



NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS  
CELEBRATING 100 YEARS

# Engaging Families in Math Fact Fluency

April 27, 2020

*Jennifer Bay-Williams*

University of Louisville



# OUR PLAN

- Big Picture
- Fundamentals & Families
- Messaging to Families
- Family Messaging
- Family Face to Face Activities
- Summing Up





# Big Picture

## From This

Student Name: \_\_\_\_\_

Multiplication Basic Facts Test - 3 minutes - 30 problems

1. $7 \times 7 =$	2. $2 \times 7 =$	3. $3 \times 7 =$
4. $2 \times 7 =$	5. $7 \times 7 =$	6. $3 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$	9. $7 \times 7 =$
10. $7 \times 7 =$	11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$	15. $7 \times 7 =$
16. $7 \times 7 =$	17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$	21. $7 \times 7 =$
22. $7 \times 7 =$	23. $7 \times 7 =$	24. $7 \times 7 =$
25. $7 \times 7 =$	26. $7 \times 7 =$	27. $7 \times 7 =$
28. $7 \times 7 =$	29. $7 \times 7 =$	30. $7 \times 7 =$

Multiplication Basic Facts ...  
teacherspayteachers.com

Student Name: \_\_\_\_\_

Grade 4 - Basic Facts Progress Test

1. $4 \times 4 =$	2. $4 \times 4 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Grade 4 - Basic Facts Progr...  
teacherspayteachers.com

Student Name: \_\_\_\_\_

Basic Facts Quiz | Highland...

1. $4 \times 4 =$	2. $4 \times 4 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Basic Facts Quiz | Highland...  
highlandnumeracyblog.wordpress.com

Student Name: \_\_\_\_\_

Basic Facts Practice Pages - St...

1. $4 \times 4 =$	2. $4 \times 4 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Basic Facts Practice Pages - St...  
smartkids.co.nz

Student Name: \_\_\_\_\_

Multiplication Basic Facts ...

1. $4 \times 4 =$	2. $4 \times 4 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Multiplication Basic Facts ...  
pinterest.com

Student Name: \_\_\_\_\_

Addition Facts Grid

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Basic Facts: the Last Addition Facts ...  
focusonmath.wordpress.com

Student Name: \_\_\_\_\_

Division Basic Facts Test by ...

1. $4 \div 4 =$	2. $4 \div 4 =$
3. $7 \div 7 =$	4. $7 \div 7 =$
5. $7 \div 7 =$	6. $7 \div 7 =$
7. $7 \div 7 =$	8. $7 \div 7 =$
9. $7 \div 7 =$	10. $7 \div 7 =$
11. $7 \div 7 =$	12. $7 \div 7 =$
13. $7 \div 7 =$	14. $7 \div 7 =$
15. $7 \div 7 =$	16. $7 \div 7 =$
17. $7 \div 7 =$	18. $7 \div 7 =$
19. $7 \div 7 =$	20. $7 \div 7 =$

Division Basic Facts Test by ...  
teacherspayteachers.com

Student Name: \_\_\_\_\_

Stage 4 Basic Facts Practice - St ...

1. $4 \times 4 =$	2. $4 \times 4 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Stage 4 Basic Facts Practice - St ...  
sites.google.com

Student Name: \_\_\_\_\_

Room 24 - Year 2: Basic ...

1. $4 \times 4 =$	2. $4 \times 4 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Room 24 - Year 2: Basic ...  
room24arahoe.blogspot.com

Student Name: \_\_\_\_\_

Multiplication  $\times 2$   $\times 5$   $\times 10$

1. $4 \times 2 =$	2. $4 \times 2 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Student Name: \_\_\_\_\_

Division  $\div 2$   $\div 5$   $\div 10$

1. $4 \div 2 =$	2. $4 \div 2 =$
3. $7 \div 7 =$	4. $7 \div 7 =$
5. $7 \div 7 =$	6. $7 \div 7 =$
7. $7 \div 7 =$	8. $7 \div 7 =$
9. $7 \div 7 =$	10. $7 \div 7 =$
11. $7 \div 7 =$	12. $7 \div 7 =$
13. $7 \div 7 =$	14. $7 \div 7 =$
15. $7 \div 7 =$	16. $7 \div 7 =$
17. $7 \div 7 =$	18. $7 \div 7 =$
19. $7 \div 7 =$	20. $7 \div 7 =$

Student Name: \_\_\_\_\_

10s & 100s

1. $4 \times 2 =$	2. $4 \times 2 =$
3. $7 \times 7 =$	4. $7 \times 7 =$
5. $7 \times 7 =$	6. $7 \times 7 =$
7. $7 \times 7 =$	8. $7 \times 7 =$
9. $7 \times 7 =$	10. $7 \times 7 =$
11. $7 \times 7 =$	12. $7 \times 7 =$
13. $7 \times 7 =$	14. $7 \times 7 =$
15. $7 \times 7 =$	16. $7 \times 7 =$
17. $7 \times 7 =$	18. $7 \times 7 =$
19. $7 \times 7 =$	20. $7 \times 7 =$

Student Name: \_\_\_\_\_

Addition to 10

1. $4 + 2 =$	2. $4 + 2 =$
3. $7 + 7 =$	4. $7 + 7 =$
5. $7 + 7 =$	6. $7 + 7 =$
7. $7 + 7 =$	8. $7 + 7 =$
9. $7 + 7 =$	10. $7 + 7 =$
11. $7 + 7 =$	12. $7 + 7 =$
13. $7 + 7 =$	14. $7 + 7 =$
15. $7 + 7 =$	16. $7 + 7 =$
17. $7 + 7 =$	18. $7 + 7 =$
19. $7 + 7 =$	20. $7 + 7 =$

Student Name: \_\_\_\_\_

Subtraction to 10

1. $4 - 2 =$	2. $4 - 2 =$
3. $7 - 7 =$	4. $7 - 7 =$
5. $7 - 7 =$	6. $7 - 7 =$
7. $7 - 7 =$	8. $7 - 7 =$
9. $7 - 7 =$	10. $7 - 7 =$
11. $7 - 7 =$	12. $7 - 7 =$
13. $7 - 7 =$	14. $7 - 7 =$
15. $7 - 7 =$	16. $7 - 7 =$
17. $7 - 7 =$	18. $7 - 7 =$
19. $7 - 7 =$	20. $7 - 7 =$



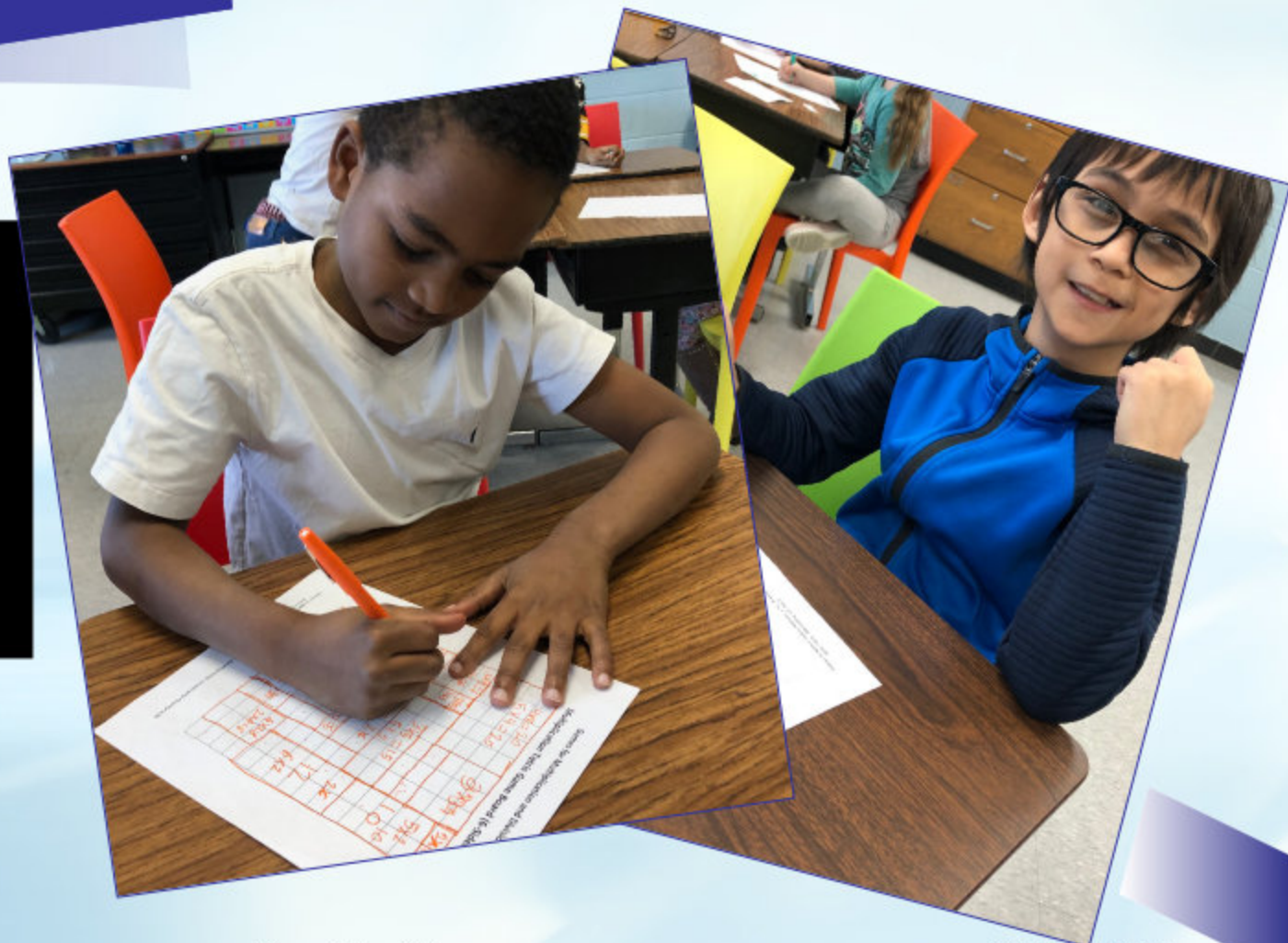




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CELEBRATING 100 YEARS

# Big Picture

To  
This



@JBayWilliams

#mathfactfluency

#NCTM100





# Big Picture

From  
This

## Addition Table

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

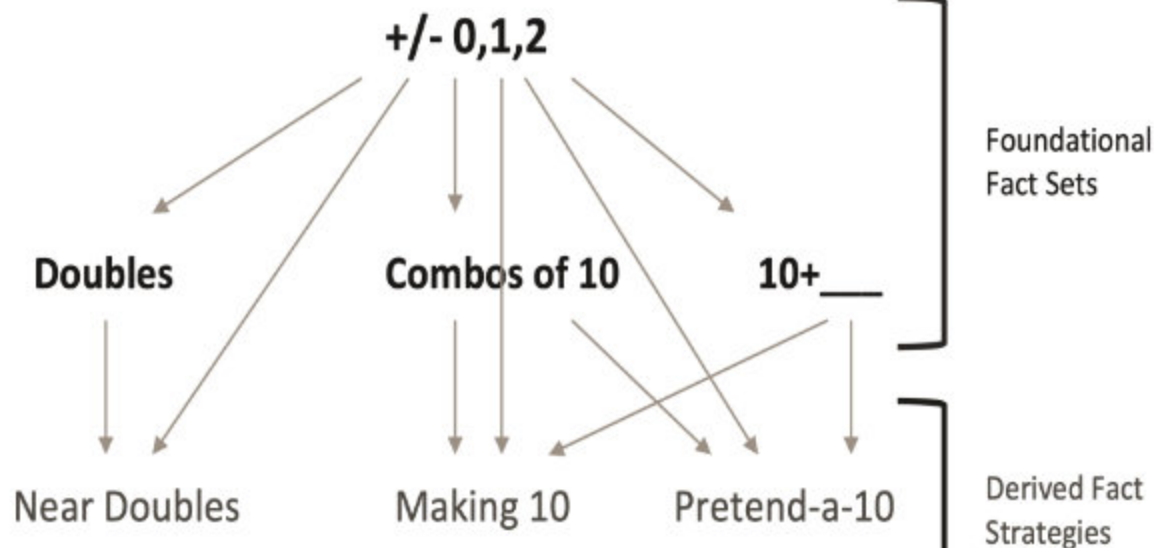




# Big Picture

To  
This

## Addition Facts Mastery Chart Flexible Learning Progression





# Big Picture

## The Bottom Line

1. What do families most **value** about their child's learning of mathematics?
2. What **expectations** do they have about how math is taught?
3. What were their **experiences** in learning the basic facts?







**It is not sufficient nor effective to  
justify a change in math because of...**

- **Standards**
- **Assessments**
- **District policies**
- **“Research says”**

**It must be because the change will  
better serve the needs of their child.**





# 5 Fundamentals of Math Fact Fluency

**#1: Mastery must focus on fluency!**

**#2: Fluency develops in three phases.**

**#3: Knowing foundational facts must precede derived facts.**

**#4: Timed tests do not assess fluency**

**#5: Students need substantial and enjoyable practice.**



Bay-Williams, J., & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD and Reston, VA: NCTM.





# 5 Fundamentals of Math Fact Fluency

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**How can we best help families understand and embrace these fundamentals of fact fluency?**







# 5 Fundamentals of Math Fact Fluency

#1: Mastery must focus on fluency.

Families must understand that while we do want every child to master their facts for life, the way to reach this goal is through a focus on fluency, not rote memorization.





# 5 Fundamentals of Math Fact Fluency

#1: Mastery must focus on fluency.	Families must understand that while we do want every child to master their facts for life, the way to reach this goal is through a focus on fluency, not rote memorization.
#2: Fluency develops in three phases.	Families need opportunities to learn strategies (Phase 2), why the strategies matter, and questions to ask to help their children progress through the phases.





# 5 Fundamentals of Math Fact Fluency

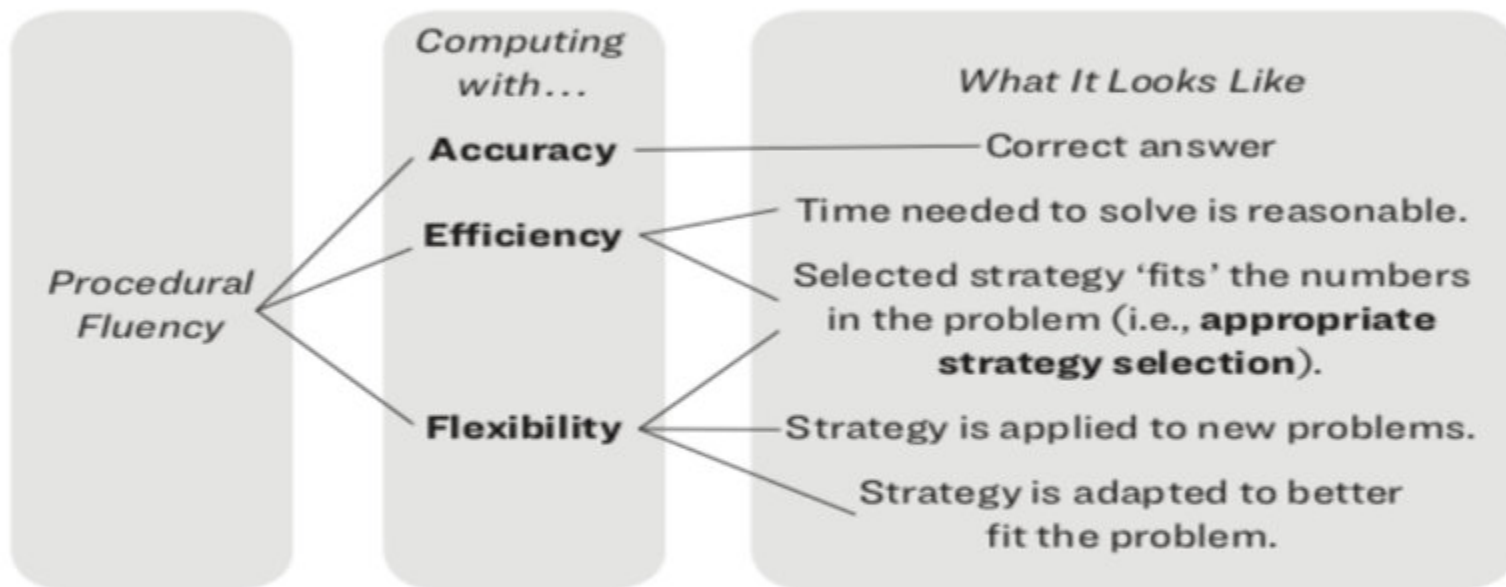
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#2: Fluency develops in three phases.	Families need to learn strategies (Phase 2), why the strategies matter, and questions to ask to help their children progress through the phases.
#3: Knowing foundational facts must precede derived facts.	Because traditionally facts are taught in order from smallest number to largest, families need to have, understand, and use these progressions.





# Procedural Fluency

FIGURE 1.1 What Procedural Fluency Is and What It Looks Like



The four components (**bolded**) are interrelated. Appropriate strategy selection is required for efficiency and flexibility.



# Learning Progression

## Phase 1: Counting

Counts with objects or mentally



## Phase 2: Deriving

Uses reasoning strategies based on known facts



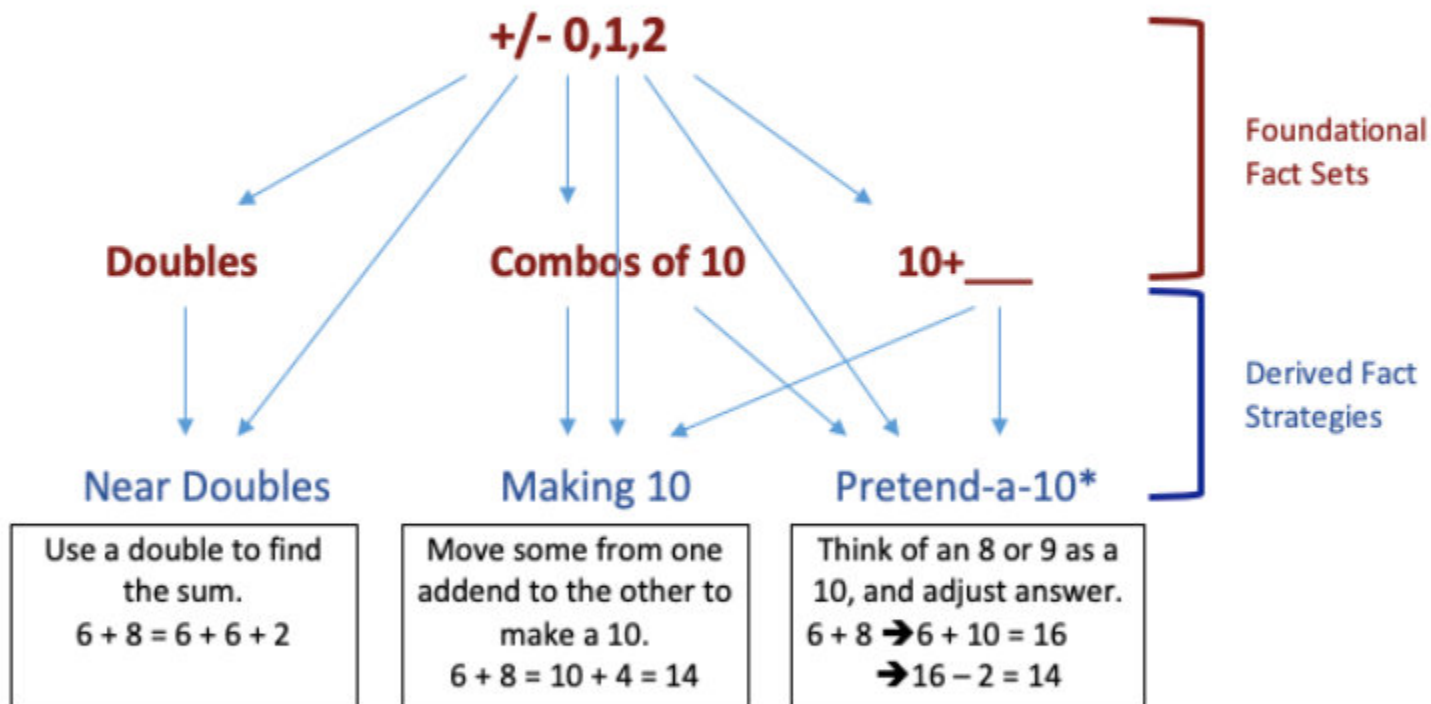
## Phase 3: Mastery

Efficient production of answers

Based on Baroody, 2006



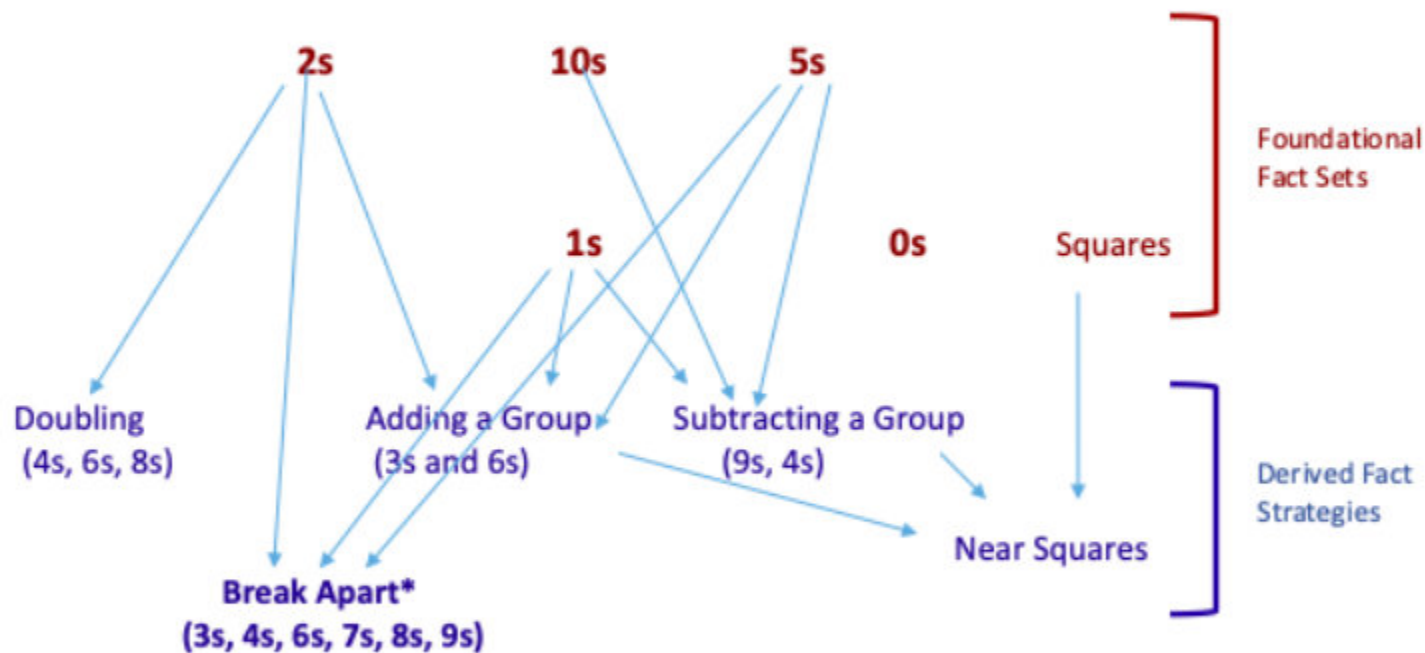
## Addition Fact Fluency Flexible Learning Progression



\*Also called Compensation and Use 10; we have found that young learners remember the strategy and distinguish it from Making 10 when we use this name. Research indicates that this strategy is more accessible than Making 10, and therefore should be explicitly taught (Baroody, Eiland, Reid, & Paliwal, 2016).



## Multiplication Fact Fluency Flexible Learning Progression



\*We acknowledge that all the derived fact strategies are break apart (distributive property) strategies. We focus on specific ways to break apart (e.g., adding a group) and move towards generalizing the Break Apart strategy.



# QUIZ 1



1.  $5 + 7 =$

2.  $9 + 6 =$

3.  $4 \times 7 =$

4.  $6 \times 3 =$

5.  $5 \times 9 =$





# Quiz 2



1.  $95 + 7 =$

2.  $90 + 60 =$

3.  $4 \times 15 =$

4.  $35 \times 3 =$

5.  $5 \times 49 =$







# Building a Case for Fluency

## Quiz 1

1.  $5 + 7 =$

2.  $9 + 6 =$

3.  $4 \times 7 =$

4.  $6 \times 3 =$

5.  $5 \times 9 =$

## Quiz 2

1.  $95 + 7 =$

2.  $90 + 60 =$

3.  $4 \times 15 =$

4.  $35 \times 3 =$

5.  $5 \times 49 =$





# 5 Fundamentals of Math Fact Fluency

#1: Mastery must focus on fluency.	Families must understand that while we do want every child to master their facts for life, the way to reach this goal is through a focus on fluency, not rote memorization.
#2: Fluency develops in three phases.	Families need to learn strategies (Phase 2), why the strategies matter, and questions to ask to help their children progress through the phases.
#3: Knowing foundational facts must precede derived facts.	Because traditionally facts are taught in order from smallest number to largest, families need to have, understand, and use these progressions.
#4: Timed tests do not assess fluency.	Families may themselves not like timed tests, but think they have some value. We need to help them see that timed tests and quick flashing of cards are not as effective as working on strategies.







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#5: Students need substantial and enjoyable practice	Practice at home is critical, but it also needs to be meaningful and stress-free! Games, strategy talk, and self-assessing progress are important “home work.”





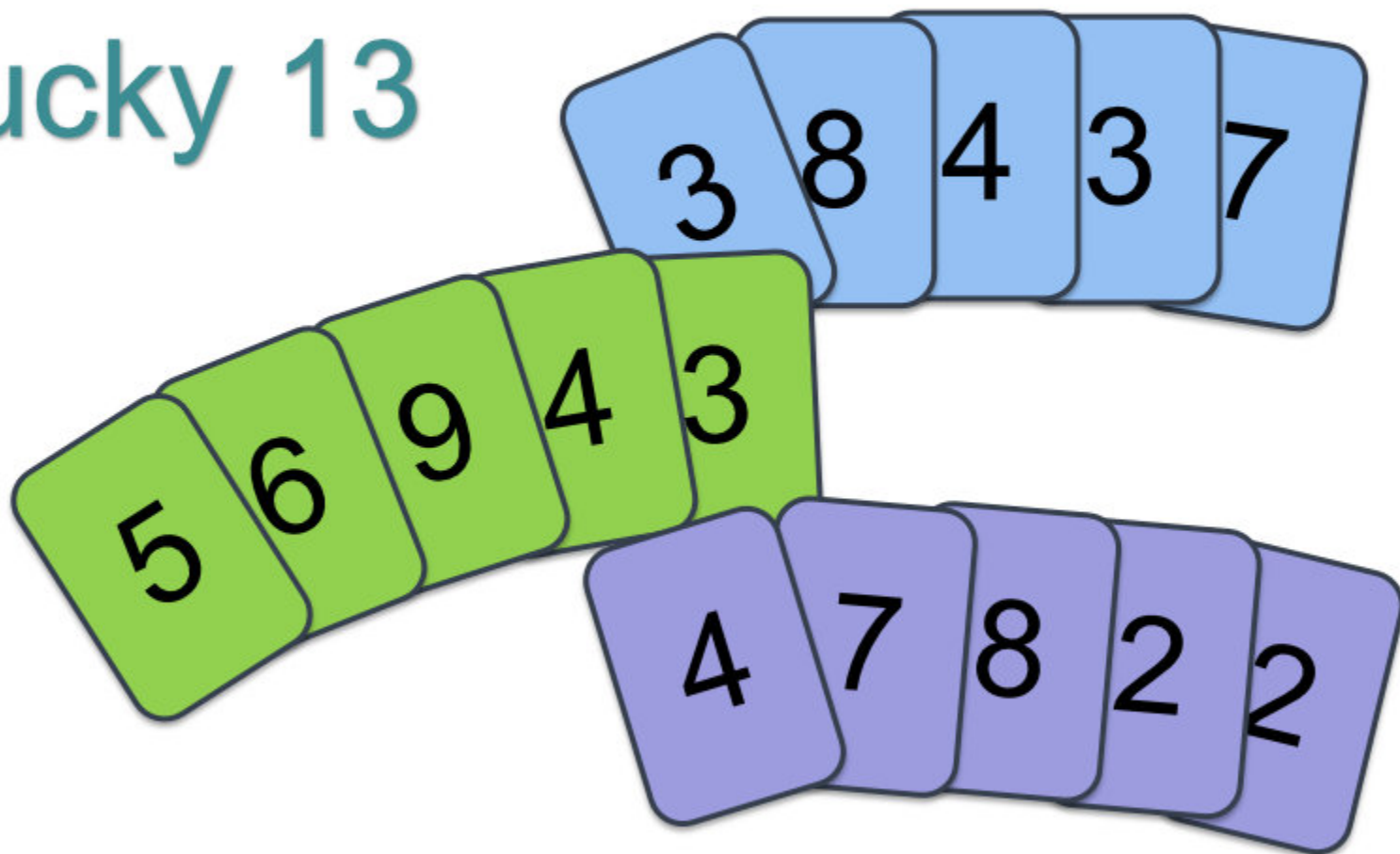
# Game: Lucky 13



Pick a color:  
blue, green, or  
purple

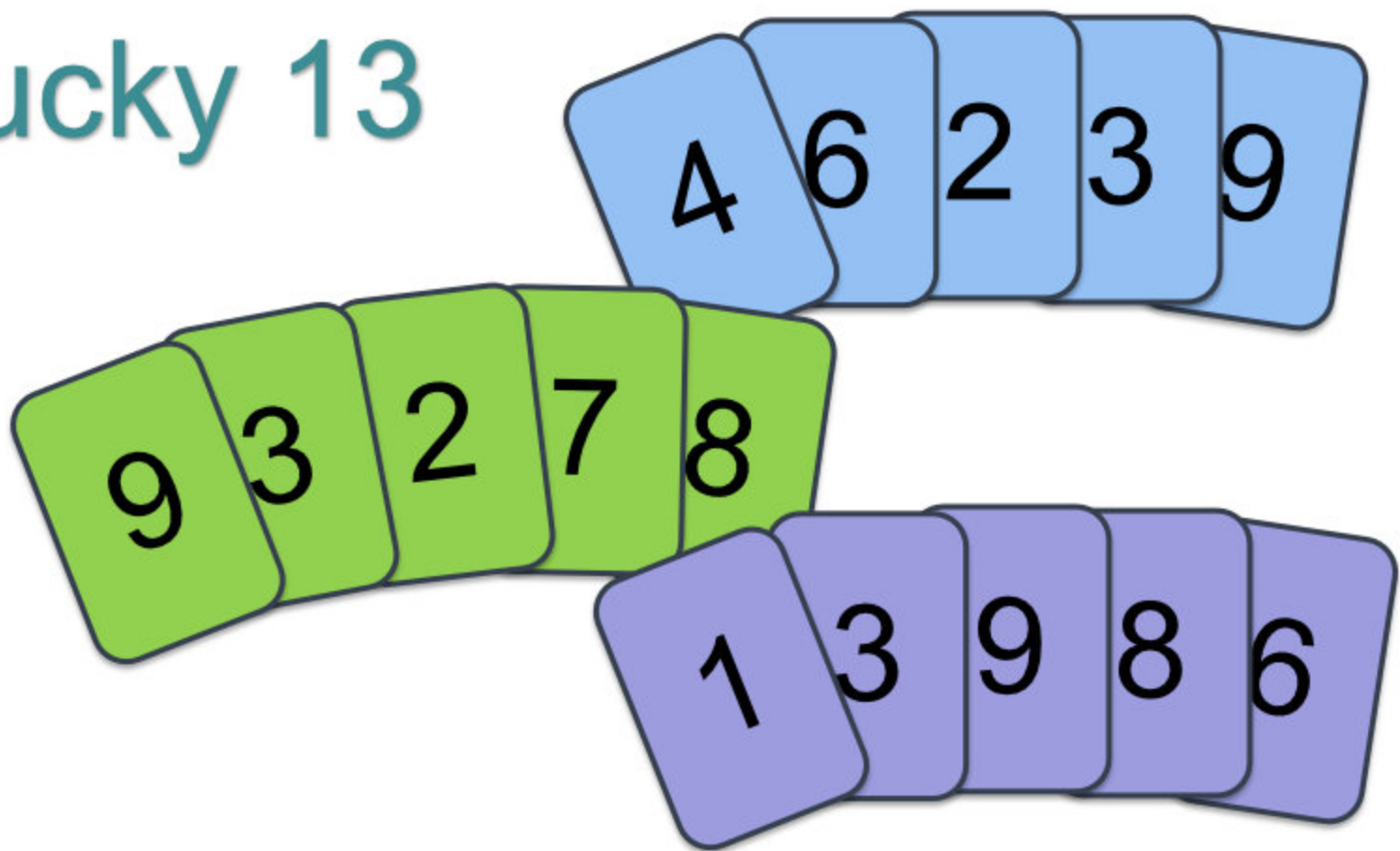
# Game:

## Lucky 13





# Game: Lucky 13

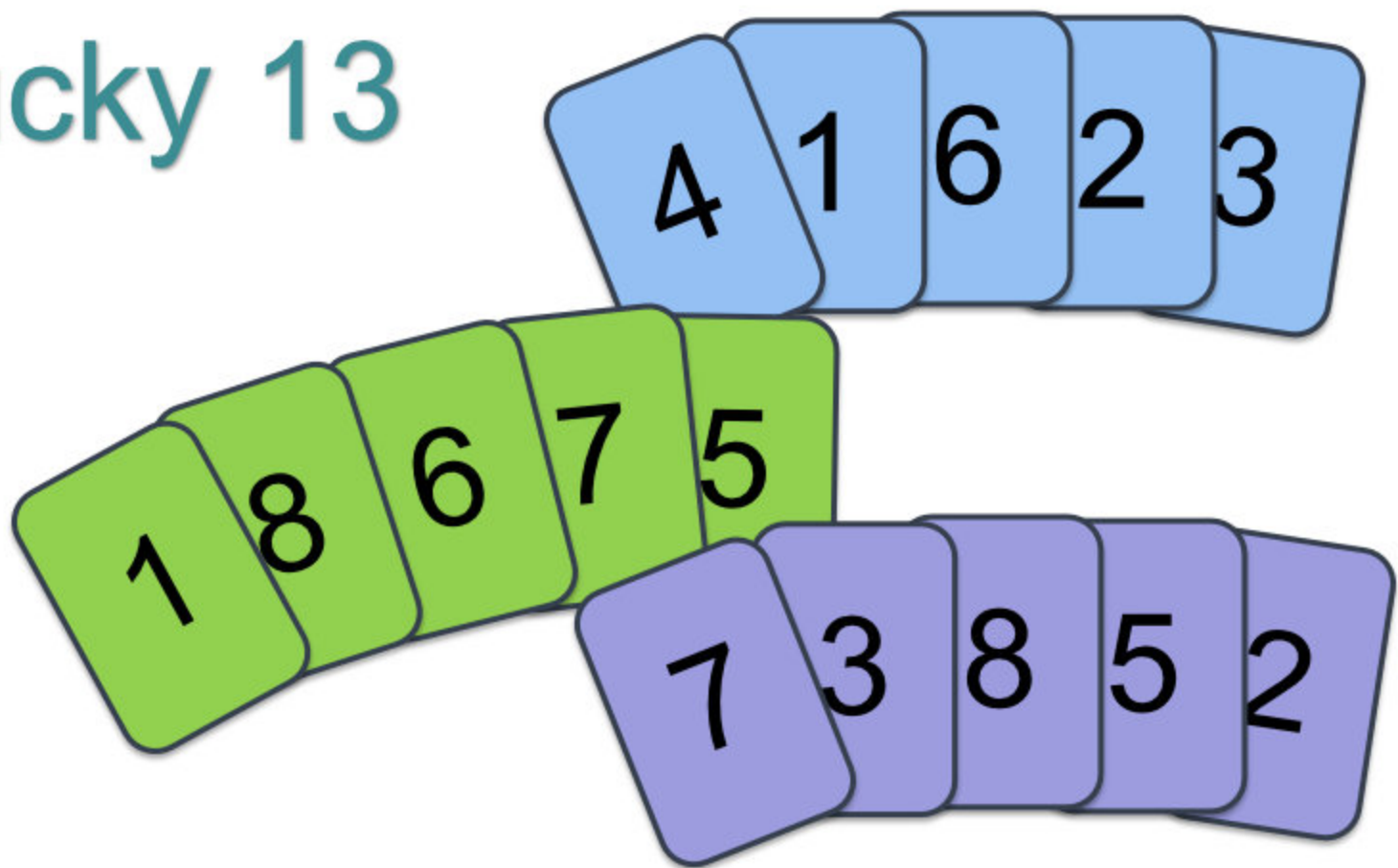






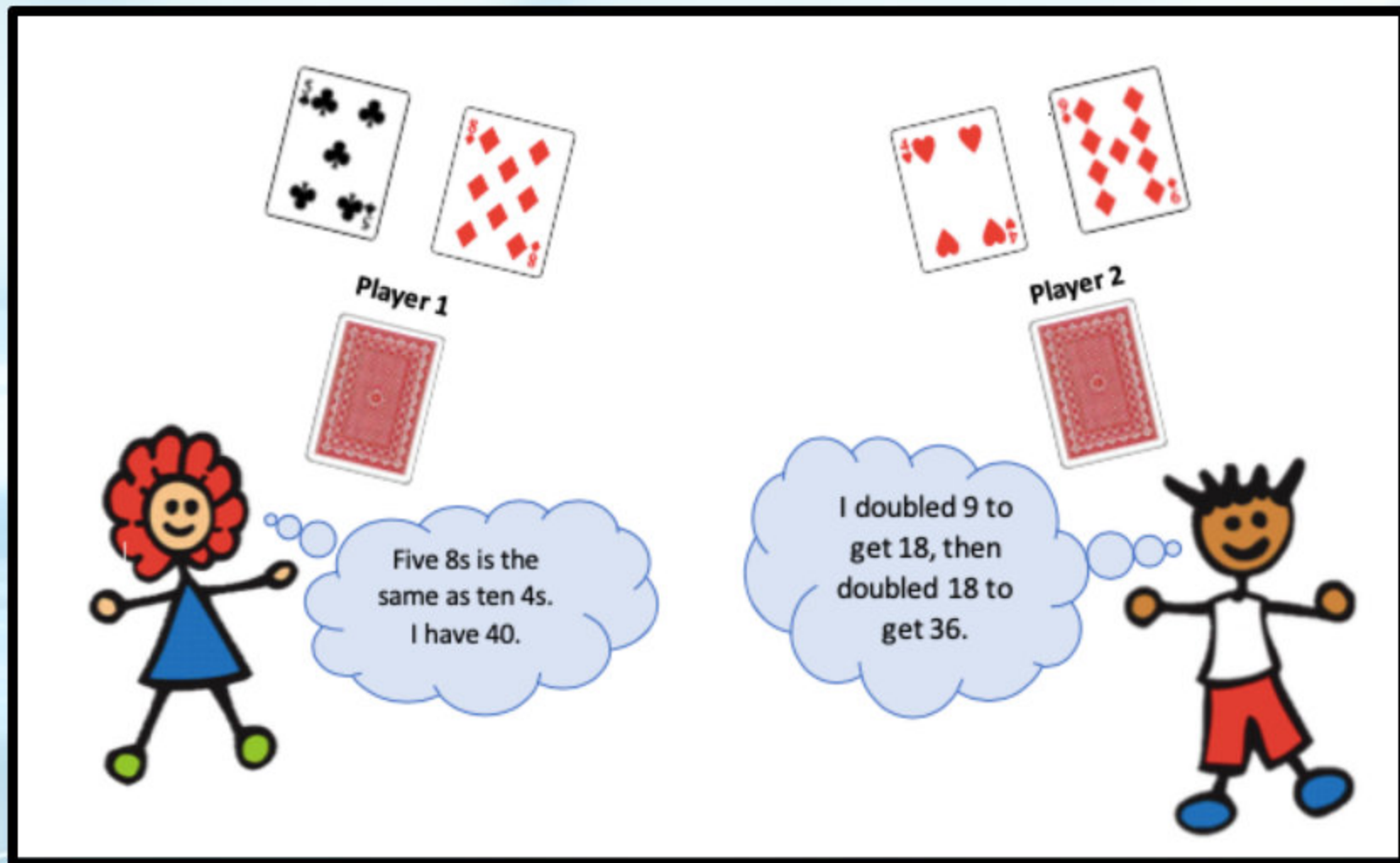
# Game:

## Lucky 13





# Game: Factor War





# Strategy Games...

## DO...

- Focus on fact sets and strategies children are ready to practice
- Lead to students **TALKING** about the strategies selected.

## DO NOT...

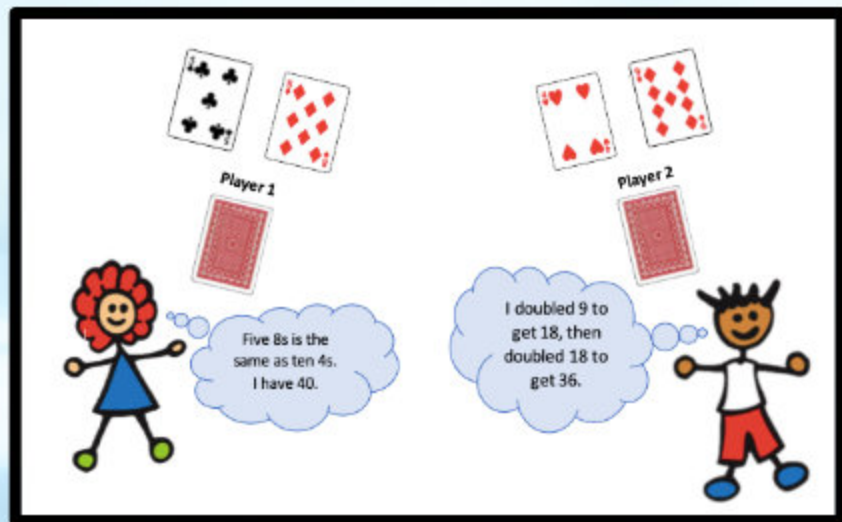
- Include a speed component.
- Have students solving the same fact.







# Bookmarks for Families



## Questions to Ask for Fact Fluency Games

- How did you solve that fact?
  - Why did you choose that strategy?
  - Are there other ways you could solve for that fact?
  - What other facts might be solved with that\* strategy?
  - When do you like to use that\* strategy (when is that strategy a good idea)?
- \*you can replace 'that' with the name of a strategy, like 'When do you use the doubling strategy?'





# 5 Fundamentals of Math Fact Fluency

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#2: Fluency develops in three phases.

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#4: Timed tests do not assess fluency

#5: Students need substantial and enjoyable practice.

**How can we best help families understand and embrace these fundamentals of fact fluency?**





# OUR MESSAGING TO FAMILIES



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# MESSAGING

“We are not going to be doing timed tests this year.”

“To do a better job determining which facts each child has learned and which ones they need to work on, we are going to be using several different assessment tools.”

“Is she saying speed doesn't matter? Are they not working on mastering facts?”

From *Math Fact Fluency*, p. 162







# MESSAGING

"We are not going to be memorizing facts. Instead, we

"For students to be able to say all the facts within seconds, we are going to be focusing on strategies. This will eventually lead to automaticity, and this way your child won't forget the facts they learned."

"Why aren't they just memorizing the facts? They have to have them memorized eventually, so why waste time with strategies."

*From Math Fact Fluency, p. 162*





# MESSAGING

"We are gonna take all year

"Our goal is not just to master the facts this year, but also develop fluency. This takes extra time but has big pay-offs for working with bigger numbers, fractions, decimals and even algebra!"

"All year? What about other topics? My child knows their facts, will she be challenged?"

*From Math Fact Fluency, p. 162*



## Math Fact Fluency

Three **KEYS** to Helping Your Child Learn Basic Facts for Life AND Like Math

### Learn

**Focus on Real Math Fluency... use STRATEGIES**

(rather than just memorize the facts with worksheets or flash cards)

#1

Why?

- Your child is *much more* likely to remember facts later on
- Your child is *much less* likely to have stress and anxiety
- The strategies will be used with greater numbers, fractions, and higher-level mathematics to support your child as a confident mathematician.

### Life

#2

**Help your child 'see' the reasoning STRATEGIES that generalize to numbers beyond basic facts.**

Developing fluency involves children building understandings from within and taking an active part in constructing number sense. Essential to this development is children deriving strategies to approach problems and recognizing that they are capable of reasoning and finding relationships.

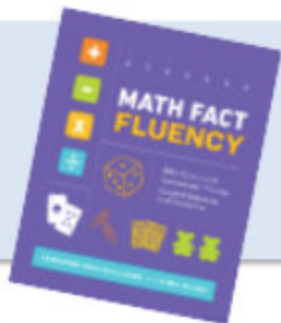
To get to that point, though, a child needs multiple opportunities to interact with number sense ideas, use number sense, and discuss number sense ideas and strategies.

### Like

#3

**Make practice enjoyable and meaningful.**

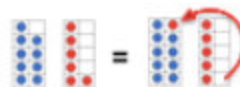
- Play games.
- Talk through STRATEGIES.
- Focus on strategy selection, not speed. Speed will come with strategy practice.



## Addition Strategies

### Making 10

Imagine shifting some counters over to make a full 10, then add.



$$9 + 6 = \underline{10} + 5 \\ = 15$$

Example  
**9 + 6**

Talk

Ten Frames

Numbers

### Pretend-a-10 [Compensation]

Pretend the biggest number is 10. Add. Adjust your answer to remove the extra you added.



$$9 + 6 = \underline{10} + 6 = 16 \\ \text{Subtract the extra one(s) you "pretended," so} \\ 9 + 6 = 15$$

### Home Made Ten Frame for Hands-On Learning

Cut off two cups of an egg carton so that you have ten cups. Use any [safe] household objects as counters (e.g., erasers, Lego, coins, game pieces, candies, etc.).



### Why Strategies Matter:

**Addition Strategies Extended to 3-digit Addition and Subtraction**

A child with math fluency looks to see when these strategies can save them from doing the more time-consuming standard algorithms.

"Making 10"

$$198 + 237 = \square \\ +2 \quad -2 \\ 200 + 235 = \underline{435}$$

"Pretend-a-10"

$$198 + 237 = \square \\ +2 \quad -2 \\ 200 + 237 = \underline{437} \\ -2 \\ \underline{435}$$

"Pretend-a-10"

$$504 - 98 = \square \\ -2 \quad +2 \\ 504 - 100 = \underline{404} \\ +2 \\ \underline{406}$$



## Why Strategies Matter:

### Addition Strategies Extended to 3-digit Addition and Subtraction

A child with math fluency looks to see when these strategies can save them from doing the more time-consuming standard algorithms.

*"Making 10"*

$$198 + 237 = \square$$

$$\begin{array}{r} +2 \quad -2 \\ 200 + 235 = \underline{\underline{435}} \end{array}$$

*"Pretend-a-10"*

$$198 + 237 = \square$$

$$\begin{array}{r} +2 \\ 200 + 237 = 437 \\ -2 \\ \hline 435 \end{array}$$

*"Pretend-a-10"*

$$504 - 98 = \square$$

$$\begin{array}{r} -2 \\ 504 - 100 = 404 \\ +2 \\ \hline 406 \end{array}$$





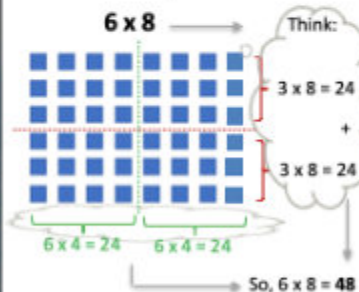
## Multiplication Strategies

### Doubling (and Halving)

With any even factor, I can use half that number to multiply and then double my answer.



If both factors are even – pick either one to halve, then double!



Talk

Numbers  
&  
Models

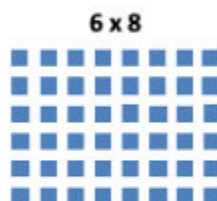
Tip: To help students see why these strategies work, use concept language like...

4 groups of 7,  
4 sets of 7,  
4 rows of 7

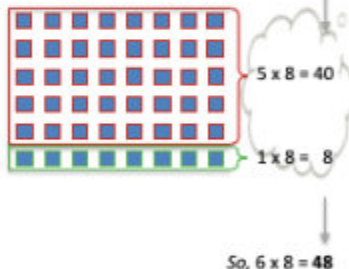
to help them understand the meaning of 4 times 7.

### Break Apart [Distributive Property]

I can take either factor and break it into two 'friendly' numbers, find the product of each part, and then join the parts together to find the product.



Think: Which 8 facts do I know?



### Home Made Materials for Hands-On Learning

**Egg Carton Ten Frames** (see above). Use small objects (jelly beans) as counters. For  $4 \times 7$ , fill 4 cups with 7 beans each.

**Cupcake Cups (or Bowls).** Start with counters, then eventually place numbers in each cup (using post-its). See how you can arrange the cups to show Doubling.



## Games

### Lucky 13

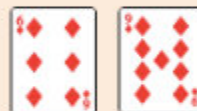


1. Deal 4 cards to each player.
2. Players use 2 of their cards to get a sum as close to 13 as they can.
3. Your score matches how far you are from 13 (e.g., if your two cards add to 15, your score is 2. If you get lucky and get 13, your score is 0!).
4. Play 5 rounds. Lowest score wins!

More ways to play: Deal 5 cards, or play Lucky 10 or Lucky 15.

### Factor War

[Think classic game of war.]



1. Share the cards equally among the players.
2. Each player flips over 2 cards and announces their product.
3. The greatest [correct] product wins those cards.
4. A tie means – War. Repeat steps 2 and 3.
5. Winner has most cards when time is up!

**More ways to play:** *Fixed Factor War.* You pick a number (e.g., 9 if you are working on your 9 facts). Place it in center for reference. Players only flip one card and multiply their card by the fixed factor. The greatest [correct] product wins. *Play Sum (or Fixed Addend) War* to practice addition facts.

### Questions to Ask for Fact Fluency Games

- How did you solve that fact?
- Why did you choose that strategy?
- Are there other ways you could solve for that fact?
- What other facts might be solved with that strategy?
- When do you like to use that strategy (when is that strategy a good idea)?

\*you can replace 'that' with the name of a strategy like 'When do you use the doubling strategy?'



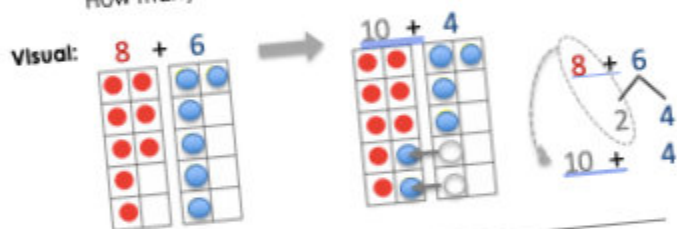


# ONE-STRATEGY LETTERS

## Success with MATH FACT FLUENCY Teachers + Students + Families Making 10 Strategy

### What does Making 10 look like?

**Story:** Eight red tulips and six blue tulips bloomed in the garden.  
How many tulips in the garden?



### Home Made Ten Frame for Hands-On Learning

Cut off two cups of an egg carton so that you have ten cups.  
Use any [safe] household objects as counters (e.g., erasers, Lego, coins, game pieces, candies, etc.).



### When is Making 10 useful?

This is a great question to ask your child! For basic facts, it is useful when we are adding two numbers that have a sum greater than 10.

*We know that strategy development is necessary for fluency. And fluency is developing automaticity with basic facts.*

## Extending Making 10: Beyond Basic Facts

Making 10 might be the most useful reasoning strategy beyond the basic facts. Using Making 10 can eliminate the need to regroup or use other error-prone and more time-consuming steps. Compare the before and after of these four examples to see how the strategy creates an easier-to-solve problem.

### Making 10s

"Move 1 over."

$$\begin{array}{r} 29 + 15 \\ \hline 30 + 14 \end{array}$$

### Making 100s

"Move 4 over."

$$\begin{array}{r} 278 + 496 \\ \hline 274 + 500 \end{array}$$

### Making 1s Fractions

"Move  $\frac{1}{4}$  over."

$$\begin{array}{r} 3\frac{3}{4} + 5\frac{3}{4} \\ \hline 4 + 5\frac{2}{4} \end{array}$$

### Making 1s Decimals

"Move 0.1 over."

$$\begin{array}{r} 21.56 + 42.9 \\ \hline 21.46 + 43 \end{array}$$

## Addition Strategy Brief

1. Students start learning addition by counting all, then learn more efficient strategies such as counting on. For facts with +1 or +2, counting on continues to be efficient; however, for facts like  $8 + 6$ , counting is not efficient.
2. Students who discover and learn reasoning strategies remember, retain, and outperform their peers who simply memorize their facts. Additionally, students who apply strategies develop confidence, not anxiety!
3. Instruction should begin with stories and visuals to help students make sense of a reasoning strategy.
4. Learning and using reasoning strategies initially takes more time than counting, but with sufficient, meaningful practice, it is eventually more efficient.
5. When we focus on "fast" (memorization), we are encouraging students to *not* reason and *not* think, but rather just recall. Watch them revert to counting. That's why the games below have no speed component!
6. Reasoning strategies themselves are important to learn because they generalize to larger numbers. Learning the strategies builds stronger math skills for life!
7. Playing purposeful math games is a great way for students to practice their reasoning strategies and learn their facts (see pages 3 and 4)!



## Games for Making 10 and Learning

### Game: **Lucky 13** (Game 11 in Math Fact Fluency)

#### Materials:

- ✓ Deck of cards, with Kings and Jacks removed; Queens = 0; Aces = 1

#### How to play:

1. One player is the dealer. The dealer gives each player 5 cards face up (see fig. 1).
2. Each player selects 2 cards which, when added together, produce a sum as close to 13 as possible (see fig. 2).
3. Players find how far their total is from "Lucky" 13 and record that difference.

#### Examples:

- Cards add to 9: Score is 4 (9 is 4 away from 13)
- Cards add to 15: Score is 2 (15 is 2 away from 13)
- Cards add to 13: Score is 0 (13 is 0 away from 13) (see fig. 2)

4. Players discard the two cards that were used and draw two new cards.

5. Repeat for 5 rounds. Lowest score wins!

**More ways to play:** Deal only 4 cards; change the each; find a *Lucky Difference* (lucky number can be

Figure 1:



Figure 2:

### Game: **Sum War** (Game 12 in Math Fact Fluency)

(2 players)

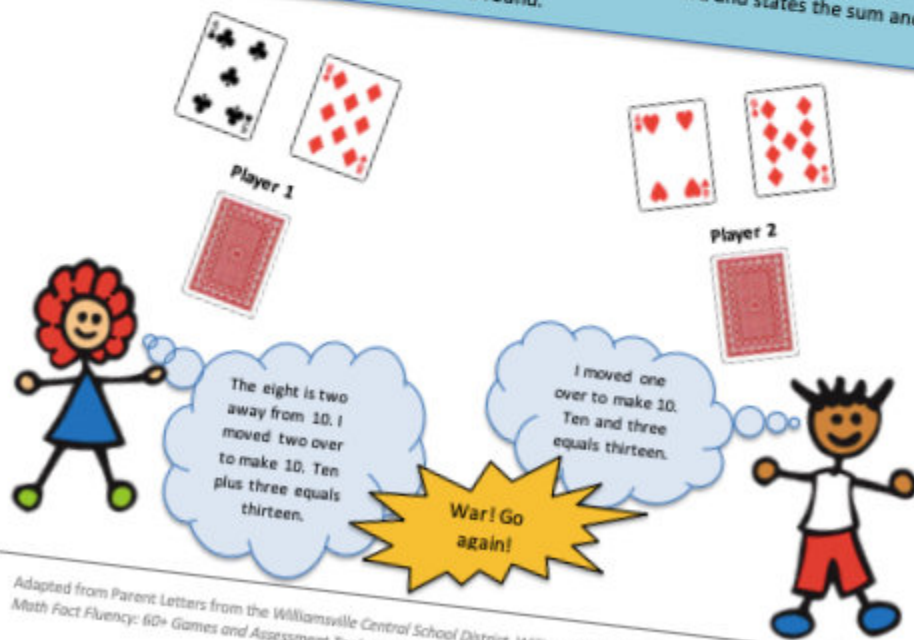
#### Materials:

- ✓ Deck of cards, with Kings and Jacks removed. Queens = 0; Aces = 1.

#### How to Play:

1. Split the deck in half so that each person has about the same number of cards.
2. At the same time, partners turn up their top two cards and say the sum.
3. Each player takes turns saying their answer, and sharing their thinking strategy. Both players decide if sums are correct:
4. The player with the larger (correct) sum gets the cards.
5. If there is a tie, it is a "war" and partners repeat steps 2 - 4.
6. Optional: Play for a set time; player with the most cards wins.

**More ways to play:** Play *Fixed Addend War* (see variation of Game 32 in Math Fact Fluency). Identify a "fixed addend" card and place face up between two players (e.g., use 9 as your fixed addend if working on the strategy Making 10). Players divide the remaining cards equally, shuffle them, and place them face down. Each player draws one card and states the sum and how they found it. The larger sum wins the round.





# FAMILIES MESSAGING TO THEIR CHILDREN







# PARENT MESSAGING

“Math is **Hard**”

“I was never good at  
**math.**”

“I don’t **like math.**”





# PARENT MESSAGING

When adults say things to their child like “I am not good at math”, it can impede their child’s success in mathematics.

In fact, a parent’s emotions are connected to the student’s emotions, and positive emotions are connected to better performance.

(Else-Quest et al., 2008)





# PARENT MESSAGING

"I was never good at math."

**Be positive!**





# MATH IN OUR DAILY LIVES

## What ideas fit your families in your setting?

- As socks are folded, ask: *How many socks did we wash?* [doubles] *How did you figure that out?* [counting, skip counting, grouping]
- At dinner, there may be 12 dinner rolls. Ask: *How might we share these rolls?*
- As bedtime or other event is approaching, ask: *How many more minutes until \_\_\_\_\_?*
- For chores, ask how long will it take to [clean each bathroom, bake all the cookies] if there are  $x$  number of [bathrooms, batches of cookies].







# MATH IN OUR DAILY LIVES

**What ideas  
fit your  
families in  
your  
setting?**

- Today's Date. Ask child to take the *day* (e.g., 12) and think of different addition (or multiplication) facts that have that number as an answer (e.g.,  $10 + 2 = 12$ ,  $3 \times 4 = 12$ ).
- In the car, ask: *Do you see a basic fact on that license plate?* or *Can you add the numbers on that license plate?*
- In the car, select a target number (e.g., 20 for younger children or 100 for older) and ask the child to use the numbers on a license plate to reach that target number (Hildebrandt, Biglan, & Budd, 2013)





# FAMILY FACE-TO-FACE EVENTS



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# Kindergarten Orientation

## The Bottom Line

What do parents do to get their  
child academically ready for  
Kindergarten?







# Kindergarten Orientation

Numeracy is as important as literacy!

7





## Numeracy is as important as literacy!



- ✓ Counting word
- ✓ 7 is a number,
- ✓ 7 is a quantity
- ✓ 7 comes after six in counting
- ✓ 7 is one more than 6
- ✓ If you have 6 bears and get one one, you will have 7.
- ✓ 7 is before 8
- ✓ 7 is one less than 8
- ✓ 7 is 5 and 2
- ✓ 7 can be shown with fingers





# Kindergarten Orientation and/or Registration

## Sleeping Bears

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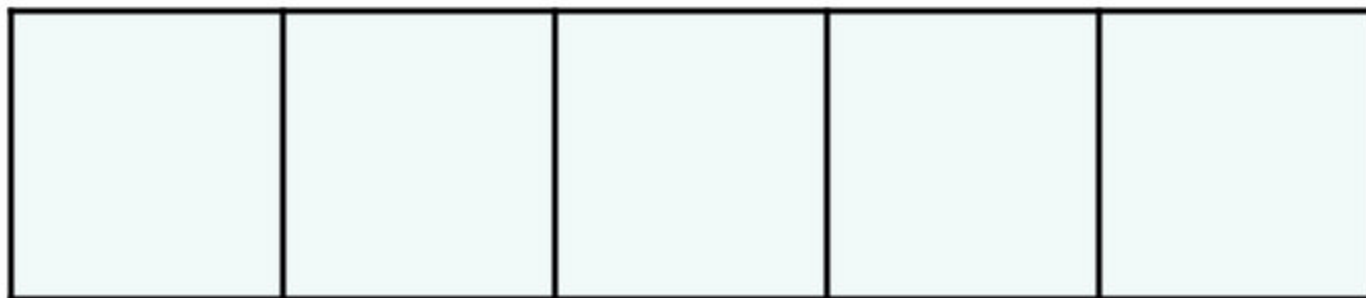






# Kindergarten Orientation and/or Registration

## Sleeping Bears



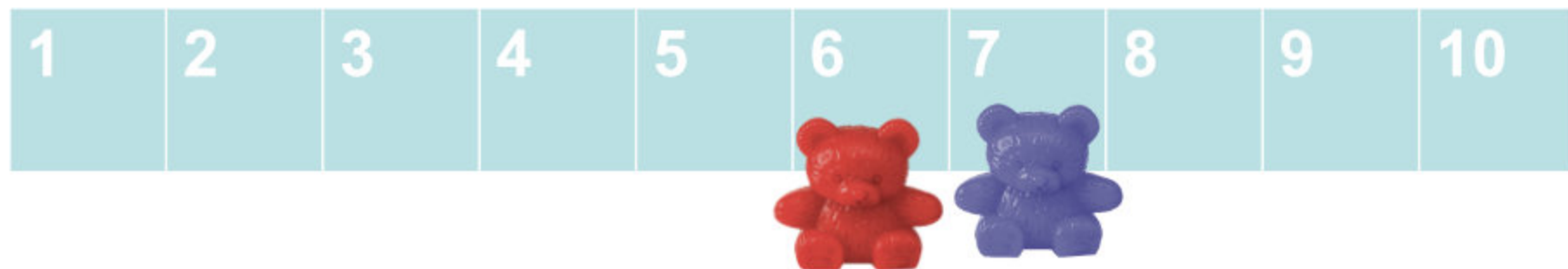


# Sleeping Bears





# Racing Bears



## Version 1: Bears Race to 10

Label the sides of a die  $+0$ ,  $+0$ ,  $+1$ ,  $+1$ ,  $+2$ , and  $+2$ . Students start their bears at 0. They then take turns rolling the die and moving their bear the appropriate number of spaces. The first bear to 10 wins the race.

Game adapted from Bay-Williams, J., & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD and Reston, VA: NCTM.







# FAMILY FACE-TO-FACE EVENTS





## For your most important topic!

### Procedural fluency includes:

1. **Flexibility**
2. **Accuracy**
3. **Efficiency** and
4. **Appropriate strategy selection**

(CCSSO, 2010, NCTM, 2014, NRC, 2001)





## For your most important topic!

### Procedural fluency includes:

1. **Flexibility**
2. **Accuracy**
3. **Efficiency** and
4. **Appropriate strategy selection**

(CCSSO, 2010, NCTM, 2014, NRC, 2001)







# Fluency

- Solve this problem any way you choose

$$306 - 298 =$$





# Fluency

- Solve this problem any way you choose

$$48 + 39 =$$





# Fluency

- Solve this problem any way you choose

$$4.8 + 3.9 =$$







# Fluency

- Solve these problems any way you choose

$$12 \times 15 =$$

$$17 \times 29 =$$





# Procedural Fluency

FIGURE 1.1 What Procedural Fluency Is and What It Looks Like

Procedural  
Fluency

Computing  
with...

Looks Like

Debrief with the #4fluency  
list, an illustration or just  
chat about being 'fluent'.

is reasonable.

its' the numbers  
**appropriate**

to new problems.

ted to better  
fit the problem.

The four components (**bolded**) are interrelated. Appropriate strategy selection is required for efficiency and flexibility.





# FAMILY FACE-TO-FACE EVENTS







# Family Math Night Plan



1. Have parents share what fluency means to them and/or engage in Truth or Myth activity, facts sort, or other activity.
2. Engage parents in a parent “quiz” or show a video of a classroom where children are demonstrating fluency (for example, using or sharing strategies during class discussion or game play).
3. Develop a robust definition of fluency (flexibility, accuracy, efficiency, and appropriate strategy use) and share the three phases of fact mastery.
4. Demonstrate fluency-building activities, such as Quick Looks and facts games.
5. Close with sharing the implications of this for assessment (timed testing doesn’t align with fluency) and/or revisiting the myths/facts sort.





# Truth or Myth Sort

Truth	Myth

Fluency is the same thing  
as Mastery.





# Truth or Myth Sort

Truth	Myth
	<div>Fluency is the same thing as Mastery.</div> <div>Automaticity is the same thing as Mastery.</div>







# Truth or Myth Sort

Truth	Myth
<div data-bbox="399 615 828 818">Automaticity is the same thing as Mastery.</div>	<div data-bbox="1058 568 1437 763">Fluency is the same thing as Mastery.</div>





# Truth or Myth?

Children eventually need to be automatic with their facts (coming up with an answer in about 3 seconds).

Timed tests help students master their facts.

Games are not adequate substitutes for fact drill.

Calculators can support students in learning their facts.

The order in which the basic facts are learned matters.

Basic fact fluency is essential to operations with fractions.

Memorization is the best technique to master the basic facts.





# Truth or Myth?

Children eventually need to be automatic with their facts (coming up with an answer in about 3 seconds).

Timed tests help students master their facts.

Calculators can help students in learning facts.

More ideas  
(myth or fact)?

Basic fact fluency is essential to operations with fractions.

Memorization is the best technique to master the basic facts.





# ENGAGE FAMILIES IN ASSESSMENT DISCUSSION







# Quiz 1



1.  $5 + 7 =$

2.  $5 \times 9 =$

3.  $9 + 6 =$

4.  $6 \times 8 =$

5.  $7 \times 3 =$





## Quiz 2



1.  $95 + 7 =$

2.  $5 \times 49 =$

3.  $90 + 60 =$

4.  $8 \times 15 =$

5.  $35 \times 3 =$



Name \_\_\_\_\_

Date \_\_\_\_\_

(Key 1-27266)

$$\begin{array}{r} 9 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 0 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 0 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$$

# Timed Test: Multiplication

for teacher to fill out

48 problems

\_\_\_\_\_ wrong

\_\_\_\_\_ correct

for teacher to fill out

48 problems

\_\_\_\_\_ wrong

\_\_\_\_\_ correct

# Share a Fluency Rubric

Beginning	Fluency Developing	Fluency Emerging	Mastery Accomplished
Knows some foundational facts but counts or skip counts for derived facts.	Demonstrates automaticity with all foundational fact sets and uses at least one of the derived fact strategies for other facts.	Demonstrates automaticity with all foundational fact sets and uses several derived fact strategies for most or all other facts, though may require processing time to implement some strategies.	Demonstrates automaticity with all or most facts, selects efficient strategies and implements them easily, or just knows the facts.

Rubric from Bay-Williams, J., & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD and Reston, VA: NCTM.







# FACT SORT



From Bay-Williams, J., & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD and Reston, VA: NCTM.



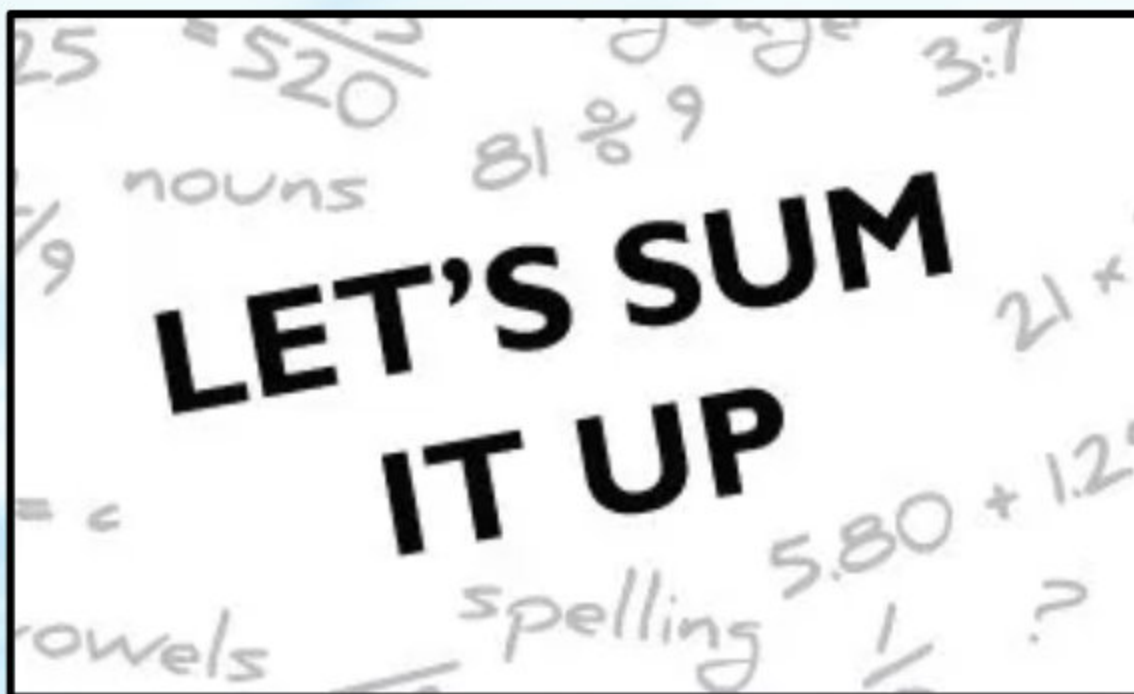


# FACT SORT

Skip Counted

Used a  
Strategy

Just Knew





# 5 Fundamentals of Math Fact Fluency

**#1: Mastery must focus on fluency!**

**#2: Fluency develops in three phases.**

**#3: Knowing foundational facts must precede derived facts.**

**#4: Timed tests do not assess fluency**

**#5: Students need substantial and enjoyable practice.**

**How can we best help families understand and embrace these fundamentals of fact fluency?**







# The Bottom Line

*Revisited*

In what ways do these ideas and others address family values, expectations and experiences?

1. What do families most **value** about their child's learning of mathematics?
2. What **expectations** do they have about how math is taught?
3. What were their **experiences** in learning the basic facts?





# Engaging Families in Math Fact Fluency





NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS  
CELEBRATING 100 YEARS

# Thank You

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@JBayWilliams

#mathfactfluency

#NCTM100