Sense Making: Is It at the Core of Your Classroom?

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NCTM Webinar, Sponsored by McGraw Hill

A PDF of the slides will be available later, along with access to the recording.
There are 125 sheep and 5 dogs in a flock. How old is the shepherd?

Robert Kaplinsky (@robertkaplinsky) gave this to 32 eighth grade students. How many do you think attempted to find a numerical answer? (Put your guess in the chat)
How Old is the Shepherd?

75% gave numerical answers.

100% of his sixth graders gave numerical answers.

In the original research paper [Reusser 1986], “…three out of four school children will produce a numerical answer to this problem.”

http://robertkaplinsky.com/how-old-is-the-shepherd/
https://www.youtube.com/watch?v=kibaFBgaPx4
Sample Grade 3 Test Question

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

A. 6 18%
B. 9 46%
C. 18 31%
D. 24 4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.
**Integrated Math 2 (Grade 10)**

<table>
<thead>
<tr>
<th>Wear Sunscreen</th>
<th>Boy</th>
<th>Girl</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>84</td>
<td>133</td>
<td>217</td>
</tr>
<tr>
<td>Do Not Wear Sunscreen</td>
<td>170</td>
<td>118</td>
<td>288</td>
</tr>
<tr>
<td>Total</td>
<td>254</td>
<td>251</td>
<td>505</td>
</tr>
</tbody>
</table>

\[
P(\text{wears sunscreen}) \quad P(\text{wears sunscreen and is a boy})
\]
\[
P(\text{is a boy}) \quad P(\text{wears sunscreen or is a boy})
\]
\[
P(\text{wears sunscreen }| \text{ is a boy}) \quad P(\text{is a boy }| \text{ wears sunscreen})
\]

\[
P(\text{is a boy or a girl})
\]
“Doing Math” or Sense Making?

12 – p = 5

12 – ? = 5

[Michelle's son] was struggling to “remember” 28/4. When [she] asked him, “How do you think about 28/4?” He replied, “Mom, you aren’t supposed to think about it, you are just supposed to do it!!”
Teacher Knows Best

\[
\frac{3}{4} + \frac{5}{8} = \frac{8}{12}
\]
Jekyll and Hyde?

Bud, Not Buddy
Student Perceptions of Math and Sense Making

1. You aren’t supposed to sense-make when doing math.

2. You are supposed to use rules and algorithms and accept whatever answer results.

3. You are supposed to do what your teacher said, even when it doesn’t seem like a good idea.

4. Right answers NOW are what matters.
Sample Test Question Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.
CCSS Math Practice 1

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt**.

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.
The Five Strands of Mathematical Proficiency

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

“Productive disposition is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.”

We teach humans.
“Draw a picture of the story,” and what else?
Encouraging Sense Making

Q: What’s one way to cultivate a classroom focused on sense making rather than answer-getting?

A: Get rid of the question. Literally.
Get Rid of the Question

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

<table>
<thead>
<tr>
<th>I Notice</th>
<th>I Wonder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Get Rid of the Question

Mr. Gavin has a ladder that is 100 centimeters tall.
Ms. Cornell has a ladder that is 2 meters tall.
Get Rid of the Question

Male and Female Medical Doctors

Percent of All Medical Doctors

Year


Male

Female
Highlight, when fellow teacher sends this text: “Look what I tried today! Not bragging—just excited to share. It wasn’t as scary or crazy as I thought it would go. The Notice and Wonder was very effective for what I was teaching on today.” @MFAnnie @ElementarySalem @HobaughTeach2
Tina Cardone
@TinaCardone

Replying to @MFAnnie

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout

5:36 PM · Nov 24, 2014 · Tweetbot for iOS

Replying to @MFAnnie

@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it)
drawingonmath.blogspot.com/2014/11/distan...

5:37 PM · Nov 24, 2014 · Tweetbot for iOS

What’s Going On in This Graph? | March 23, 2022

Refugees from Ukraine as compared to other countries' largest one-year increases in refugees

- Venezuela: 2017–2018, +2,920,000
- Ukraine: Feb. 24–March 13, +2,820,000 refugees
- Rwanda: 1993–1994, +1,807,000
- Syria: 2012–2013, +1,754,000
- Iraq: 2005–2006, +1,201,000
- South Sudan: 2016–2017, −1,003,000
- Ethiopia: 1979–1980, −944,000
- Liberia: 1999–1990, +735,000

Each dot represents 1,000 people.

from the NYTimes Learning Network
What’s Going On in This Graph? | March 23, 2022

Since the start of the Russian invasion of Ukraine on Feb. 24, 2022, almost three million of the country’s 44 million residents have left the country. How big is this crisis?

1. After looking closely at the graphs above (or at these two full-size images), answer these four questions:

   • What do you notice?
   • What do you wonder?
   • How does this relate to you and your community?
   • What’s going on in this graph? Create a catchy headline that captures these graphs’ main idea.

The questions are intended to build on one another, so try to answer them in order.
Encouraging Sense-Making

Q: What’s another way to cultivate a classroom focused on sense making rather than answer-getting?

A: Get rid of the question and the numbers.
Get Rid of the Question and the Numbers

Raul had some pet mice. Xavier gave him some more mice.
Raul had some pet mice. Xavier gave him 3 more mice.
Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice.
Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

A Numberless Word Problem from Brian Bushart, bstockus.wordpress.com
Caitlyn is still trying to make brownies for the class. She has the mix and milk but needs to go get eggs. A carton of eggs weighs some 24 pounds. Each carton has 12 eggs. Each carton costs a certain amount. How much does one egg weigh? (in ounces)

Caitlyn realized that she needs one more ingredient. She forgot the vegetable oil. The oil is sold in 2p bottles. She needs a certain number of bottles for the brownies. Each bottle cost a certain amount. Caitlyn brings $20 with her to the store.

From Kat Kulis, grade 5, Windham Center School, Windham, CT
Get Rid of the Question and the Numbers

A store has the floor plan shown. The area of the women’s department is 175 square feet. The area of the total store is 750 square feet.
Tina Cardone @tinacardone
You Don’t Even Need Words
You Need To Start a Conversation
Encouraging Sense Making

Q: What’s another way to cultivate a classroom focused on sense making rather than answer-getting?

A: Give the answer.
Give the Answer (or Several!)

Rachel bakes cookies and delivers them to her friends.

• It takes 8 minutes to mix the batter.
• The cookies bake for 9 minutes.
• For 6 minutes they cool.

If the answer is 23 minutes, what is the question?
If the answer is 3 minutes, what is the question?
If the answer is bake, what is the question?

from Joe Schwartz, @JSchwartz10a
Encouraging Sense Making

Q: What’s another way to cultivate a classroom focused on sense making rather than answer-getting?

A: Ask about ideas, not answers.

This can be really simple:

“Tell me something about number 7.”

*instead of*

“What’s the answer to number 7?”
Ask About Ideas, Not Answers

It can be a little more complex:

Tell me everything you can about this figure.

Find the volume of the rectangular prism.

(instead of)

(from Joe Schwartz’s blog, exit10a.blogspot.com, October 10, 2016)
Ask About Ideas, Not Answers

It can be a little more complex:

\[ f(x) = x^3 - 4 \cdot x + 4 \]

Tell me everything you can about the derivative of this function.

instead of

Find the derivative of this function.
Ask About Ideas, Not Answers

Explain everything you can about the derivative of this function.
Teacher Questions

“Why?”

“How do you know?”

“How did you decide?”

“Tell me more about that.”
Are you asking *idea-focused* questions or *answer-focused* questions? Record yourself and find out!

#ToVForRatio
Ways to Encourage Sense Making Rather Than Answer Getting

• Get rid of the question.
• Get rid of the question and the numbers.
• Give the answer.
• Ask about ideas, not answers.
But Wait! There’s More!
Q: Another?

A: Launch by asking for their ideas instead of telling them things. (Just hush up for a bit!)
Gathering Ideas as a Launch

**Relate Pictures to Tens and Ones**

**MP.1 Make Sense of Problems** Analyze the Problem Discuss the pictures in Exercises 1 and 2. Count the number of cars in the first row. 10 cars Explain that drivers may be directed to fill a row before parking in the next row of a parking lot. In the same way, people may be asked to fill a row of seats before sitting in the next row at a theater.

- How can a filled row help you count the number of cars or the number of people? Possible response: A filled row shows ten, so I can use the picture to count tens and extras.

- How do the cars in Exercise 1 show tens and ones? 2 filled rows show tens and 3 extra cars show ones.

- How do the people in Exercise 2 show tens and ones? There are 4 rows of ten with 6 extra ones. This time the ones are at the top and the tens are shown below.
What Do They Notice? Wonder?
Stop Talking So Much!

**Equality on the Number Line**

To illustrate the equation $3t + 12 = 5t + 6$, you can draw $t$ as an unknown length. Whatever length you choose for $t$, you cannot compare it to the length of 6 or 12, because you do not yet know the value of $t$. You do know that every $t$ has the same length.

![Number line diagram](image)

The symbols above the number line show $3t + 12$. The symbols below the number line show $5t + 6$. The equation $3t + 12 = 5t + 6$ tells you that the two expressions are equal. So, when you draw the two expressions, they can start and end at the same point on the number line.

Look at the $3t$'s on the left above and below the line.

![Number line diagram](image)

$3t$ is in each expression.

Suppose you ignore the $3t$'s on both the top and bottom. The 12 above the line and the $2t + 6$ below the line start and end at the same point on the number line. So they must be equal.

Ignoring the $3t$'s above and below the line is the same as subtracting $3t$ from both sides of the equation. Above the line, 12 units are left over, and below the line $2t + 6$ units are left over. Now you have an equation, $12 = 2t + 6$, that you can solve using backtracking.
Let Them Make Sense of Things
"We" don't give students enough credit! I had my Ss graph points for sinx & cosx, then #noticewonder about their graphs. They noticed EVERYTHING I wanted to teach them and the discussion was amazing! Thanks @saravdwerf & @MFAnnie for inspiring me! #MTBoS #iteachmath #NWMNmath
Lesson 6-7
Multiply Two 2-Digit Factors

Be Curious

How are they the same?
How are they different?

48 ft

48 ft

16 ft

16 ft

Math is... Mindset
How can a plan help you solve a problem?
Warm Up
1.1 Notice and Wonder: Two Graphs

What do you notice? What do you wonder?

[Graphs of two linear functions showing distance traveled over time]
Ways to Honor Students’ Ideas

• Get rid of the question.
• Get rid of the question and/or the numbers.
• Ask for questions.
• Ask about ideas, not answers.
• Gather their ideas as a launch instead of talking at them.
Your students all have valid mathematical ideas about pretty much every problem or story.

Your job is to help them believe that.
In addition to eliciting and honoring students’ ideas, your job is to monitor for sense making All. The. Time.
Are We All Doing This?

Susie Hakånsson: *Increase Underserved Students’ Mathematical Agency by Using Equity Commentators in Lesson Study*

“Teachers are designing lessons for students to show their brilliance.”
Learn More, Start Conversations

- **Sense-Making: Aren’t We Already Doing That in Literacy?** (5 minute Ignite video)
- **The Hurrier I Go, the Behinder I Get** (5 minute Ignite video)
- **Emily and the Kindergartners** (blog post)
- **#NoticeWonder Love** (blog post)
- **Noticing and Wondering in Elementary School** (blog post)
- **Noticing and Wondering in High School** (blog post)
- Search Twitter for #NoticeWonder
- Search the web for Notice and Wonder
- NCTM Members can check out my June 2021 webinar, “It All Starts with Noticing and Wondering”
Take a Moment to Reflect

Write down:
What’s one thing you noticed in these ideas?
What’s one thing you’re wondering?
Sharing Reflections

Share your ideas in the chat.
You could format them:

NOTICE: [your idea]
WONDER: [your idea]
On the next couple of screens I’ve compiled things from the chat that folks noticed and wondered. I’ve responded to the wonderings, but would love to continue conversations with people.
Many Things Y’All Noticed

• Empowers students—gives them voice—supports positive math identity development!
• So many concepts have been brought to life - cognition, polya's problem solving, deep understanding.
• There are so many little things that can be done daily to increase students’ sense making
• I noticed the students have much more of a voice and are way more of a voice in their math education
• Levels playing field
• ALL STUDENTS have ideas about the problems we use.
• I notice that it seems so easy to tweak for such a major change
• This approach to math is very student centered!! So much different from the math instruction I experienced in elementary school
• The more quiet I am in the Notice portion of the Launch the more voices I start to hear. HS kids don’t always like the “void” (to which someone wondered: is kids’ dislike of the “void” actually an opportunity for getting them involved? Can teacher silence be a prompt for kids?)
• I noticed you focus on the journey vs. the destination and that the destination will come!
• Student's dispositions have a great impact on their mathematical development.
• When you do notice and wonder, everyone has access. I have even done it with non-verbal students who will point and gesture. Especially with graphs and images.
• What are equity commentaries?
  • I was learning about equity commentators at the Indianapolis NCTM Regional last week from Susie Håkansson and her work with the California Action Network for Mathematics Excellence and Equity. You can learn more about the project at (https://cmpso.org/canmee/). One of the resources they link to is their Equity Commentator Course (https://guides.loft.io/equity-commentator-course/)

• I noticed that pictures are really powerful. I wondered how I can get teachers excited. This was amazing!
  • Maybe share an Ignite video or two with them (5-minute videos) and just start a conversation. You might try one on the Learn More slide five slides back. Or you could even tweet out and ask that question - “Hey, how did you get your teachers excited about this sort of shift?” Maybe related is this blog post from Sara Van De Werf: https://www.saravanderwerf.com/how-kobe-bryant-challenged-me-to-be-a-better-math-teacher/

• About the resources you shared - eager to get to them to get others in the journey with me.
  • If the materials on the Learn More aren’t enough to get you started, let me know what you’re looking for.

• I notice that it seems so easy to tweak for such a major change
  • Okay, this wasn’t a wonder, but YES!! In fact, a put in a proposal to speak at NCTM in LA with a high school teacher who started trying Notice and Wonder and that snowballed, over time, to basically becoming a totally different person. From students in rows and her telling them stuff to students working in groups and her eliciting and leveraging their thinking. You can see the tweet that started this at https://twitter.com/ClaireVerti/status/1398860285574672384
And Wondered...

• I’m wondering how to know if my question prompt or task is going to be a good one for them
  • Try one! Or try it out on some adults, or random children. See what they mention. Ask on Twitter. Just try it and see. What’s the worst that can happen? And what’s the BEST??

• Where could I take out the numbers to get kids thinking even more and making sense for themselves?
  • Everywhere and anywhere! Just pick something you were going to do tomorrow and block or blur out the numbers. See what happens. Or search the internet for Numberless Word Problems or Numberless Graphs. If you’re in elementary, look at the link on slide 25 to Brian’s Numberless Word Problem blog posts and resources.

• What do you mean by “monitor for sense making”?
  • Does their explanation of how they came up with an answer make sense, or is it full of magical thinking, or “I just did what you said” or “I copied the book”. Ask about their decisions, not their steps. Not what they did, but why they did it. And ask a LOT. Then ask more.

• I wonder at what point do we go symbolic, and do we go back and forth.
  • I don’t know exactly what you mean, but I’ll make up an answer anyway. Try both. Introduce symbolic notation by showing a worked out problem/situation/something with new symbols in it and ask them what they notice and wonder about it before you “tell” them anything about the notation.

• I wonder, about coming up with ideas.
  • Try adapting something from your current materials using one of the ideas I suggested - just hack off the question, take out the numbers, something like that. Then do some web searching. But most importantly, just try it!

noticed that you focused on student’s attitudes and dispositions, and i’m wondering how to interrupt learned
And Wondered...

• I noticed that you focused on student's attitudes and dispositions, and i'm wondering how to interrupt learned helplessness in students who don't believe they can be successful in math
  • One strategy is when you have the whole class notice and wonder and you start to collect ideas from different tables/groups, call on some of those students early on (after you’ve checked in and you know they have stuff written on their paper). For some, just realizing they have ideas that you care about is enough of a jumpstart to get them going. Also, for some the shift to sense-making instead of answer getting helps them engage. My friend Melynee is a junior high special education teacher in Oklahoma who teaches a lot of those students, and she tweets and blogs about things she does to change her students’ perceptions of math. She’s amazing and believes in her kids SO HARD - and they reward her by being amazing. I recommend you check out her blog at https://mnmmath.wordpress.com/.

• How long to share their responses?
  • I’m thinking you’re wondering how long you should spend gathering students’ ideas when you do something like a Notice and Wonder routine. The answer is as long as you want. Sometimes you just ask them to tell you something, have each kid tell you one thing, and you move on. Other days they have noticed and wondered so much that you never get to the main part of the lesson you planned because they already had that conversation.

• How do we encourage thinking when kids think they “know” it since they’ve memorized an answer?!!??
  • You have to convince them that the “why” matters. Because if they don’t believe it does, then they won’t want any part of “more work”. You can convince them of it if you consistently ask. Check out one of the stories in this 5-minute ignite I did: https://www.youtube.com/watch?v=JHuijasExC4 Also, maybe you need to give them things that require more than memorization.

• How do we navigate noticing and wondering in our lessons when our schools' math curriculum is very scripted?
  •

And RWondered…
And Wondered...

• How do we navigate noticing and wondering in our lessons when our schools' math curriculum is very scripted?
  • Be subversive. Just try it. I mean, how's the other thing working out? Does everyone love math now? (Sorry, not helpful.) Honestly, try presenting something you’re supposed to tell/show them and ask them to notice and wonder. Maybe they'll like it and it will help? Or maybe have them do it as homework, for something completely different?

• How can we adapt our classroom environment so that we can help students develop a more positive outlook towards Math?
  • Make “doing math” mean something different - with a focus on mathematical ideas, rather than answers, math can become interesting to a lot more students! Start by making a commitment to ask about ideas at least X times a day for a while. Then X + 2 times a day…then more. See what happens.

• I noticed that I can modify my CGI word problems that I start my math groups with by adding the numbers later I wonder if this will be ok with my school because they are back to basics, traditional school.
  • No way to know other than to try it. Maybe your students will take to it and it will snowball from there.

• I wonder how we get parents on board when they too grew up with straight algorithms and focused on calculating…
  • Linda Gojak’s reply in the chat was a lot like what I might say: “I gave them the opportunity to do this kind of thinking…at open house or even in conferences…or have the kids take home something to notice and wonder about it. They loved it!” You could also share this blog post (really a “letter to parents”) from Wendy Ward Hoffer that addresses this issue head on: https://www.pebc.org/math-why-doesnt-yours-look-like-mine/
And Wondered!

• I do notice and wonder in my class already, but how do I engage my students more to look more in depth into the image that may be presented. I work in a city school that students give up very easily and don’t put effort in.
  • You’re getting at the idea of getting better at Noticing and Wondering - there’s a whole world out there that goes beyond just doing the routine. I’m guessing maybe they don’t see the point of trying hard. I wonder if it would help to have them notice and wonder about something (like the example I shared of Tina Cardone’s Foundations of Algebra students noticing and wondering about a graphic), then giving them questions about it. Perhaps they’ll find that it’s easier than usual to answer the questions? Or have them notice and wonder, then ask them to vote on which of the wonderings they want to explore or that they think would be most challenging? So that goes a bit beyond the routine, and maybe leverages their thinking.
• My colleague Steve Weimar was supporting an 8th grade teacher whose students weren’t very successful at school mathematics. But with enough Noticing and Wondering experience, this one group of boys had figured out that if they did a thorough job of Noticing and Wondering about something, they could pretty much answer anything you asked them. In fact, one time Steve said, “You guys are doing a great job of Noticing and Wondering! Maybe you should start exploring the questions.” The boys said, “No! No! We’re not done Noticing and Wondering!” They knew that if they didn’t do a thorough job, it was a lot harder to answer the questions, so they disciplined themselves to Notice and Wonder until they decided they were “done”. Pretty magical, huh?

Thanks for all your great wondering! I hope some of these ideas are helpful.
Thank you!

Annie Fetter

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Reminder that you’ll receive a link to the recording and a PDF of the slides.