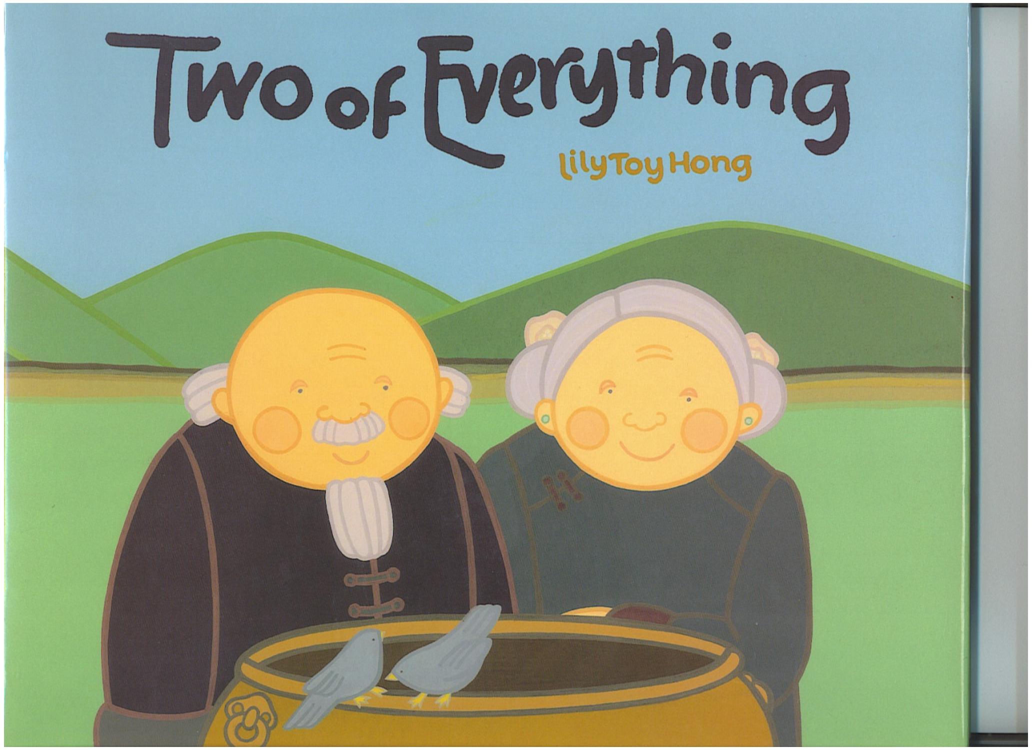
$$\begin{array}{r} ^{1}\,99 \\ +99 \\ \hline \cancel{18}8 \end{array}$$

he fogot to regroup

$$\begin{array}{r} 99 \\ +99 \\ \hline 198 \end{array}$$

Two of Everything – Lily Toy Hong

- Tell the story of Mr. Haktak's magical doubling pot – for all grades



In pot – and Out pot

Fraction Models

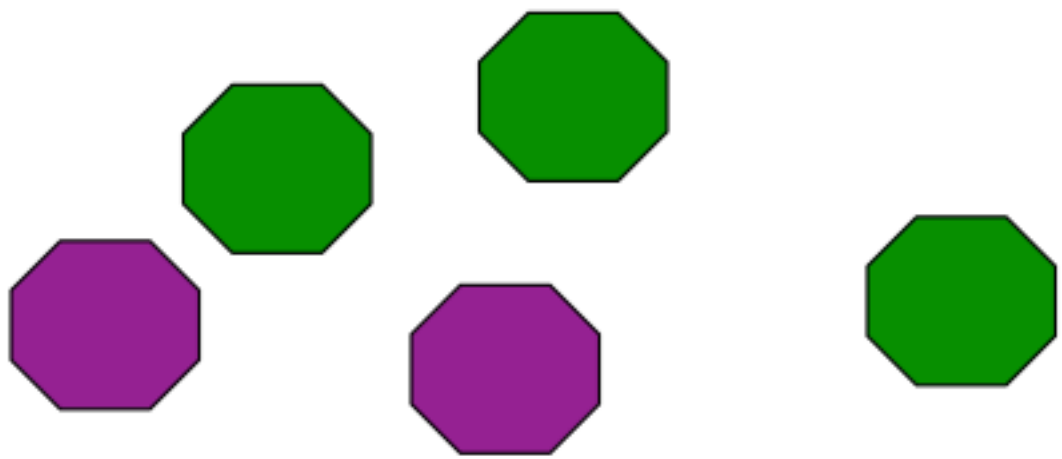
Continuous models for fractions:

Continuous models could be related to length, area, volume or mass.

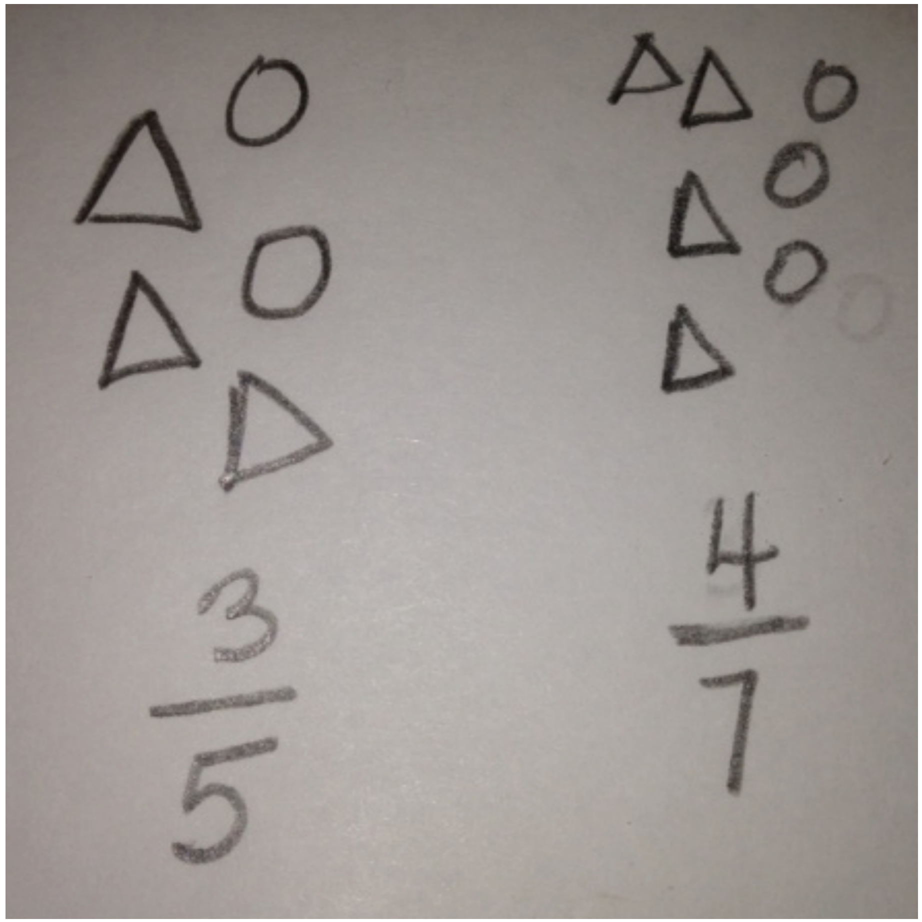
Fraction Models

Discrete models:

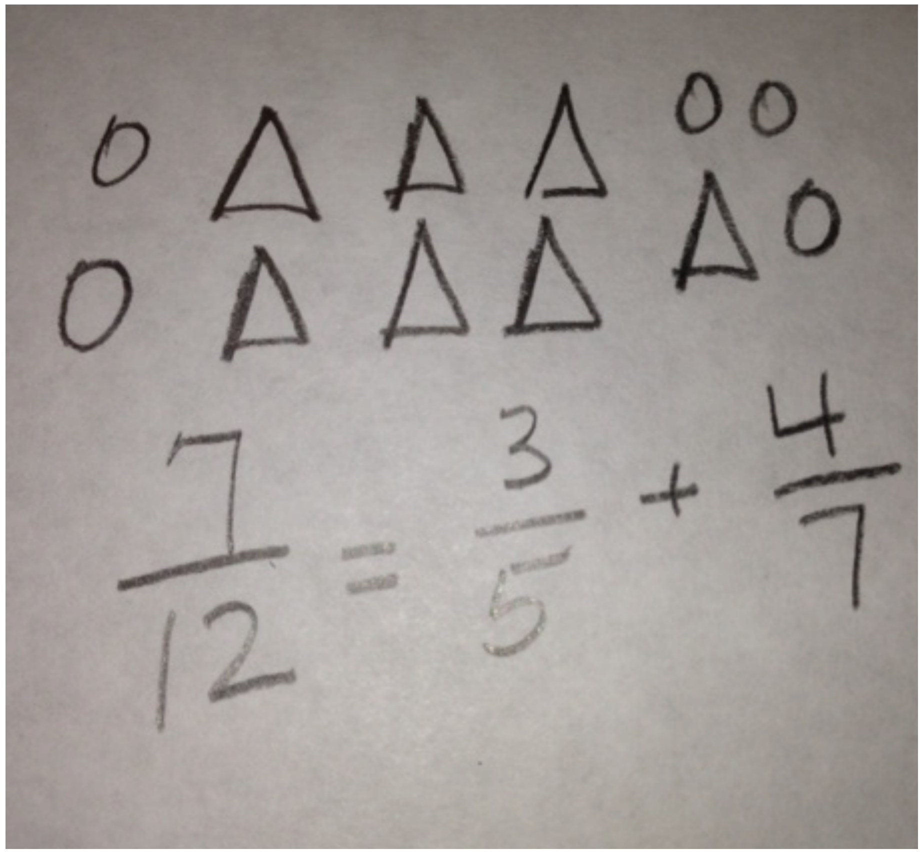
Discrete models are typically sets of objects.



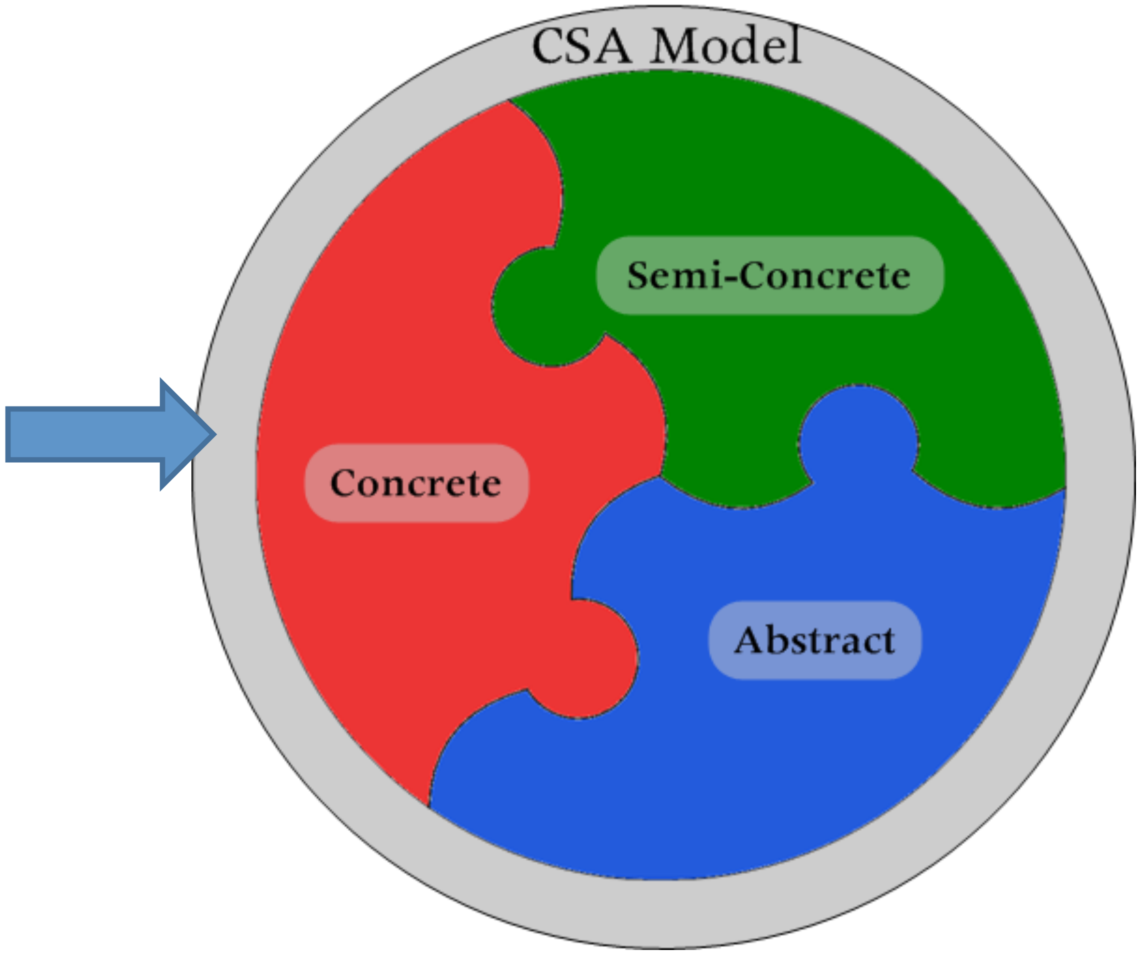
Fraction Model Problem



Fraction Model Problem: PROOF!



CSA: Concrete—Semi-Concrete—Abstract



Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Intervention Recommendations from Research

- **C**oncrete—**S**emi-concrete—**A**bstract (CSA) representations
- Explicit instruction (not direct instruction)
- Underlying mathematical structures
- Examples (and counterexamples)
- Feedback – Not only teacher to student but students' feedback to other students and teacher on what they know and don't know

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* New York: Routledge.

CSA: Concrete—Semi-Concrete—Abstract

- Does not suggest a linear sequencing of representations
- Use representations concurrently
- Links representations, leading to the ability to working eventually with abstract symbols

Create Mental Residues

- Establishes foundational understanding
- Models the physical action is the important
- Does not fade away or disappear
- Supports their thinking about the operation

Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahweh, NJ: Erlbaum.

Characteristics of learning

- Introduce every topic with problem solving
- Every lesson includes five forms of communication
  - Reading
  - Speaking
  - Critical listening
  - Writing
  - Multiple representations
- Topics are connected
- Students have 8–15 days to move a concept to a skill
- Challenging problems for all students

Change teaching to change learning

- When concepts and skills are taught through problem solving, students learn deeper without reteaching. (Schoen & Charles, 2003)

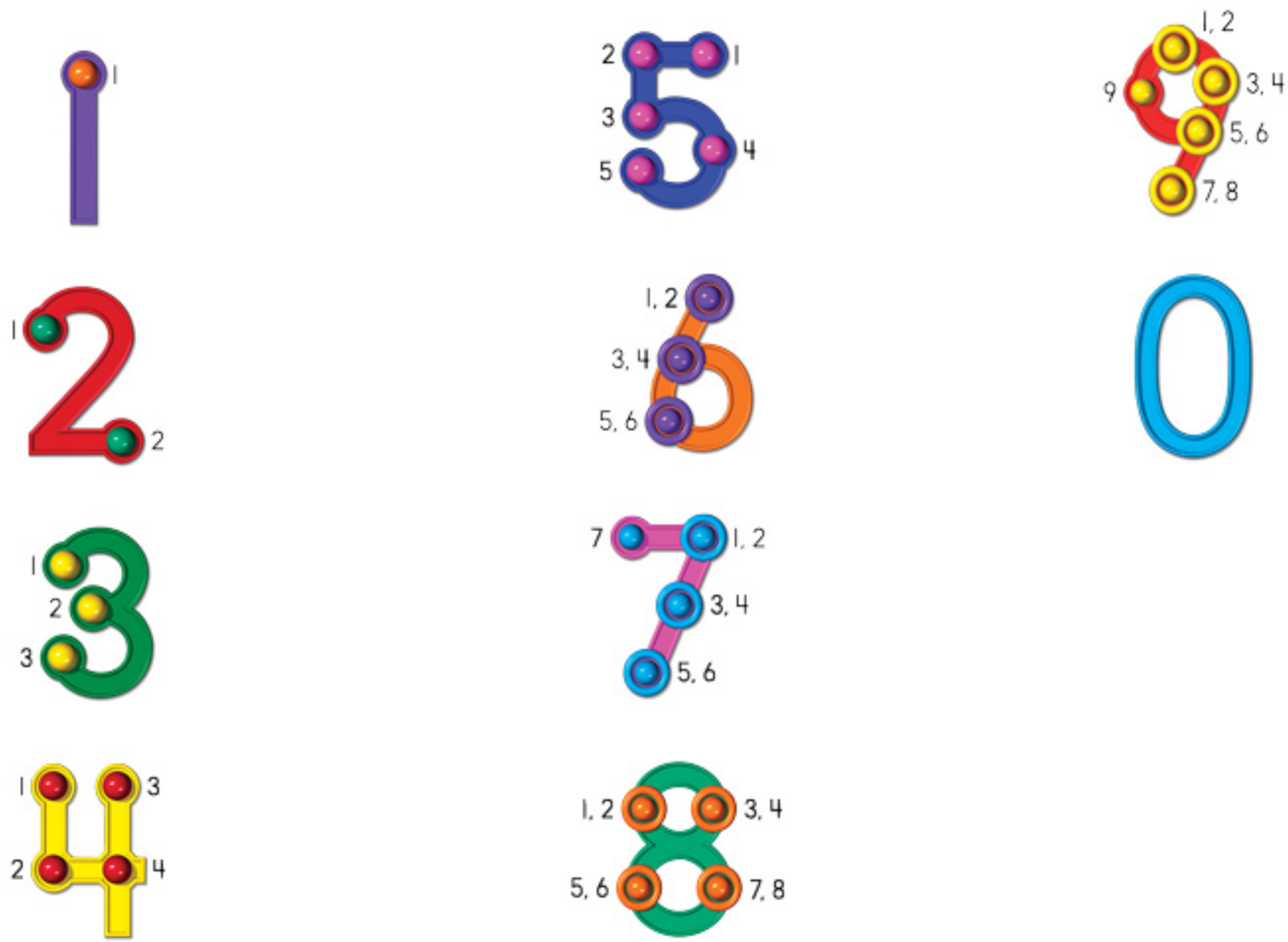
Change teaching to change learning

- Significant amounts of practice with repetition of a skill may not promote stronger learning in the long term. (Dougherty & Foegen, 2011)

Curriculum Materials

- Curriculum materials matter.
- Not all curriculum materials are created equal.

Touch Math



Curriculum materials

239 - 248	796 + 718	191 + 770
782 + 482	479 + 783	460 + 360
139 - 208	713 + 667	235 + 291

- When students practice only a skill it places a large cognitive demand on their memory.

Curriculum materials

- When the materials break skills down into small pieces, it requires students to put the pieces together to form the whole.

Intervention Materials

Two recent studies revealed that teachers providing Tier 2 mathematics instruction to elementary and middle grades students largely used worksheets (Foegen & Dougherty, 2010; Swanson, Solis, Ciullo & McKenna, 2012)

One-size-fits-all computer programs

What is explicit instruction?

- Focusing students attention on particular structures or ideas
  - Asking questions so that students ‘see’ the mathematics
  - Providing tasks that allow students to explore the topic

What does it look like?

- Teacher introduces a problem that links to previous learning.
- Students work in pairs or small groups to solve.
- Students share their thinking with the class, critiqued by others and teacher.
- Teacher scaffolds tasks based on misconceptions that are evident in thinking.

Explicit Instruction

- Try to elicit the information from students (see Never Say Anything a Kid Can Say, *Mathematics Teaching in the Middle School*)

-Developing Concepts and Generalizations to Build Algebraic Thinking: The Reflectivity, Flexibility, and Generalization Approach

Dougherty, Bryant, Bryant, Darrough, & Pfannenstiel, *Intervention in School and Clinic*, May 2015 projected publication.

Food for Thought

- Critical thinking questions should be asked in every class, every day
- Consistency helps students understand the expectations and move toward higher proficiency

Can I be excused? My brain is full.

Closing the Gap

- Changing the way tasks are posed
- Creating high expectations and accountability

Questioning Techniques

- Factual questions comprise the majority of questions asked in a mathematics class
  - More than 145 questions in 48 minute class period
  - Less than 2 seconds for response

Dougherty & Foegen, 2010

Traditional Tasks

$$\begin{array}{r} 458 \\ \underline{+\ 397} \end{array}$$

Change the Task

Reversibility question:

- Find 2 three-digit numbers whose sum is 855
- Find another pair
- Find 3 more pairs

Change the Task

Generalization question:

- What is the maximum number of digits you can get in the sum when you add 2 three-digit numbers? Why?
- What is the minimum number of digits you can get in the sum when you add 2 three-digit numbers? Why?

Change the Task

Flexibility question:

- Add 458 and 397 in two different ways.
- How are the ways you added them alike?
- How are they different?

Change the Task

Flexibility question

Add:

458 + 397 = ?

463 + 397 = ?

463 + 407 = ?

Questions to Promote Problem Solving and Generalizations

- Reversibility questions:
  - Promote the ability to think in different ways
  - Give the answer, students create the problem

Patterns: Times 2 Problem
Grade 4

Sophie wrote the following equations:

$$14 = 7 \times 2$$
$$12 = 6 \times 2$$
$$10 = 5 \times 2$$
$$8 = 4 \times 2$$

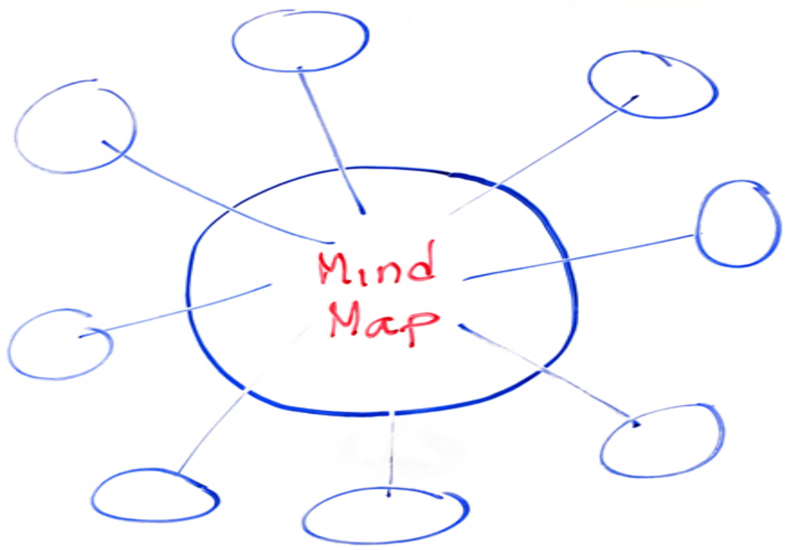
She said, “I notice something.” What do you think Sophie noticed?

Times 2 Problem

Generalizations:

- Multiplying by 2 results in an even product.
- When one factor (2) stays the same, and the other decreases by 1, the product decreases by 2.
- Multiplying by 2 gives a product equal to the sum if you add a number to itself.

What are general classroom strategies?



Routines

- Response routines
  - Think-pair-share
  - Pair think/response
  - Random calling
  - Unison response
  - Display student work
    - Whiteboard or other display of answers

May I be excused? My brain is full.

Routines

- Response routines
  - Wait time
  - Give students time to think (3 – 5 seconds?)

Student responses

- If discussing, honor student responses by writing them
- Focuses students' attention

Feedback

- Self monitoring
- Language-based (hearing one's thinking, metacognition)

Feedback

- Self monitoring
- Language approach

Assessment

Assessment

- Assessments should measure more than skill.
- Assessments should be sensitive to conceptual development and to show student growth.

Diagnostic Interviews

Diagnostic Interviews – Knowing students' thinking

- Select tasks that are close to what you expect the children to be able to do – start easy and ramp up if you are not sure
- Ask questions to prompt why they are doing something – or to ask them to explain their thinking or use models or drawings to demonstrate their ideas
- Be neutral – watch your body language - avoid clues or leading questions
- Wait silently - Do not interrupt – Do not teach
- Take notes, keep student work, take photos
- Say – show me, tell me, do…, try…, can you do that another way?

Diagnostic Interviews

- Collect **in-depth** information about an individual student's knowledge and mental strategies.
- Provide evidence of **students' prior knowledge, naïve understandings** and **ways of thinking**
- Focus on a task/problem where students are asked to verbalize their thinking and/or demonstrate ideas through **multiple representations**
- Is **not a teaching opportunity**
- Use errors to **identify barriers to understanding** and to **inform instructional decisions**

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Diagnostic Interview Task

Ask the student to write the number that shows:

3 ones, 1 hundred and 5 tens

Student Work

3 ones 1 hundred 5 tens

315

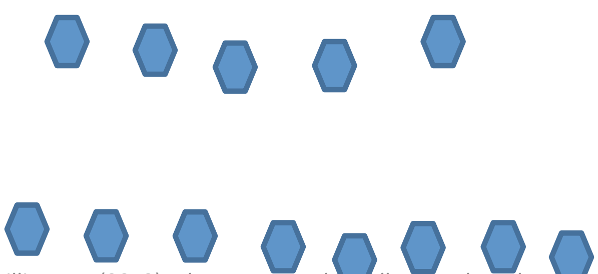
153

Link Sheet

Equation	Word Problem
Model/Illustration	**Explanation**

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Link Sheet – Student work

Equation	Word Problem
5 + 8 = 13	**There were 5 birds on the feeder. Then 8 more birds flew to the feeder. How many birds are now on the feeder?**
Model/Illustration	**Explanation**
	**First I had 5 birds. Then I needed to combine them with 8 more birds. I added so all together I have 13 birds.**

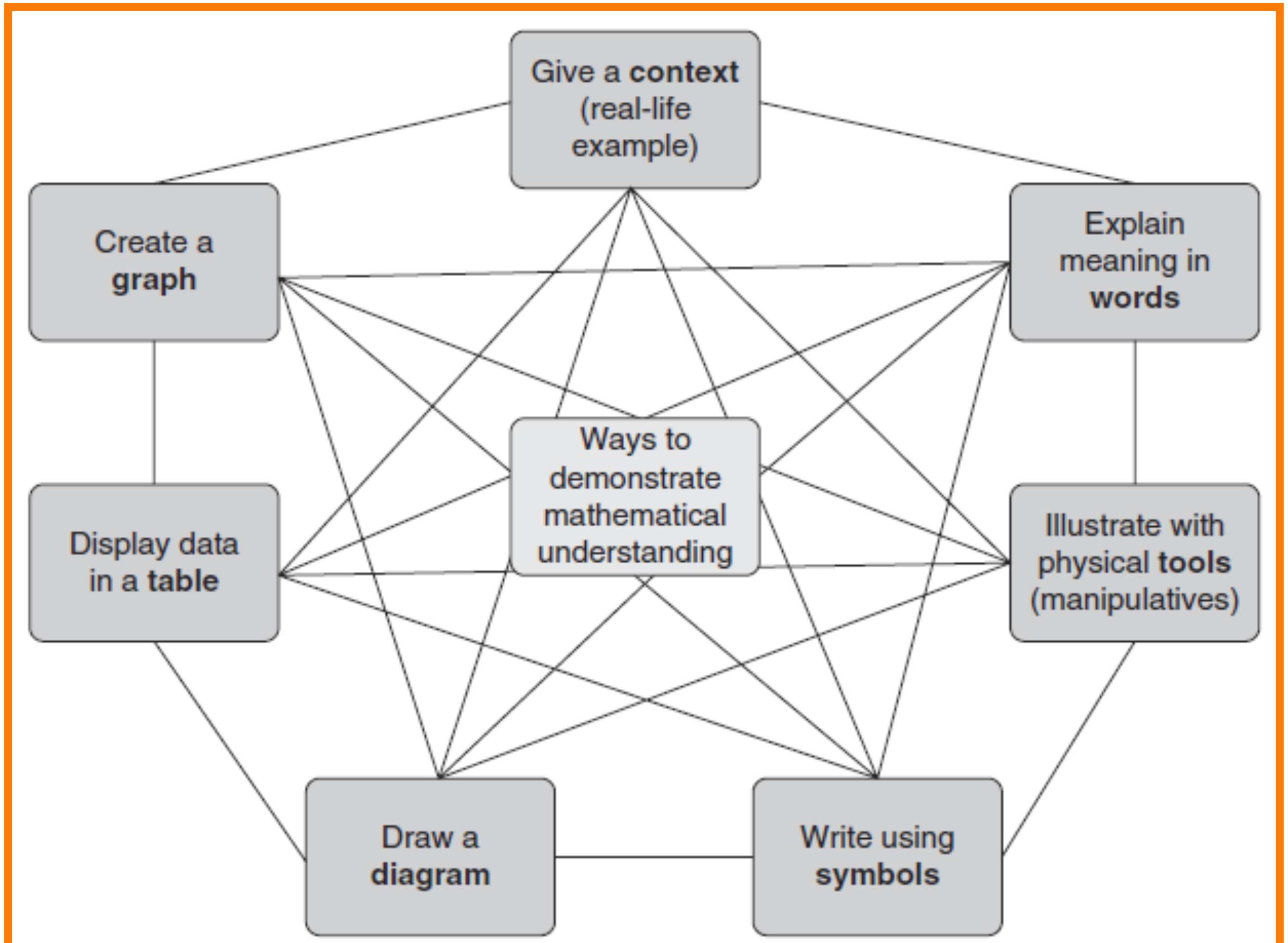
an de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

At the Middle School Level

Algebraically	**Graphically**
Numerically in Tables	**Verbal Description**

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Multiple Representations



From *Teaching Student Centered Mathematics* (2014) (Volumes I - III)

What do ALL students need?

- Understand the mathematics (why it works, the big ideas, the underpinnings)
- Do computations and processes with automaticity
- Apply the mathematics in different contexts

Shifts in Thinking

- Teacher talking and doing TO students talking and doing
- Using key words TO student understanding with mathematical communication
- Learning rules and algorithms TO struggling with rich, high-quality problems

Move toward. . .

- Start with problem solving (a rich, engaging task, not naked numbers)
- Solve it with pairs or small groups
- Discuss the solution processes, links with other problems, extensions
- Interact with teacher who facilitates with questions that explicitly identify important ideas
- Practice

How can we find a place to start?

Have students:

– Develop images and representations in story contexts to understand the behaviors of the operations

– Link to the big and LONG lasting ideas

– Avoid the use of poorly understood rules and procedures when faced with a novel problem

– Engage in “doing mathematics”

Reducing Resistance, Developing Responsibility and Building Resilience

- Give students with disabilities **choices** and capitalize on their **unique strengths**
- Nurture traits of **resilience** – rather than learned helplessness
- Demonstrate an ethic of **caring**
- Make mathematics **irresistible**
- Give students with disabilities some **leadership** in their own learning

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

What do all schools need?

- To decide on the language and models everyone will use – be precise and consistent
- To think about the level of teaching – are you teaching to help students perform at the highest level?
- To get kids "doing mathematics" so they can build mental residue and long lasting understanding

Don't Forget the Importance of Formative Assessment and Asking Questions

- I realize how valuable a well-designed, research-based assessment can be in finding evidence of student understanding. Also how this awareness of children's thinking helped me decide what they (students) actually knew versus what I thought they knew.

A teacher from the Vermont Mathematics Partnership as quoted in Petit & Zawojewski, 2010

Petit, M. Zawojewski, J.. (2010). Formative assessment in elementary school mathematics. In D. Lambdin & F. K Lester, Jr. (Eds.) *Teaching and learning mathematics: Translating research for elementary school teachers*. (pp. 73-79). Reston, VA: NCTM.

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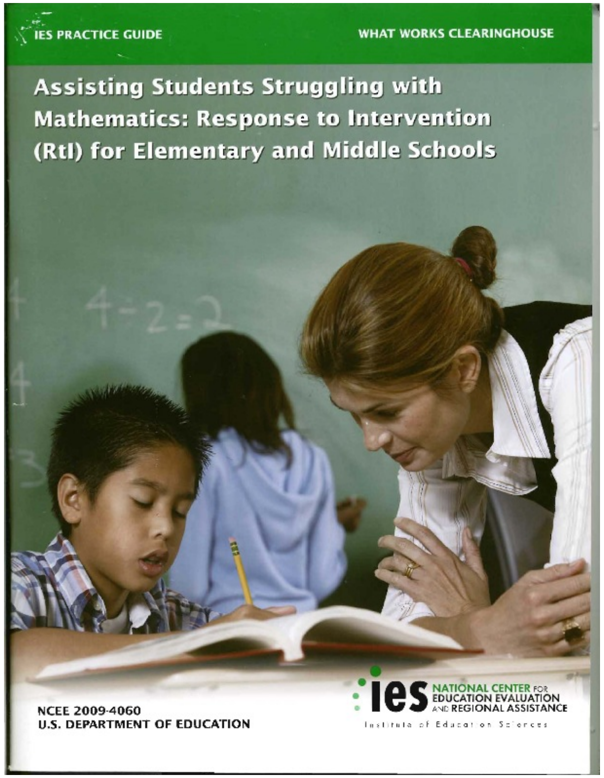
A Concluding Thought

We expect that the very best doctors will treat the most grievously ill patients. It should be no different in education. Great teachers have the skills to help the students who struggle the most. (Larson, 2011)

Larson, M. (2011).. Supporting students' achievement of the Common Core. NCTM Conference presentation. Indianapolis, IN.

Recommendations for identifying and supporting students struggling in mathematics

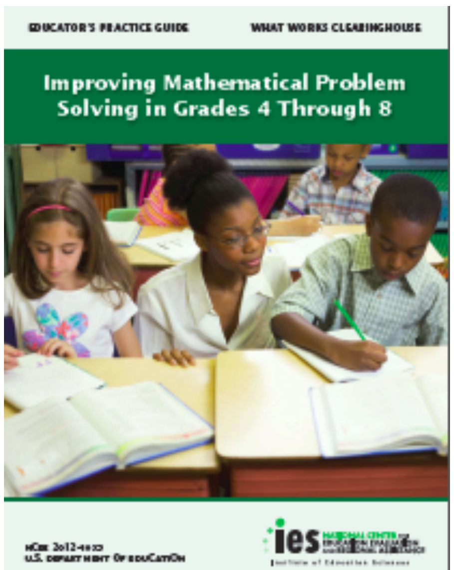
- Recommendations are based on strong and moderate levels of evidence resulting from comprehensive reviews of current research literature.

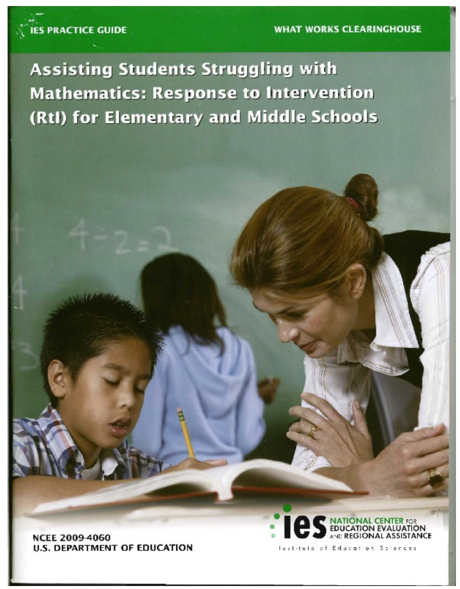


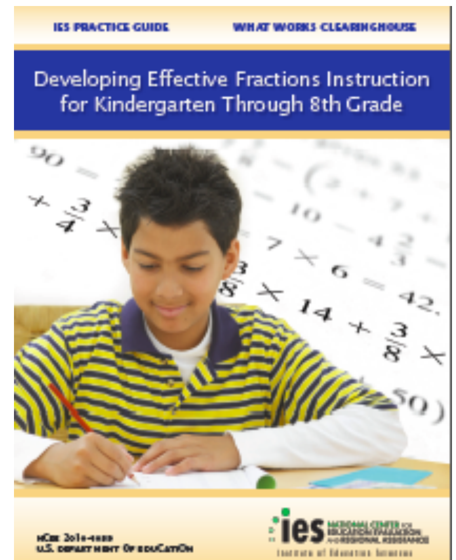
http://ies. ed.gov/ncee/wwc/publications/practiceguides/

Recommendations for identifying and supporting students struggling in mathematics

- Based on strong and moderate levels of evidence resulting from comprehensive reviews of current research

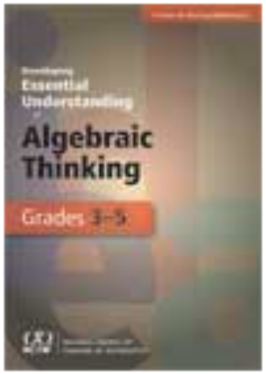


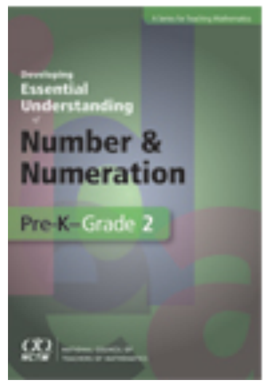


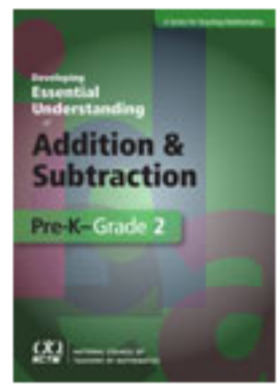


Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC:* National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies. ed.gov/ncee/wwc/publications/practiceguides/.

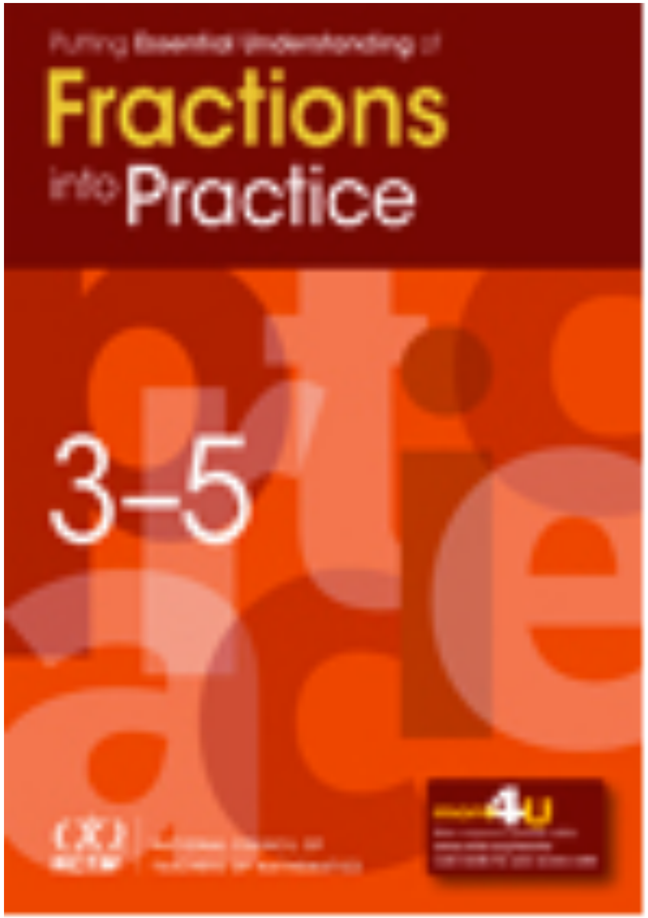
NCTM Resources: Developing Essential Understandings





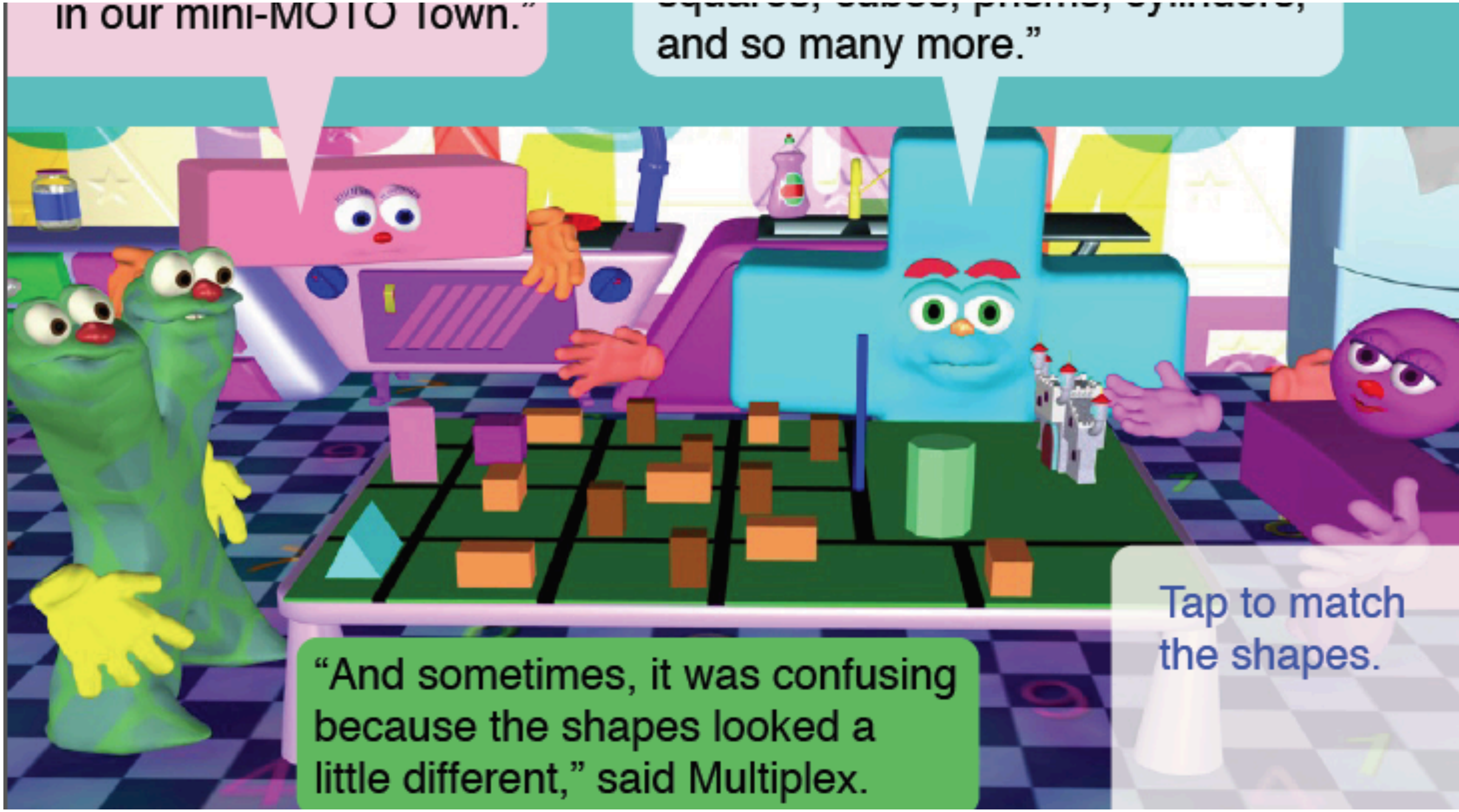


NCTM Resources: Putting Essential Understandings Into Practice



MOTO

MOTO


in our mini-MOTO Town.”
and so many more.”
“And sometimes, it was confusing because the shapes looked a little different,” said Multiplex.
Tap to match the shapes.

RtI Preconference 2015

- NCTM Annual Conference, April 15, 2015, Boston, MA

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