

Pillars & Practices

A Framework to Catalyze
Change at the Margins

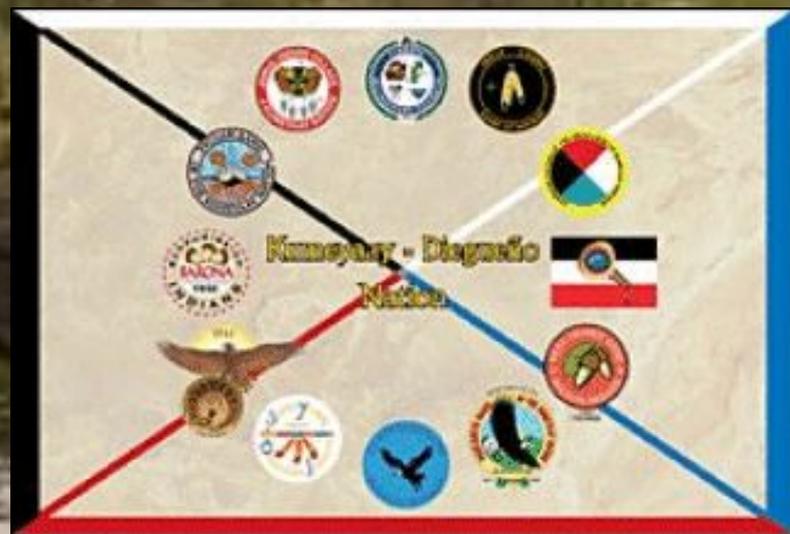
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#CatalyzingChange

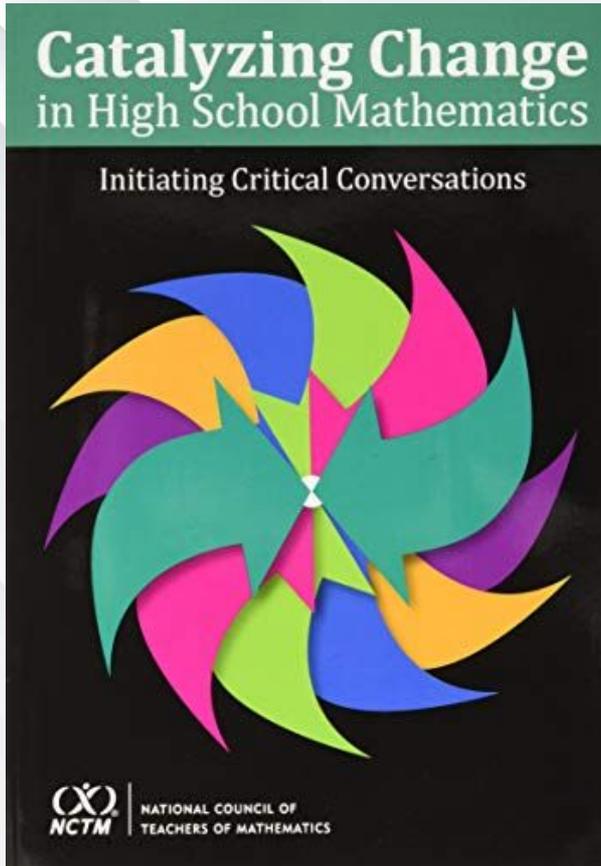
#Pillars_Practices

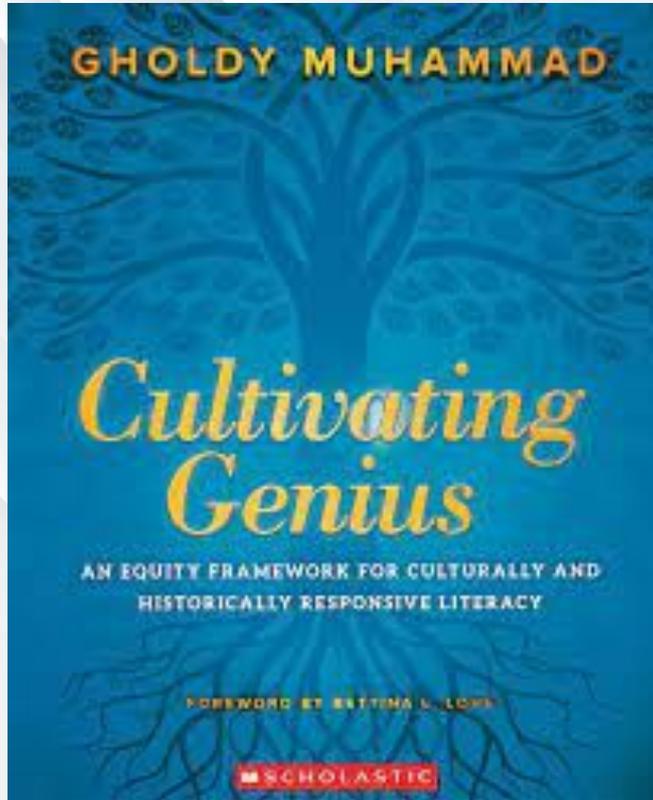


Implementing Equitable Instruction

“**Classroom instruction** should be consistent with **research-informed** and **equitable** teaching practices.”

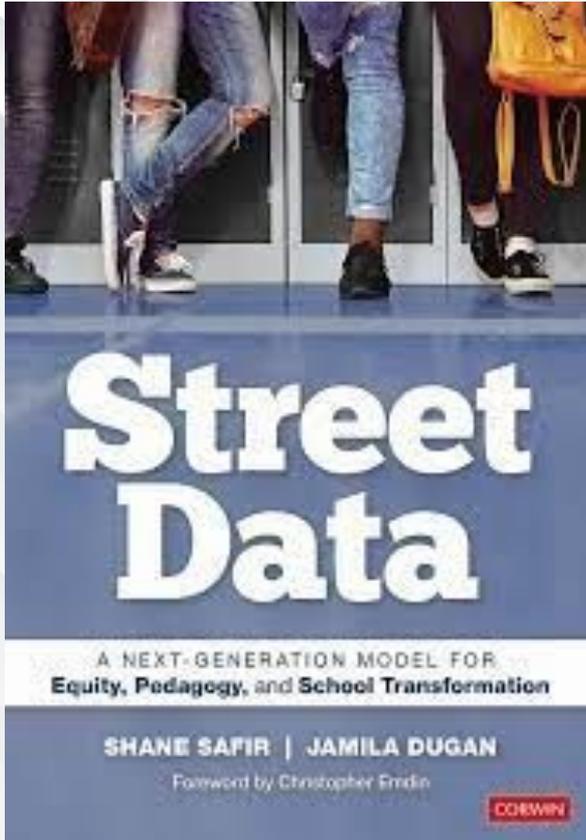
–page 25





Gholdy Muhammad

“Our Black students are not failing; it is the systems, instruction, and standards created to monitor, control, and measure a very narrow definition of achievement that are off the mark.”



Shane Safir & Jamila Dugan

“For all our talk of being student centered, we have bought into a success paradigm that robs many children of their voices, marginalizes their gifts, and prioritizes measurement and incremental improvement over learning and transformation.”

**From Argumentation
to Truth-Telling:
Critical Race Theory
in Mathematics
Teacher Education**

Mathematics
**Teacher
Educator**

a joint publication of
the National Council of Teachers of Mathematics
and the Association of Mathematics Teacher Educators



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Jett, Yeh, and Zavala

“We call the community to dive into the ongoing shift from solely prioritizing the content toward a simultaneous prioritization of context, content, and critical analysis.”

Catalyzing Change in High School Mathematics

Initiating Critical Conversations



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Mathematical Identity

“Developing students’
identities should be part
of teachers’ daily work.”
–page 28

“

Mathematical identity is the primary lens through which students engage with our classroom activities.

”

Equitable Mathematics Teaching Practices

Equitable Mathematics Teaching Practices (Bartell, et. al, 2017)

1. Draw on students' funds of knowledge
2. Establish classroom norms for participation
3. Position students as capable
4. Monitor how students position each other
5. Attend explicitly to race and culture
6. Recognize multiple forms of discourse and language as a resource
7. Press for academic success
8. Attend to students' mathematical thinking
9. Support development of a sociopolitical disposition

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2017, Vol. 48, No. 1, 7-21

Research Commentary

Toward a Framework for Research Linking Equitable Teaching With the Standards for Mathematical Practice

Tonya Bartell
Michigan State University
Anita Wager
University of Wisconsin–Madison
Ann Edwards
Carnegie Foundation for the Advancement of Teaching
Dan Battey
Rutgers University
Mary Foote
Queens College
Joi Spencer
University of San Diego

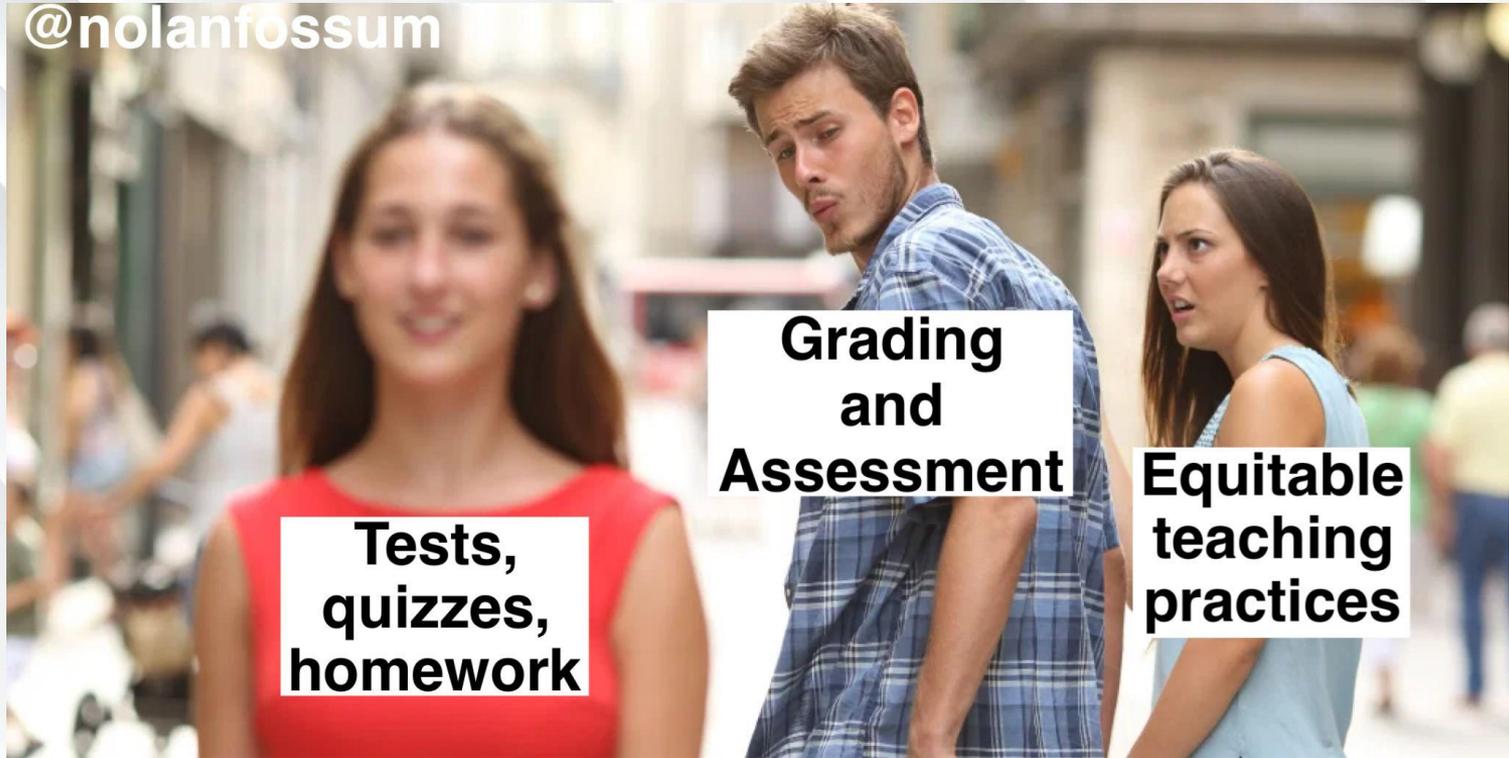
The Common Core State Standards for Mathematics (CCSSM) do not make any promises about the teaching practices that should be used to support students' enactment of the standards. Thus, equity gets framed as achievable through making the standards a goal for all students. We know from research on past reform efforts that standards without explicit (or companion) teaching practices, and teaching practices without explicit attention to equity, will inevitably result in the failure of the standards to achieve goals for students. This commentary provides a framework for future research that hypothesizes research-based equitable mathematics teaching practices in support of the CCSSM's Standards for Mathematical Practice, connecting research, policy, and practice in order to realize the equity potential of the CCSSM.

Key words: Equity; Diversity; Special needs; Common Core State Standards for Mathematics; Equitable teaching practices; Research issues

The Common Core State Standards for Mathematics (CCSSM) delineate the mathematical content all students should learn as well as eight Standards for Mathematical Practice (SMP) through which students should engage the mathematical content (National Governors' Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010). In this commentary, we consider how the CCSSM, and thus the SMP, is positioned within a larger political context and how these political forces, combined with a lack of attention

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**Tests,
quizzes,
homework**

**Grading
and
Assessment**

**Equitable
teaching
practices**

Confronting Some of My Implicit Biases

- My own student experience
- My own teaching experience
- My own lived experience
- Perspectives gained from my teaching colleagues
- My view of my ability to figure out solutions to challenges

Catalyzing Change in High School Mathematics

Initiating Critical Conversations



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Mathematical Agency

“Mathematical agency is about participating in mathematics in ways that are meaningful, both personally and socially.”

–page 29



AGENCY

IDENTITY

Let's Do Some Math!

What Do You See?

Figure 1

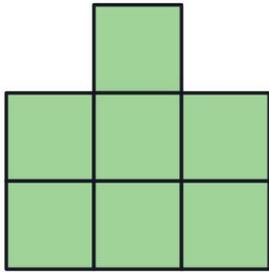


Figure 2

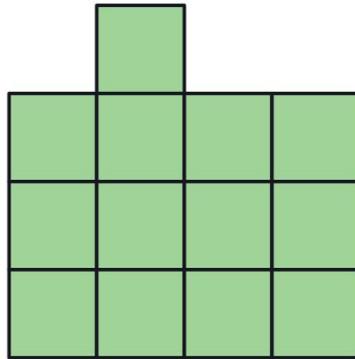
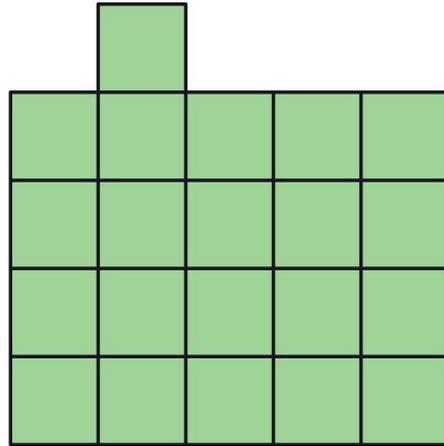
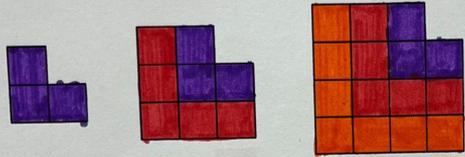


Figure 3

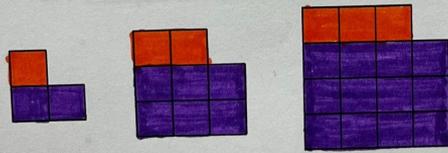


How do you see this pattern growing?



I see arrows extending

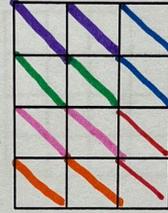
- $3 + 5$
- $3 + 5$
- $8 + 7$



I see rectangles + a smaller rectangle

- $1 \times 2 + 1 \times 1$
- $2 \times 3 + 1 \times 2$
- $3 \times 4 + 1 \times 3$

I see _____

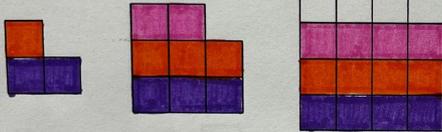


- 1 two blocks
- 3 two blocks
- 6 two blocks



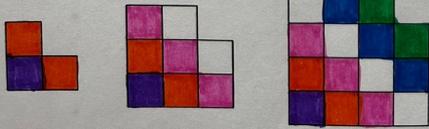
I see half

- I see 1 and 1 half
- I see 3 and 2 halves
- I see 6 and 3 halves



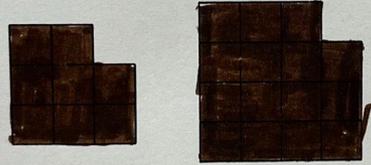
I see Rows

- 1 row of 2, 1 row of 1
- 2 rows of 3, 1 row of 2
- 3 rows of 4, 1 row of 3



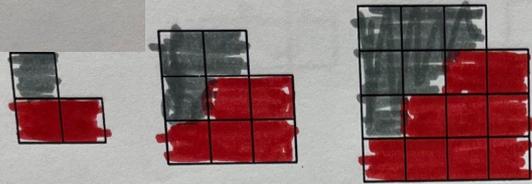
I see Diagonals

- 1 line of 2, 1 line of 1
- 2 lines of 2, 1 line of 3, 1 line of 1
- 2 lines of 2, 2 lines of 3, 1 line of 4, 1 line of 1



I see a square with a corner missing

- I see $2 \times 2 - 1$
- I see $3 \times 3 - 1$
- I see $4 \times 4 - 1$



I see L's

- $1 + 2$
- $(2 + 3) + (1 + 2)$
- $(3 + 4) + (2 + 3) + (1 + 2)$

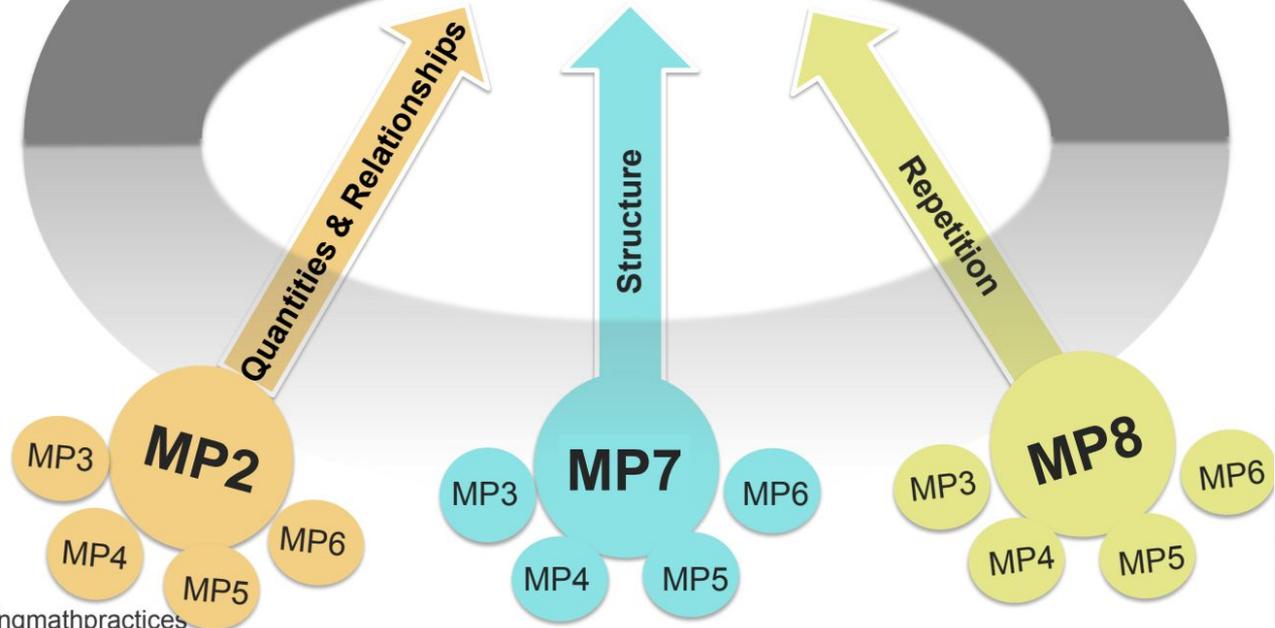
What aspects of the
“What Do You See?” activity
attend explicitly to
mathematical identity
and agency?

Equitable Mathematics Teaching Practices (Bartell, et. al, 2017)

1. Draw on students' funds of knowledge
2. Establish classroom norms for participation
3. Position students as capable
4. Monitor how students position each other
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9. Support development of a sociopolitical disposition

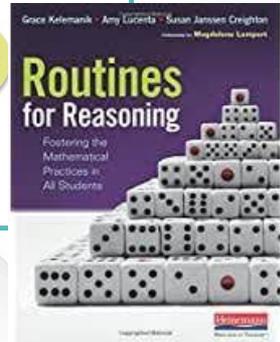
What aspects of the
“What Do You See?” activity
attend explicitly to
mathematical identity
and agency?

MP1 Make Sense of Problems & Persevere in Solving Them



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I see rectangles with a block on top.

Figure 1

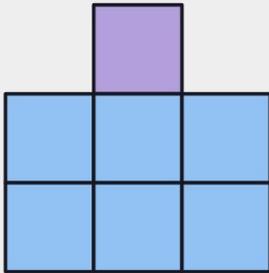


Figure 2

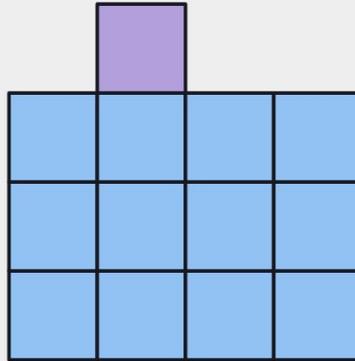
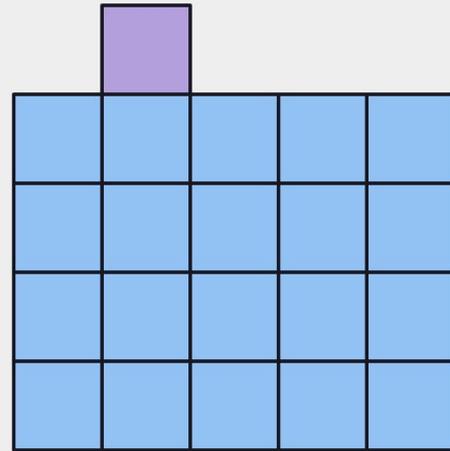


Figure 3



I see one tall stack and lots of short stacks.

Figure 1

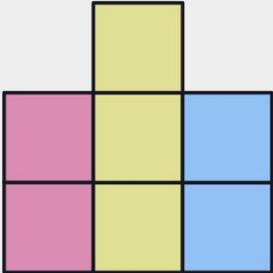


Figure 2

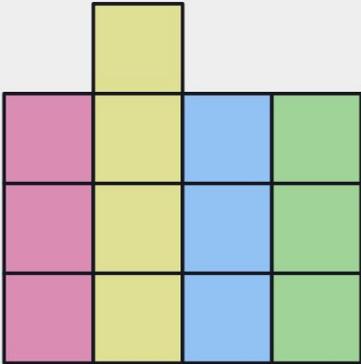
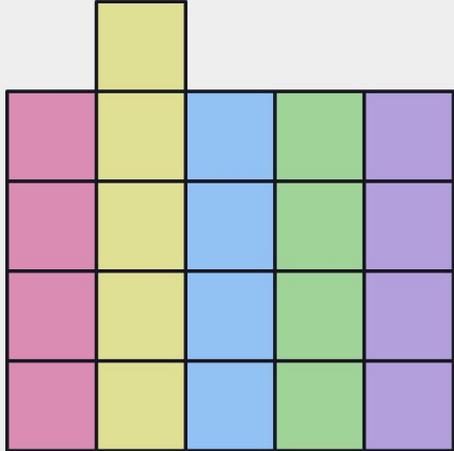


Figure 3



I see L-shapes that wrap around each other.

Figure 1

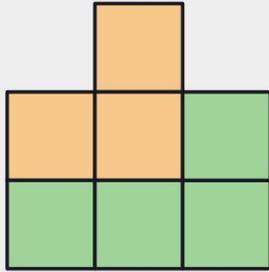


Figure 2

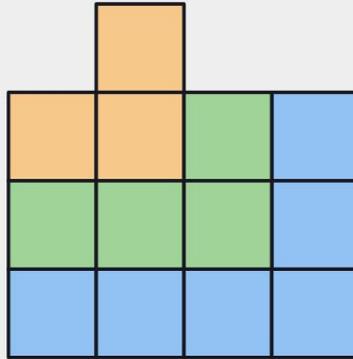
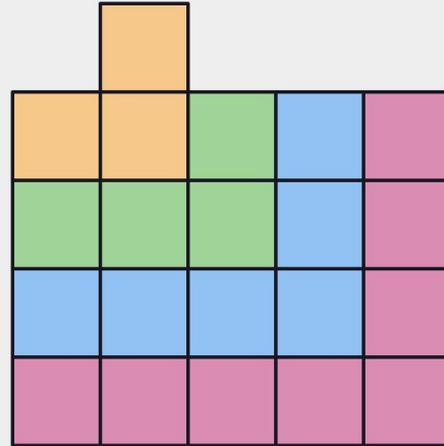


Figure 3



What aspects of the
“What Do You See?” activity
are hard to capture in your
gradebook?

“

*The act of grading actively diminishes
mathematical identity and agency in
all students.*

”

Our grading practices are a non-negligible part of the work of building students' identity and agency as learners.

The Pillars & Practices Framework

The Four Pillars

These are habits to guide student actions and interactions.

The Practices

Guidelines to align curriculum, instruction, and assessment to our values.

The Four Pillars

Engagement



Understanding



Communication



Authenticity



The Practices



Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Catalyzing Change in High School Mathematics

Initiating Critical Conversations



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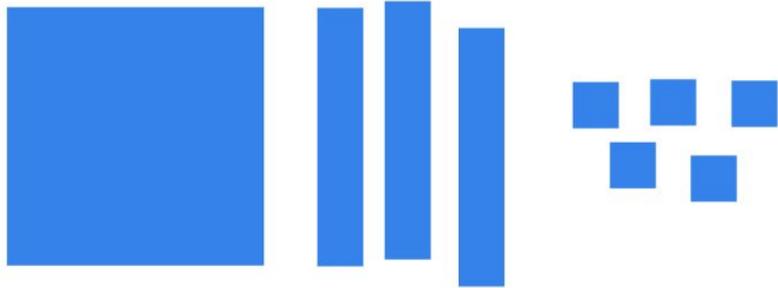
Essential Concepts

Algebra—Expressions can be rewritten in equivalent forms... to make different characteristics visible.

Functions—Key features of function graphs can be associated with and interpreted in terms of equivalent symbolic representations.

What Does
Pillars & Practices
Look Like in
Classroom Instruction?

Today's Agenda



$$x^2 + 3x + 5$$

☐ Skills—**Graphing from Tile Models**

Supplies you need today:

- Highlighter
- Pencil/Pen
- Marker
- Algebra Tiles

Learning Goals...

Let's use **tile models** to generate **graphs** of **parabolas**.

Learning Focus...

Pillar—**Engagement**

MP8—**Look for and make use of regularity in repeated reasoning.**



Using Tile Models to Graph Parabolas

Pillar: Understanding

MP8: Look for and express regularity in repeated reasoning

Learning Goal: Use tile models to generate graphs of parabolas.

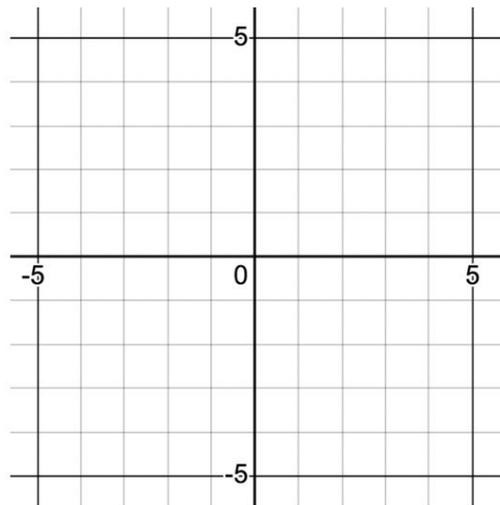
Post-Activity Process Questions

Answer these questions after you finish the three examples.

1. Write a tip that would help someone use a tile model to find the y-intercept of the parabola.
2. Write a tip that would help someone use a tile model to find the vertex of the parabola.
3. What advice would you offer to someone looking to find the x-intercepts of the parabola with a tile model?

Example 2—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 6x + 5$.

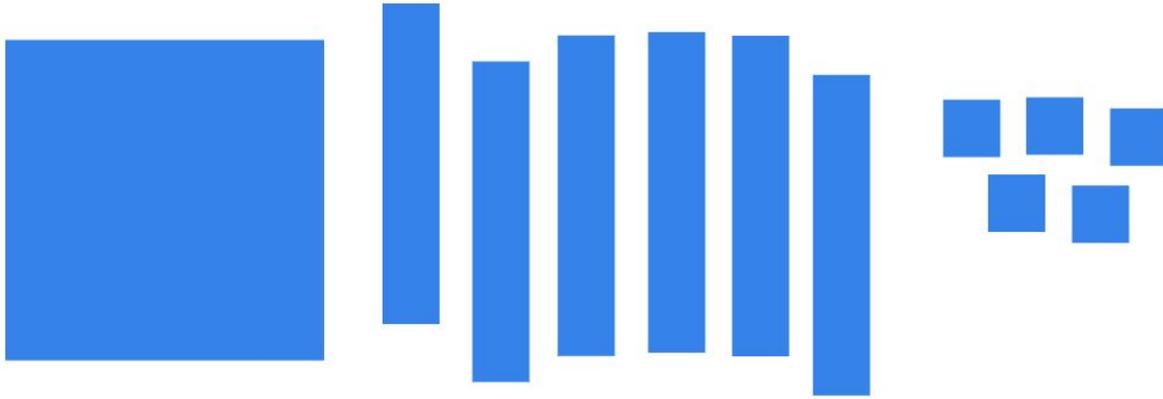
x	y



Analysis Questions

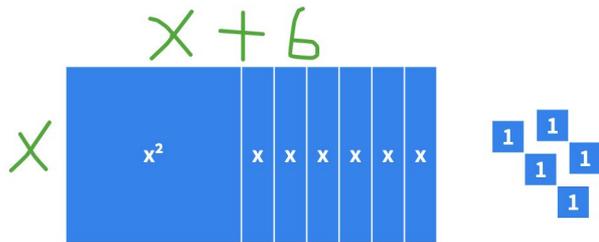
- 2a. What is the y-intercept of the graph? _____
- 2b. Where is the center of the graph's symmetry? _____
- 2c. What is the vertex of the parabola? _____
- 2d. Where are the x-intercepts of the graph? _____

Let's build arrangements for $x^2 + 6x + 5$



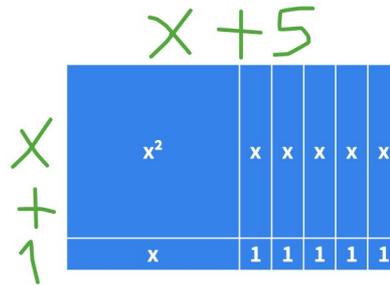
Go to bit.ly/PillarsPolypadTask1 to play around

Build arrangements for $x^2 + 6x + 5$



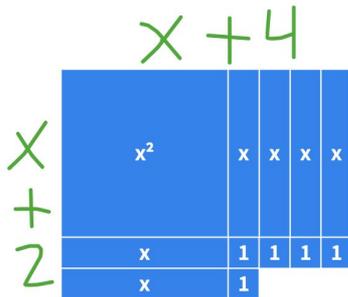
$$x(x + 6) + 5$$

Build arrangements for $x^2 + 6x + 5$



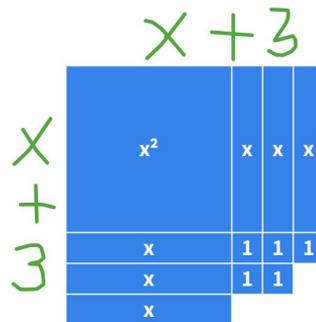
$$(x + 1)(x + 5)$$

Build arrangements for $x^2 + 6x + 5$



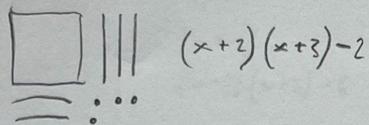
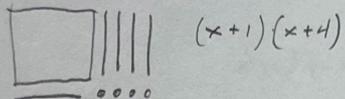
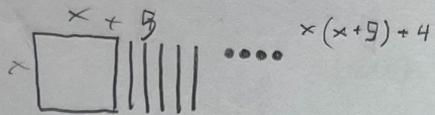
$$(x + 2)(x + 4) - 3$$

Build arrangements for $x^2 + 6x + 5$



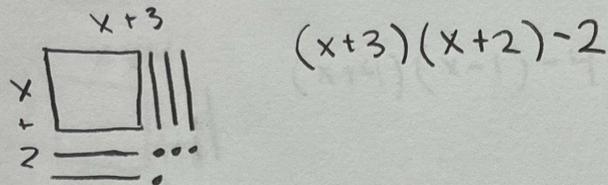
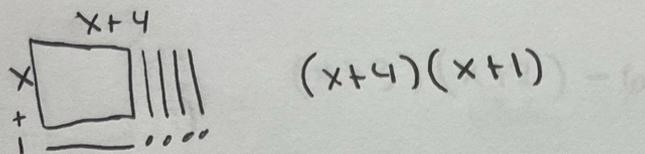
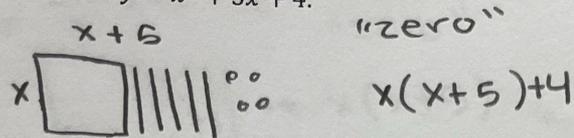
$$(x + 2)^2 - 4$$

Example 1—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 5x + 4$.



x	y
-5	4
-1	0
-4	0
-2	
-3	

Example 1—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 5x + 4$.

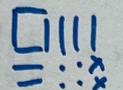


x	y
0	4
-5	4
-4	0
-1	0
-3	-2
-2	-2

Example 1—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 5x + 4$.

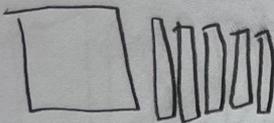
 $\therefore x(x+5)+4$

 $(x+1)(x+4)$

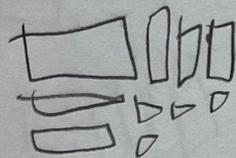
 $(x+3)(x+2)-2$

x	y
0	4
-5	4
1	6

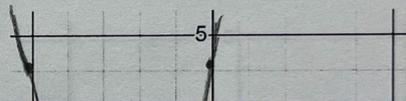
Example 1—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 5x + 4$.

 $x(x+5)+4$

 $(x+4)(x+1)$

 $(x+3)(x+2)-2$

x	y
6	4
-5	4
-4	0
-1	0
-3	-2
-2	-2



Example 1—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 5x + 4$.

$x \begin{array}{|c|} \hline x+5 \\ \hline \end{array} \begin{array}{|c|} \hline \\ \hline \end{array} \therefore x(x+5) + 4$

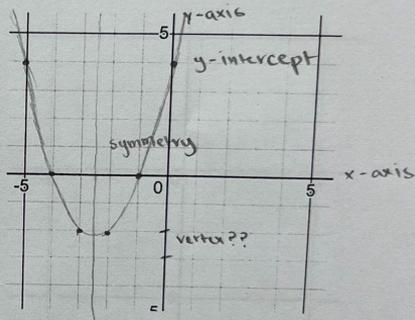
$\begin{array}{|c|} \hline x+4 \\ \hline \end{array} \begin{array}{|c|} \hline x+1 \\ \hline \end{array} \therefore (x+4)(x+1)$

$\begin{array}{|c|} \hline x+3 \\ \hline \end{array} \begin{array}{|c|} \hline x+2 \\ \hline \end{array} - 2 \therefore (x+3)(x+2) - 2$

$\begin{array}{|c|} \hline x+2 \\ \hline \end{array} \begin{array}{|c|} \hline x+3 \\ \hline \end{array} - 2 \therefore (x+2)(x+3) - 2$

$\begin{array}{|c|} \hline x+1 \\ \hline \end{array} \begin{array}{|c|} \hline x+4 \\ \hline \end{array} \therefore (x+1)(x+4)$

x	y
0	4
-5	4
-4	0
-1	0
-3	-2
-2	-2



Analysis Questions

- 1a. What is the y-intercept of the graph? (0, 4)
- 1b. Where is the center of the graph's symmetry? x = -2.5
- 1c. Where are the x-intercepts of the graph? (-4, 0) & (-1, 0)
- 1d. Are you able to find the vertex? Not sure (-2.5, ?)

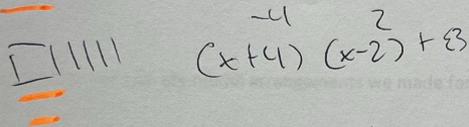
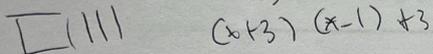
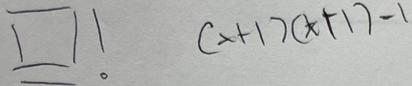
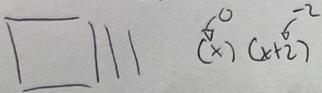
Extending the Activity

Post-Activity Process Questions

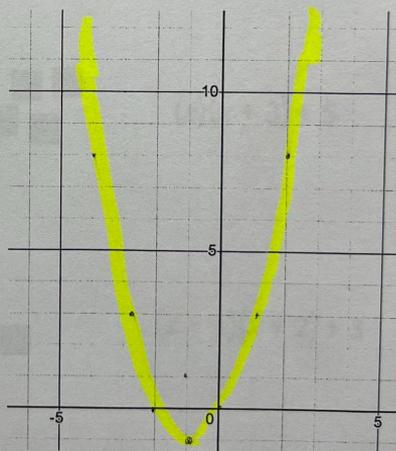
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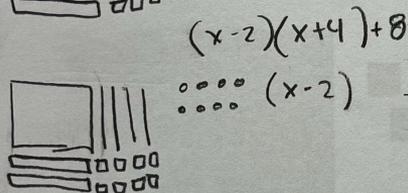
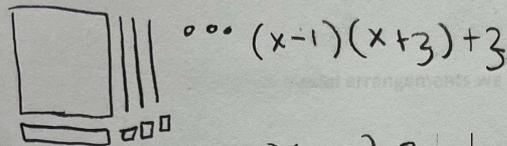
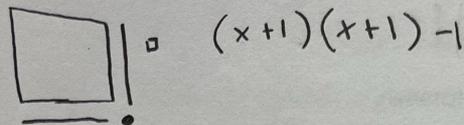
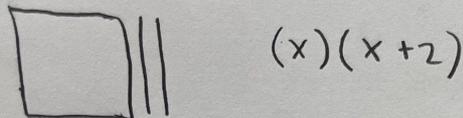
Example 2—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 2x$.



x	y
-4	8
-3	+3
-2	0
-1	-1
0	0
1	+3
2	8

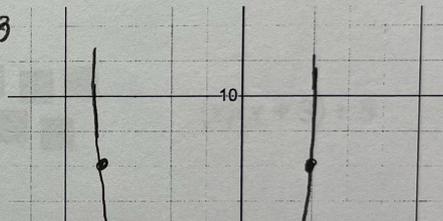


Example 2—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 2x$.

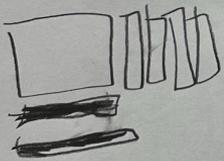
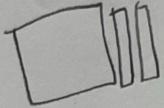


x	y
-4	8
-3	3
-2	0
-1	-1
0	0
1	3
2	8

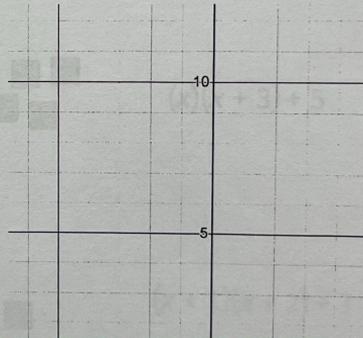
} +1
+3
+5



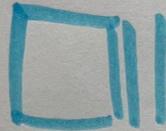
Example 2—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 2x$.



x	y
-4	
-3	
-2	
-1	
0	
1	
2	



Example 2—Use tile models to complete the table of values and graph the parabola given $y = x^2 + 2x$.



$$(x)(x+2)$$



$$(x+1)(x+1) - 3$$

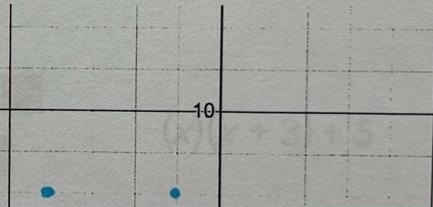


$$(x+1)(x+3) + 3$$



$$(x-2)(x+4) + 8$$

x	y
-4	8
-3	3
-2	-3
-1	-3
0	3
1	3
2	8



What aspects of the
“Tile Models to Graph Parabolas”
activity attend explicitly to
mathematical identity
and agency?

What aspects of the
“Tile Models to Graph Parabolas”
support growth in the eight
Math Practices?

What Does
Pillars & Practices
Look Like in
Curriculum Planning?

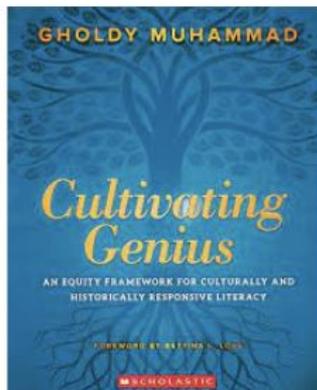
Geometry Unit Progression

Identity & Representation

Epistemology

Proximity

Angles & Perspectives



UNIT	FOCUS ON SELF AND SOCIETY	FOCUS ON CONTENT
Identity & Representation	<p>Who am I? What are my beliefs about myself and others? How do others view me? What familial, cultural, and historical factors contribute to my identity? Which narratives are dominant? Which are silenced or neglected?</p>	<p>Foundational geometric ideas—undefined terms, definition, postulate, theorem, prior geometric concepts formalized. Classifying triangles.</p>
Epistemology	<p>How are our belief systems formed? Why do we believe what we believe? How do we know what we know? What are truth, certainty, conjecture, opinion?</p>	<p>Formal and informal proof, logic, inductive and deductive reasoning. Formalizing definition, postulate, theorem. Applying different tools (compass, coordinates, analytical diagrams)</p>

<p>Proximity (Location & Distance)</p>	<p>How do power, position, and privilege impact my life and the lives of others? What access is afforded or denied to me? How have these informed my understanding of my identity and the identities of others?</p>	<p>Distance (in coordinate plane), length, simple coordinate geometry, area, and connections to circles. <u>Introduce</u> vectors. Introduction to Spherical (Non-Euclidean) Geometry.</p>
<p>Angles & Perspective</p>	<p>Challenging our own beliefs. Listening. Honoring lived experiences of others. Inclusivity and a history of exclusion.</p>	<p>Isosceles and equilateral triangles. Special segments in triangles. Interior angle sum and exterior angle theorem. Revisit Spherical Geometry (angle sum).</p>
<p>Parallels and Intersection</p>	<p>Finding common ground. Address history of groups put at odds by white culture. Rights for one are tied into rights for all. Convergence, divergence, and non-alignment (skew).</p>	<p>Angles formed by parallels with transversal. Rate of change and equations of lines. Parallel line proofs. Perpendicular theorems. Distance between lines.</p>

What Does
Pillars & Practices
Look Like in
Assessment?

Pillar 1 – Engagement

Ways I have been actively engaged this year:

I've talked more with my classmates and in my group. I also learn to expand my thinking.

Ways I could improve my engagement this year:

I can improve my engagement by participating more and asking question to get a better understanding of what we are learning.

I also need to focus on my independent work time and not talk to others.



Evidence of Struggle

Tell about one or two experiences so far this semester where you have really struggled with the classroom tasks.

In the lesson "The Rumor" I was quick to answer and I did not thoroughly read the prompt, my answer was rushed and I had to rethink and revise.

Write yourself a pep talk for next time reminding yourself that struggling is part of learning and one or two things you can do to persevere

No matter how tough the problem is or how little you know about the lesson, you must remember to never give up, to reread, and ask questions. Don't be embarrassed to ask questions, everyone needs help sometimes.

The Rumor

Andre overheard his sister and her friend talking before school. He heard a piece of juicy gossip that he was just dying to tell his friends. He told the rumor to 3 of his friends in 1st period. During 2nd period, each of his friends told 3 of their friends about the rumor.

The rumor keeps spreading like this. Each person who hears it tells three other people during the next period.

1. Make a table that represents the way this rumor is spreading. Label your columns.

P	F
0	1
1	3
2	9
3	27
4	81
5	243
6	729

2. Write an expression for the number of students who have heard the rumor ~~at the end of period X.~~ during period X.
 $(3)^x$

3. How many students will know the rumor by the end of the day?
By the end of the day 729 students will know the rumor.

The total number: $729 + 243 + 81 + 27 + 9 + 3 + 1$
Total: 1,093

4. Does your expression make sense for inputs like 50, 100, or 1000? Why or why not?
No because to get from 50 to 100 each person would have to tell 2 and to get from 100 to 1000 each person needs to tell 10 people. This wouldn't be exponential growth, it would be more like immediate growth.

Evidence of Engagement

Tell about one or two experiences so far this semester where you have impressed yourself with the way you engaged in classroom tasks

When we did the writing exponential equations from table, I understood everything. I was really set on finding the answers. And me and my tablemate Vanessa solved the problems first. We did have to ask the teacher for help but that's a good thing.

The whiteboard shows five numbered problems, each with a table of x and y values and the resulting exponential equation:

- | x | y |
|---|-------|
| 0 | 5 |
| 1 | 20 |
| 2 | 80 |
| 3 | 320 |
| 4 | 1,280 |

$$m = 4$$
$$y = 5(4)^x$$
- | x | y |
|---|----|
| 0 | 5 |
| 1 | 10 |
| 2 | 20 |
| 3 | 40 |
| 4 | 80 |

$$m = 2$$
$$y = 5(2)^x$$
- | x | y |
|---|------|
| 0 | 5 |
| 1 | 7.1 |
| 2 | 14.2 |
| 3 | 28.4 |
| 4 | 56.8 |

$$m = 1.42$$
$$y = 5(1.42)^x$$
- | x | y |
|---|-------|
| 0 | 5 |
| 1 | 30 |
| 2 | 180 |
| 3 | 1,080 |
| 4 | 6,480 |

$$m = 6$$
$$y = 5(6)^x$$
- | x | y |
|---|-------|
| 0 | 5 |
| 1 | 12.25 |
| 2 | 30 |
| 3 | 73.53 |
| 4 | 180 |

$$m = 2.45$$
$$y = 5(2.45)^x$$

A pink sticky note is attached to the board, and a playing card (Jack of Hearts) is visible at the top right.

What I Know about Linear Patterns



Figure 1

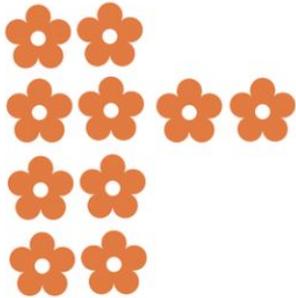


Figure 2

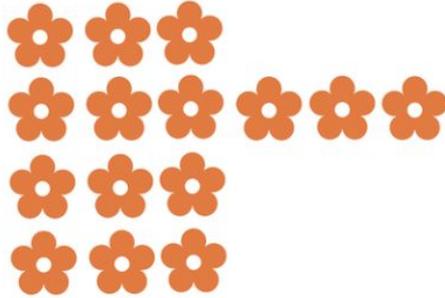
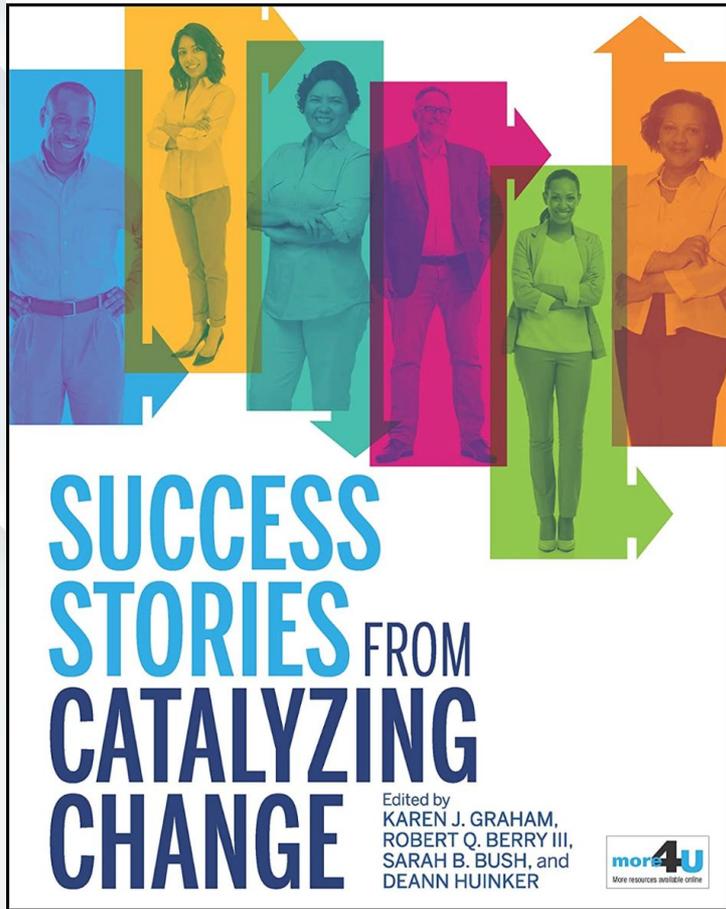


Figure 3

What I know about linear patterns is that that everytime the pattern changes it changes by a constant number for example the picture above is changing by 5 every time so like Figure 15 would have 75 and the equation would be $y=5x+5$. No, I don't have any questions about this topic I think so far I'm understanding it pretty well.



Pillars & Practices

A Framework to Catalyze
Change at the Margins

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#CatalyzingChange

#Pillars_Practices