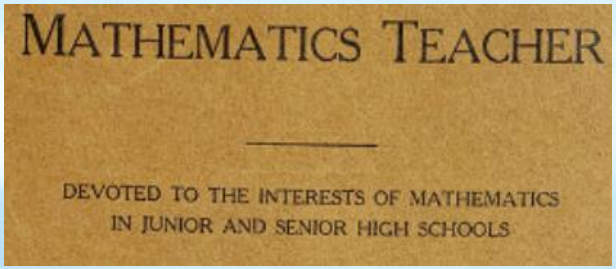
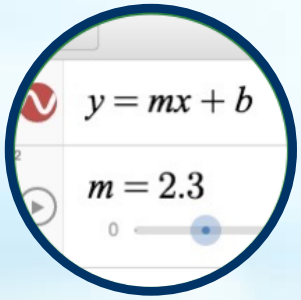
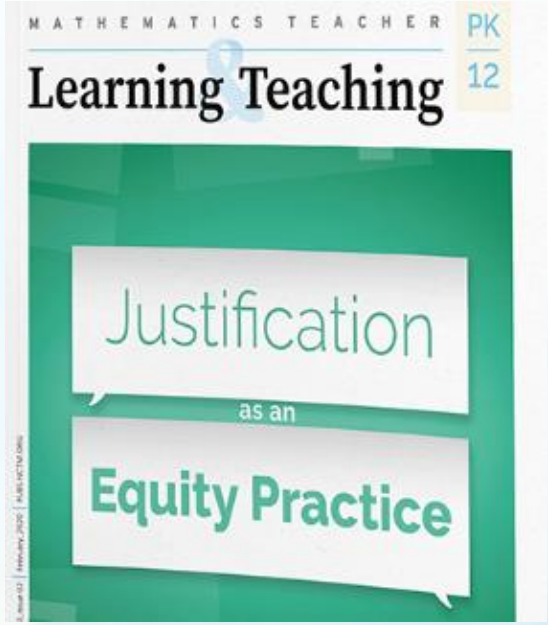
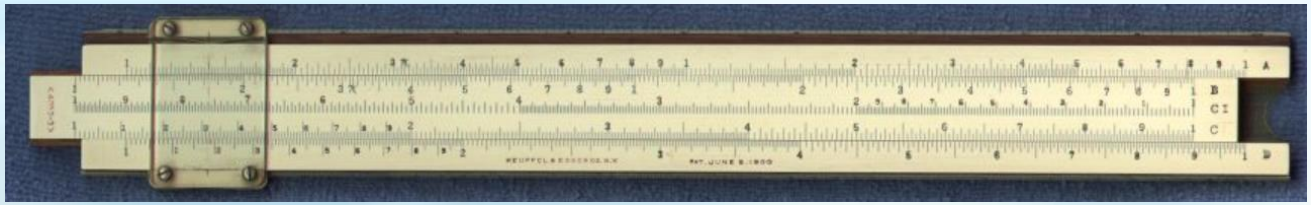
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

Teaching Algebra: A Look Back at the Materials and Methods We Have Used over the Last 100 Years









Presented by: Benjamin Sinwell, Mathematics Teacher
Pendleton High School, Anderson, SC bsinwell@anderson4.org

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

https://www.nctm.org/100timeline/

THE MATHEMATICS TEACHER

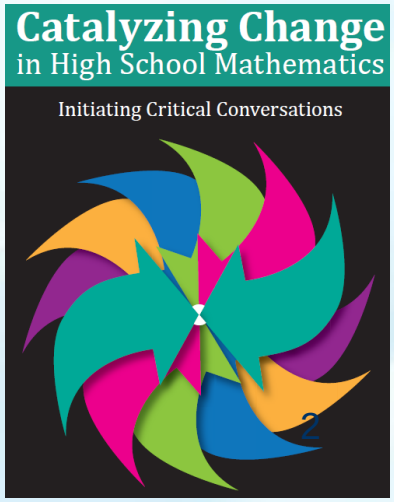
Vol. IV

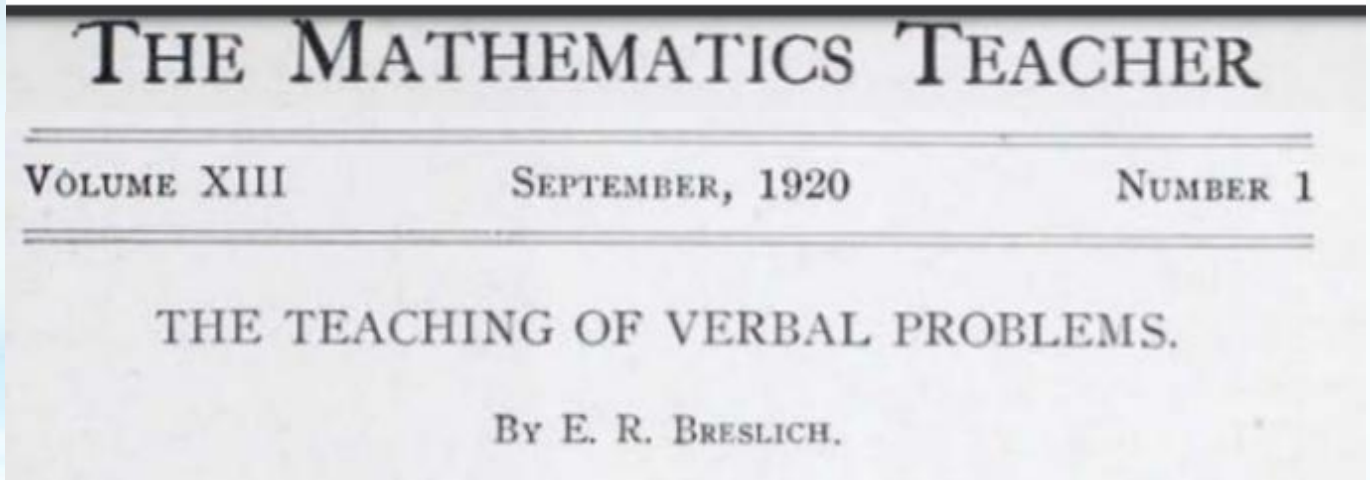
September, 1911—June, 1912

In recent years our teachers of mathematics have divided into two camps: first, those who claim that mathematics is only justified in the curriculum for its practical application; second those who claim that its cultural value, its power to train the reasoning faculties, is its greatest argument for existence in the curriculum.

The purposes of high school mathematics have to do with empowerment. High school mathematics empowers students to—

- expand professional opportunity;
- understand and critique the world; and
- experience wonder, joy, and beauty.




THE MATHEMATICS TEACHER

VOLUME XIII SEPTEMBER, 1920 NUMBER 1

THE TEACHING OF VERBAL PROBLEMS.

BY E. R. BRESLICH.

"Verbal problems in algebra are usually regarded by the pupil as the most difficult part of the course…

…the ideas involved are sometimes remote from the pupil's experience and therefore are of little interest."

1923 Modern Algebra Textbook

Based on Modern Psychology:

- Topics introduced by practical, concrete problem…to arouse interest
- Challenge to the pupil: Tell how to solve the problem or explain how to get started.
- Discovery by the pupil
- Provisions for individual difference…depending on level of independence
- Tried, tested, reviewed, revised, repeatedly

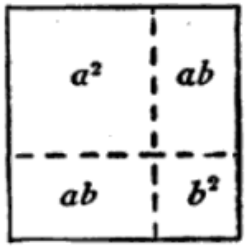
13. The highest office building in the world (the Woolworth Building, New York City) casts a shadow 1267 ft. long at the same time that a boy 5 ft. tall casts a shadow 8 ft. long. What is the height of the building?

Modern Algebra, 1923
Schorling and Clark

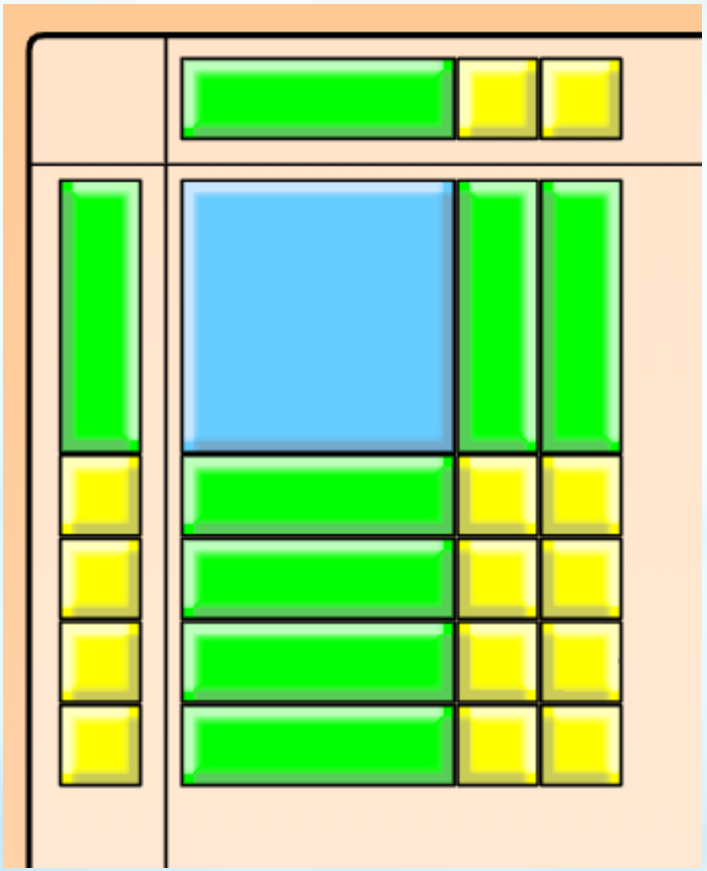
In Surrey, British Columbia, there is a flagpole that is 282 feet tall. It is casting a shadow that is 115 feet long. Justin Beiber sees it and slams on his breaks because it is so big. He gets out of the car, how long is Beiber's shadow?

The mathematical materials represented in these publications have been devised, formulated, criticized, tried in class, revised, tried, and criticized again and again.

5. This figure represents any square, each side of which is $a + b$ units. From the figure you see that the area of the square is $a^2 + 2\,ab + b^2$ square units. What does this have to do with $(f + s)^2 = f^2 + 2\,fs + s^2$?



6. Make a drawing which will show that $(10 + 5)^2 = 10^2 + 2 \cdot 10 \cdot 5 + 25$.



So long as people continue to think about young people, so long will improved ways be found for doing both new and old things.

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Algebra-Tiles/

NCTM “will at all times keep the values and interests of mathematics before the educational world. Instead of continual criticism at educational meetings, we intend to present constructive programs by friends of mathematics. We prefer that curriculum studies and reforms and adjustments come from the teachers of mathematics rather than from educational reformers.”

Charles M. Austin, 1920, First NCTM President

The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

NCTM’s Mission Statement, 2020

The subject that we teach is not an easy one to teach or to be taught; nor is it, I believe, a difficult one unless teacher, or student, or both make it so. This is frequently done, and mathematics is proverbially known as a difficult subject. It is a common belief that a few are born with mathematical powers, and that those who are not so blessed can never hope to gain them. I don't suppose that many of us believe that. I have never come in contact with a non-mathematical mind except as I have encountered the non-logical mind. So far as my experience goes, a student who can reason in any subject can reason in mathematics; and students who *reason* in mathematics succeed in it.

Lynde,
Mathematics Teacher
1920

"In mathematics classrooms, mathematical identity is mediated by how students engage with mathematics. Consequently, mathematics teaching involves not only helping students learn concepts and develop skills and understanding but also empowering students to see themselves as capable of participating in and being doers of mathematics." Catalyzing Change

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

1930

ASSOCIATION OF MATHEMATICS TEACHERS OF NEW JERSEY. REPORT OF THE COMMITTEE OF FIRST-YEAR HIGH-SCHOOL MATHEMATICS.

OUTLINE OF REPORT.

I. The Work of the Committee.
II. List of Topics Recommended for Omission.
III. Simplification.
IV. Fundamental Plan.
V. Correlations.
VI. Textbook.
VII. Teaching.
VIII. The Bright Pupil.
IX. Syllabus.
  1. Formulas and Numerical Substitution.
  2. Equations and Problems.
  3. Graphs and Negative Numbers.
  4. Addition and Subtraction.
  5. Multiplication and Division.
  6. Equations and Problems.
  7. Factoring.
  8. Fractions.
  9. Fractional Equations.
  10. Ratio and Proportion.
  11. Graphs.
  12. Simultaneous Equations.
  13. Square Root.
  14. Exponents.
  15. Radicals.
  16. Quadratics.
X. General Notes.
XI. First Year Mathematics Word List.
XII. Bibliography.
XIII. Note on Intermediate Algebra.

Mathematics Teacher, 1930

1. Connections to algebra
2. Rules of Algebra
3. Solving Linear Equations
4. Graphing Linear Equations
5. Writing Linear Equations
6. Solving and Graphing Linear Inequalities
7. Systems of Equations
8. Powers and Exponents
9. Quadratic Equations
10. Polynomials and Factoring
11. Using Proportions and Rational Equaions
12. Functions
13. Radicals

Algebra 1 text from 2000s

THE MATHEMATICS TEACHER

The Official Journal of
The National Council of Teachers of Mathematics
Incorporated 1928

· MARCH · 1930 ·

« Contents »

Some Difficulties Involved in Solving Verbal Problems in Elementary Algebra.....*Orlie M. Clem and Bertha Adams Hendershot* 141

The Value of Pupils' Examination Papers to the Teacher.....*Joseph B. Orleans* 148

4. Textbooks and teaching aids should give more assistance to teachers in teaching verbal problems. Teacher training courses in mathematics should give more specific attention to problem solving.

The Value of Pupils' Examination Papers to the Teacher

By JOSEPH B. ORLEANS, *Chairman, Mathematics Department, George Washington High School, New York City*

What are the purposes of assessment?

Use the chat box

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

The Value of Pupils' Examination Papers to the Teacher

By JOSEPH B. ORLEANS, *Chairman, Mathematics Department, George Washington High School, New York City*

What are the purposes of assessment?

- Looking at errors in students work.
- Explaining mistakes to the class.
- The teacher learns from the student work.
- To diagnose student learning *Mathematics Teacher,* March 1930

Assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.

Too much weight is placed on results from assessments—particularly large-scale, high-stakes assessments—that emphasize skills and fact recall and fail to give sufficient attention to problem solving and reasoning. *Principles to Action*, NCTM, 2014

	Right	Wrong
The number of cents in d dimes and c cents is	209	190
$2x \cdot 3x =$	312	81
$5a^2 \cdot 4a^3 =$	327	66
If $x = 3$, and $y = 2$, the value of $x^2 - 9xy + 4y^2$ is	136	257
If $a = 2$ and $b = 3$, the value of $a^2 + b^3$ is	215	184
If $c = 5$ and $d = 4$, the value of $3c^2d$ is	252	147
$a(a + 1) =$	175	218

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. 2014, NCTM, Principles to Actions

WELLS AND HART

ALGEBRAS

WHEREVER used the mercury of class achievement climbs higher. Algebra for comprehension, mastery, and appreciation—thoroughly taught, tested, and retaught.

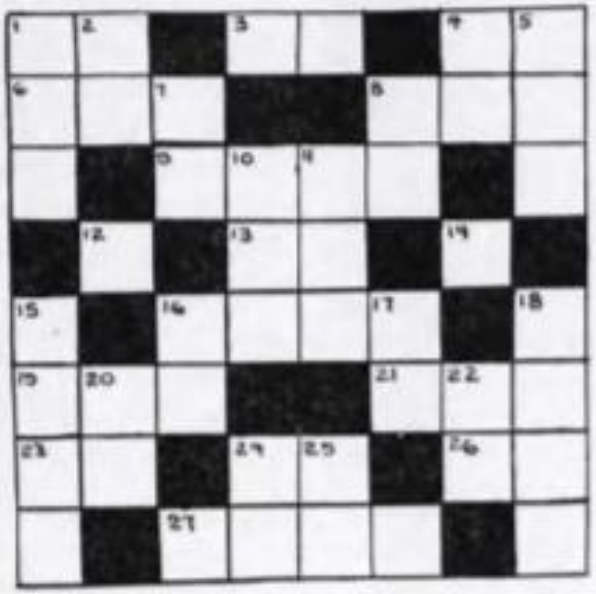
D. C. HEATH AND COMPANY

1940

THE ART OF TEACHING

Crossword Puzzle on Percentage

By C. R. Purdy
Joseph Sears School, Kenilworth, Illinois



Horizontal

1. $\frac{3}{4}$ as a per cent.
3. A salesman paid $400 for a car and sold it at a profit of 20%. What was the profit?

Vertical

1. A salesman sells $7110 worth of goods in two months. What is his commission if the rate of commission is 10%.
2. .53 as a per cent.

Horizontal

1. Express $\frac{3}{4}$ as a per cent.

Vertical

1. A salesman sells $7100 worth of goods in two months. What is his commission if the rate of commission is 10%.

Mathematics Teacher, March 1940

Today when the question of the value of mathematics is raised so often by pupils parents, and administrators, the teacher needs to have a ready answer.

Mathematics Teacher, April 1940

Presenting the Report of the Joint Commission to the National Council of Teachers of Mathematics

By E. R. Breslich
Department of Education, University of Chicago, Chicago, Ill.

The development of the mathematical curriculum of the secondary schools is divided into two periods, one from 1893 to 1923 and the other from 1923 to 1940. The first period begins with the report of the famous *Committee of Ten on Secondary Studies,* the second with the report of the equally famous *Committee on Mathematical Requirements.* Each report has made a lasting impression on the mathematical curriculum. The presentation of the report

taken an active interest in the work of the lower schools adds greatly to the value of this report.

From the beginning it has been the policy of the commission to draw the general teaching public into their discussions. Members of the commission have taken the problems to numerous local groups of teachers for criticisms and suggestions. Two years ago a tentative report was published. This was widely distribs

- In the past, much dependence was placed on mere drill. Recent psychological investigations suggest that all techniques should be based on insight.

- No new topics without providing the background knowledge or prerequisite skills required.

  -- 1940 Joint Report

- Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

- Productive Beliefs: The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others. -- 2014, NCTM, Principles to Action

Using Appropriate Tools

We should no longer expect a teacher of mathematics to depend exclusively on a textbook and a piece of chalk. Precisely as in a modern science classroom, there should be at hand illustrative and enrichment materials of many types, such as models, charts, slides, posters, source books and folders, instruments, designs, outlines, and even films. Notable progress has been made in this direction. An ever increasing number of classrooms is being equipped in this manner. Filing cabinets and projection lanterns are being installed, squared blackboards are being added, and even some apparatus for scientific experiments is being created.

THE ART OF TEACHING

A Simple Blackboard Ellipsograph

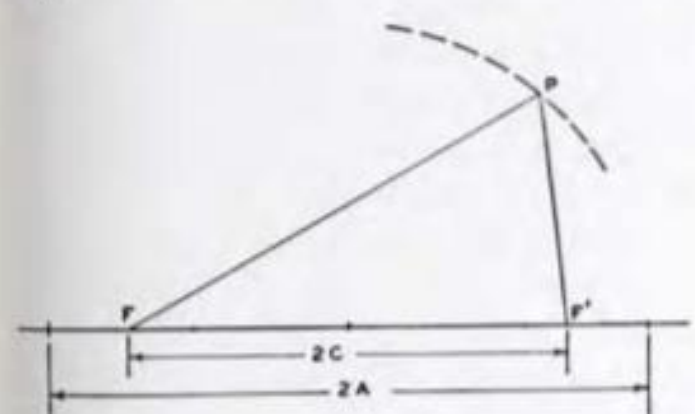
By Haym Kruglak
Milwaukee Vocational School, Milwaukee, Wisconsin

The simplest and best known method of drawing an ellipse is based on the fundamental theorem in conics that in any ellipse the sum of the focal radii is constant and equal to the major axis of the ellipse. This method is widely used by gardeners in laying out elliptical beds, by pattern-makers, metal workers, artists, and others.

The writer found that inexpensive rubber suction cups adhere well to a blackboard. A screw and nut arrangement changes the convexity of the rubber, thus facilitating attachment and removal. These cups may be bought in any auto supply store or 5-and-10 cent store.

Two such cups are attached at F and F' respectively. A looped piece of string is

string of length $2a+2c$ is made to enclose F and F'. A moving marker P pressed tightly against the string traces out an ellipse.



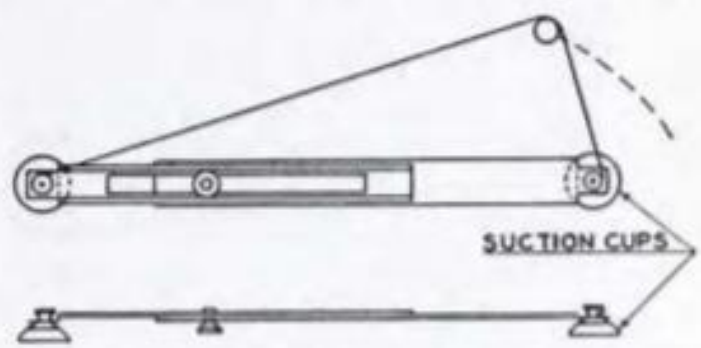
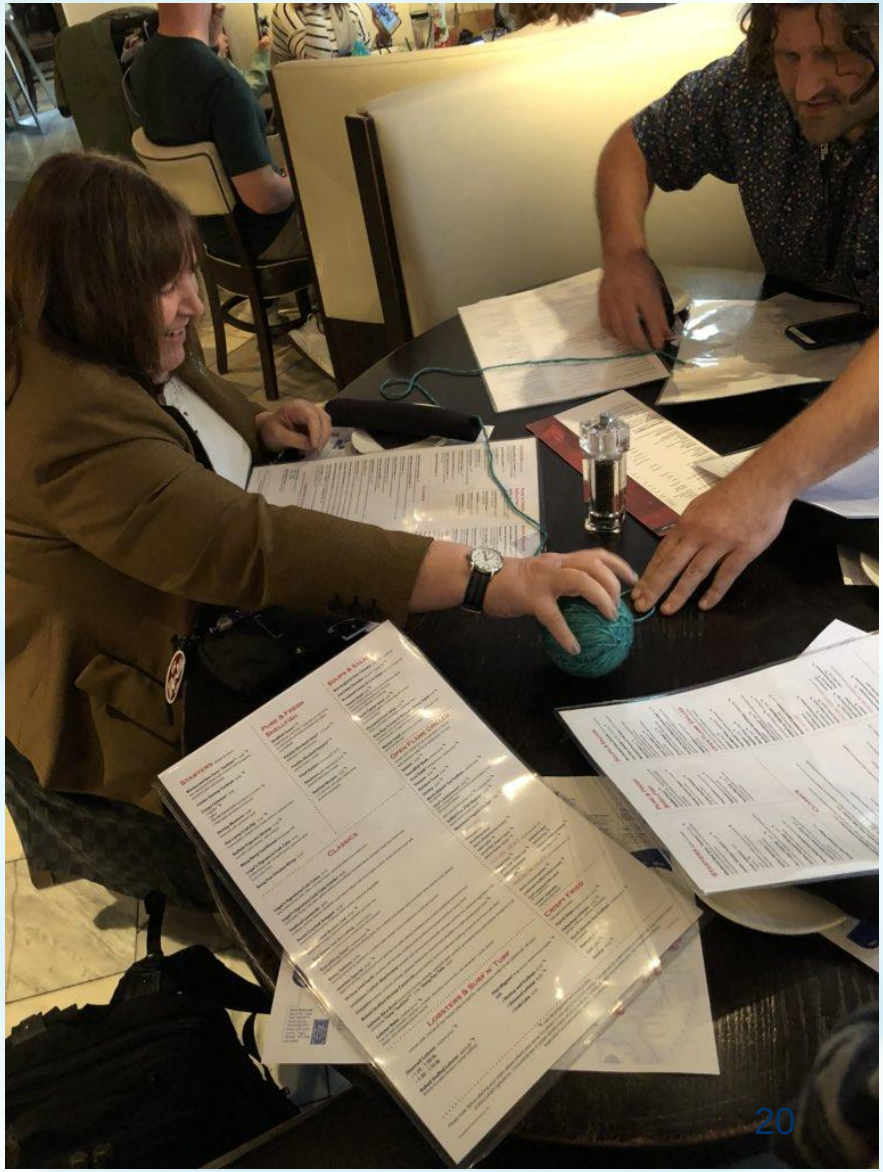


Fig. 2. Blackboard Ellipsograph

Figure 2 shows a simple ellipsograph in which the cups are mounted on a sheet metal frame. The distance between the



GeoGebra Calculator Suite

SIGN IN

A = (-8.09, -6.49)

B = (6.93, -6.49)

C = (-5.6, -1.4)

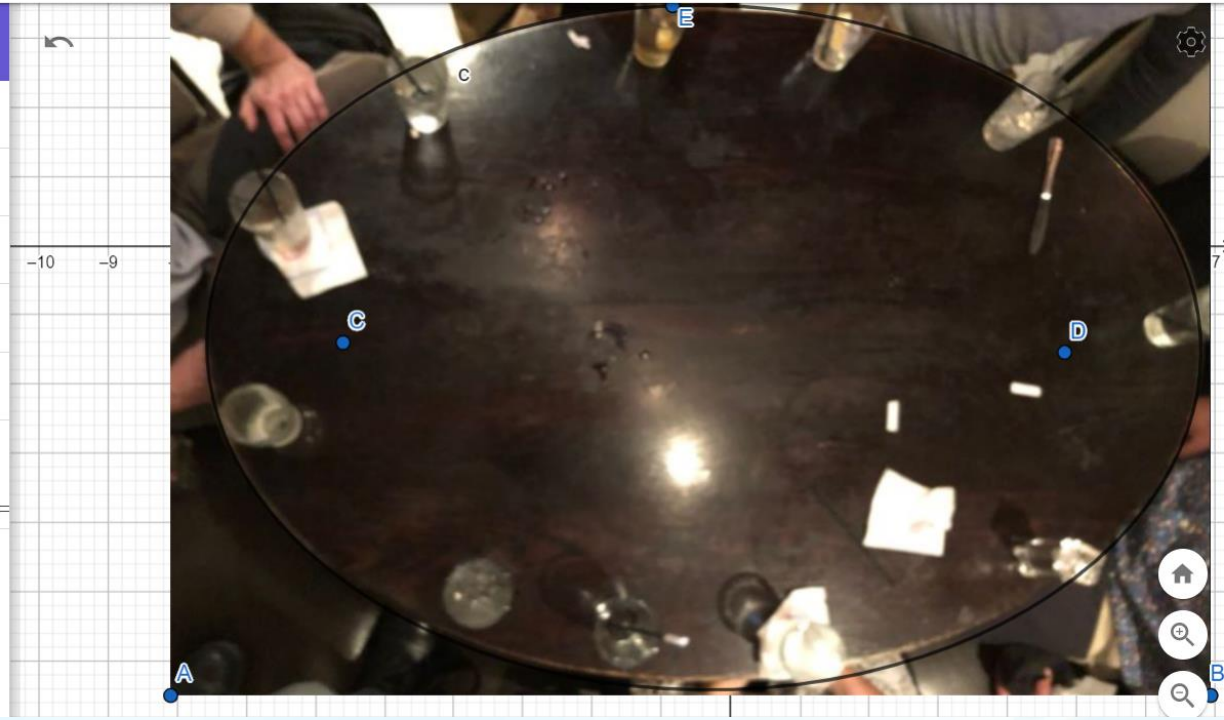
D = (4.82, -1.54)

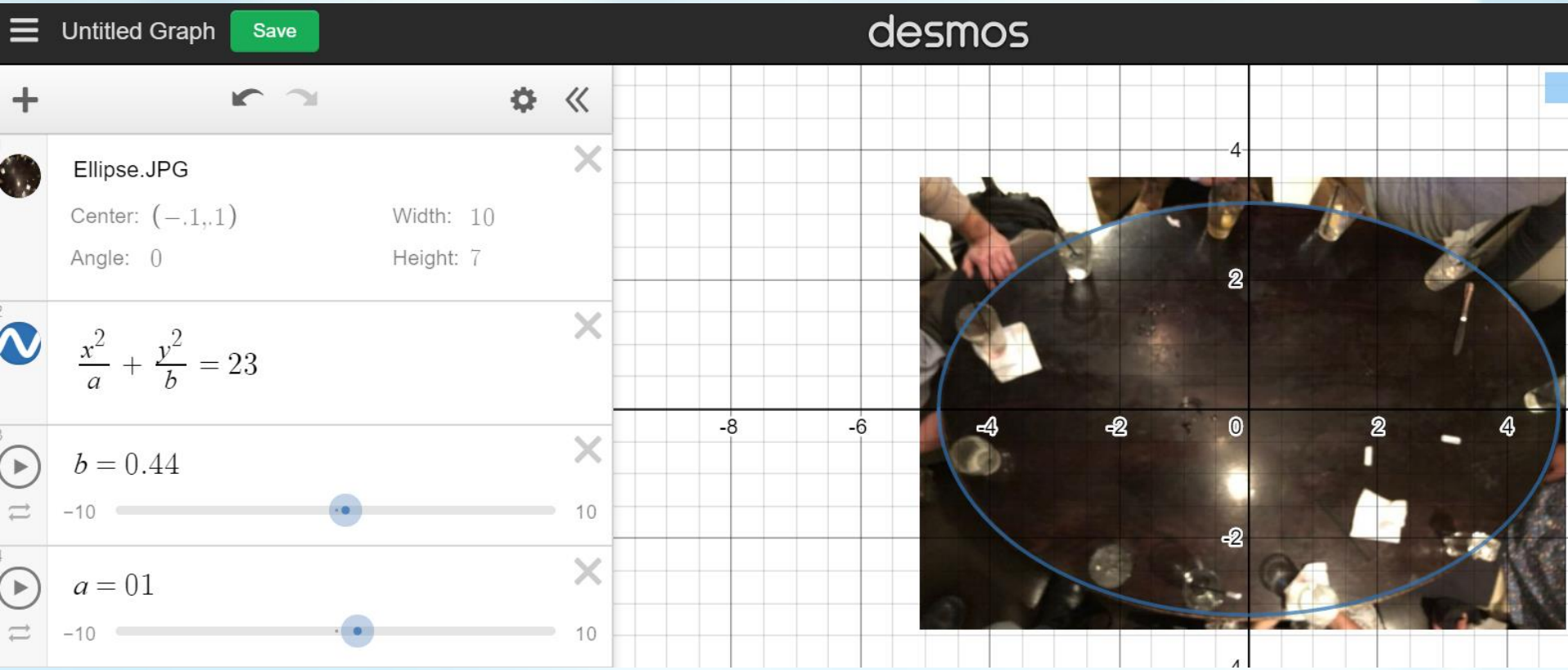
E = (-0.84, 3.46)

c : Ellipse(C, D, E)

→ $389.95x^2 + 11.67x\,y + 824.18y^2 + 321.32x + 2427.63y =$

Input...




Untitled Graph
Save
desmos
Ellipse.JPG
Center: (−.1,.1)
Width: 10
Angle: 0
Height: 7
$\frac{x^2}{a} + \frac{y^2}{b} = 23$
$b = 0.44$
−10
10
$a = 01$
−10
10
-8
-6
-4
-2
0
2
4
2
-2
4

1940 - Almost half of students will not take make beyond 8th grade

- What is the plan for the “other half” ? No one best pattern of instruction.
- It is possible to find a particular method and content for teaching mathematics as the best for all students at all times.
- Mathematics teachers are too hesitant to advertise themselves as teacher of democracy, and as promoters of social welfare, or even as promulgators of one of the social sciences. *Mathematics Teacher, 1940*
- When high school mathematics is approached as a tool to give students a better understanding of their roles as members of our democratic society, students are more likely to actively engage in their communities and to appreciate their potential power to challenge injustices and contribute to societal improvement (Gutstein and Peterson
- 2013). From Catalyzing Change in High School Mathematcs, 2018

2020 – Robert Berry, NCTM, Past-President

- Our love for mathematics, for students, and for communities is a common thread that binds teachers of mathematics together. It explains the willingness of teachers to engage in activities supportive of building community; to engage in professional networks, observing and providing feedback to one another, deepening knowledge of mathematics content and pedagogy, and taking time to learn about our students and their communities; and to engage in critical conversations on issues impacting mathematics teaching and learning.

What year was your first year of teaching?

If you remember, what were you teaching?

What is a tool or resource you used then that you no longer use?

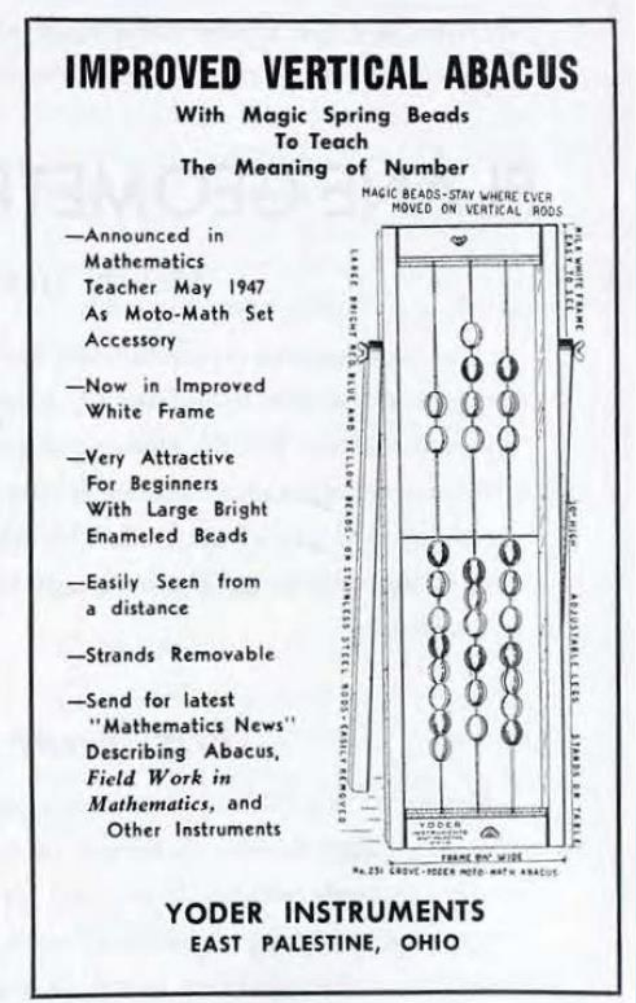
Mathematics Teacher, April 1950

- Inclusion of larger and larger numbers of children in public education has a profound effect upon the thinking of those responsible for that education.
- What would this experience in school do for all these children?
- How would the state and nation benefit, in order to justify the cost…
- Without steady progress in evaluation [standardized tests], as well as in curriculum reform, adverse criticism of mathematics instruction, by colleges and the general public, will continue.

V. Which Visual Aid Will Be Used Today?

Leader: Nelle Kitchens, Hickman High High School, Columbia, Missouri

Visual aids are "springboards for discussion," not a substitute for serious mental effort. They should be introduced early in the course before interest is lost. They reduce tension and increase interest. Specific aids mentioned are: blackboard, Bulletin board, slides, models, supplementary texts, field trips, library, films, anecdotes and quotations, files containing current articles, flash cards, mathematics club programs, resource units, and exhibits.



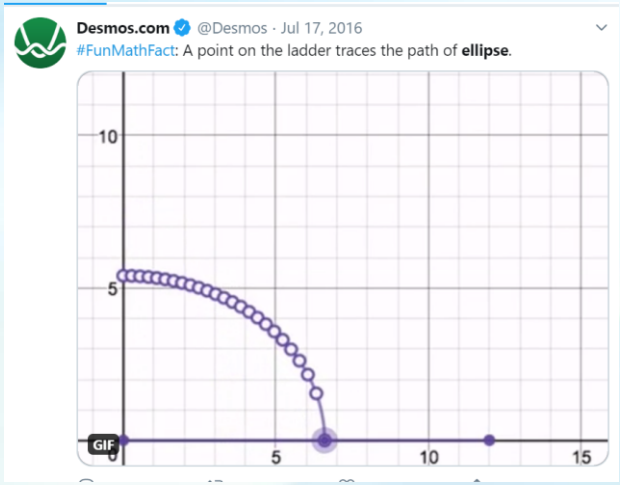
"Mathematics teachers should judiciously adopt technology that supports effective instruction but not simply for the sake of using more technology in the classroom."
Principles to Actions, NCTM, 2014

DISCUSSION groups are becoming more popular and more valuable with each meeting of the Council. It is interesting to note that teachers in distant parts of the country have the same teaching problems to solve.

Where do you go to discuss teaching mathematics?

Discussions

- https://mathcircles.org/find-a-math-circle/
- www.mtbos.org (Twitter)
- https://my.nctm.org/browse/allrecentposts
- Conferences (Local, State, Regional, National)
- Reddit

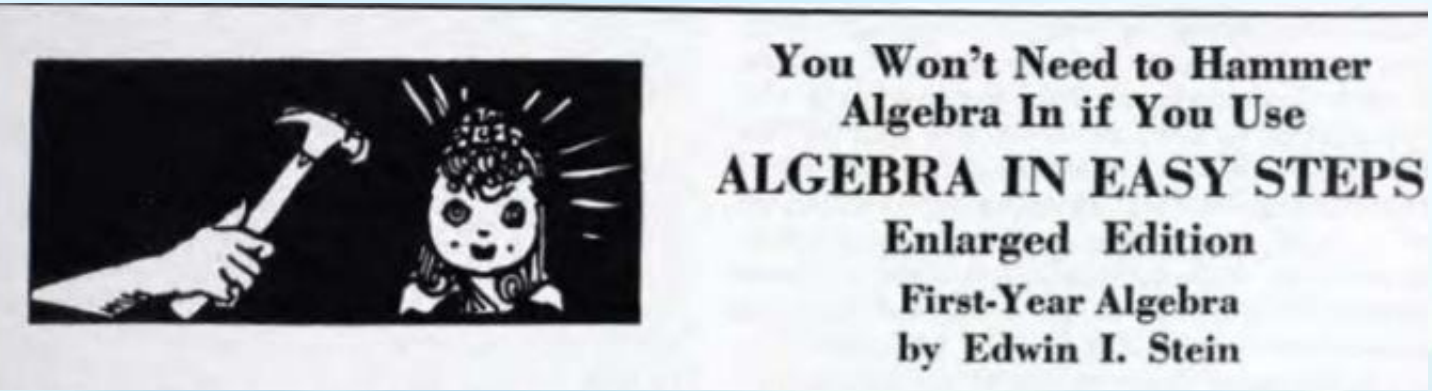


DISCUSSION groups are becoming more popular and more valuable with each meeting of the Council. It is interesting to note that teachers in distant parts of the country have the same teaching problems to solve. Here are brief summaries of a few

7. Some grade daily work in class, others check daily papers but do not grade daily work, and others correct daily work and hand it back ungraded.

8. Much of the teaching is done in the class period. It is becoming increasingly difficult to get home work done.

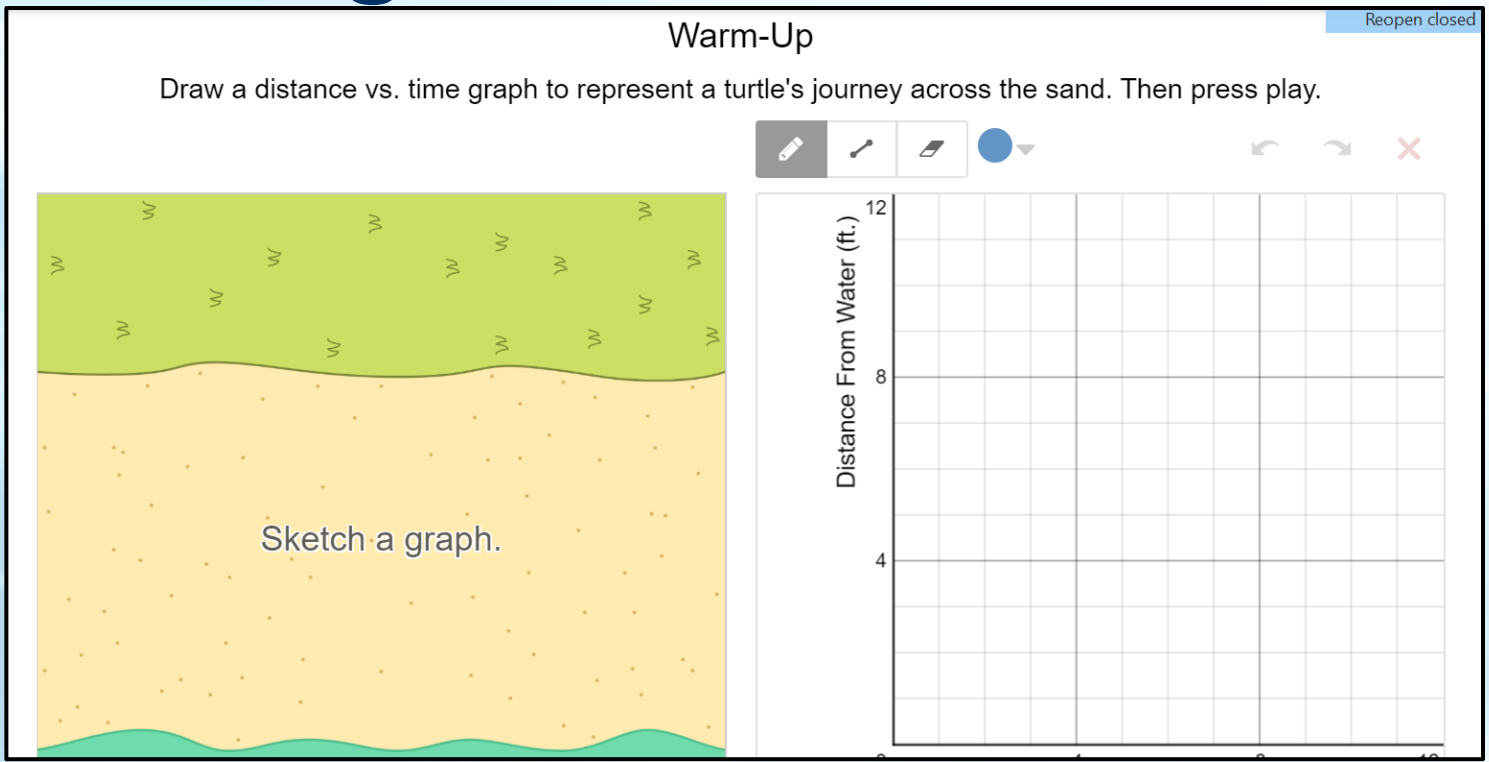
Beliefs about teaching and learning mathematics	
Unproductive beliefs	**Productive beliefs**
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.



Try ALGEBRA IN EASY STEPS with Any First-Year Pupil!

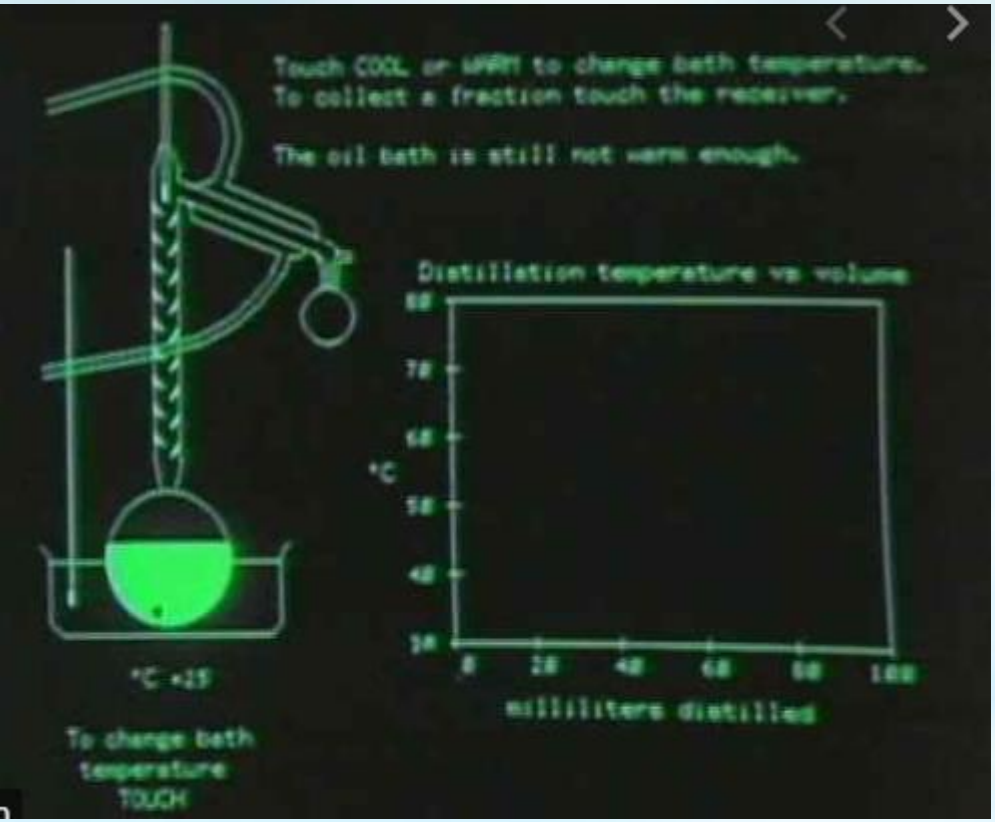
https://student.desmos.com/join/8pb2a5

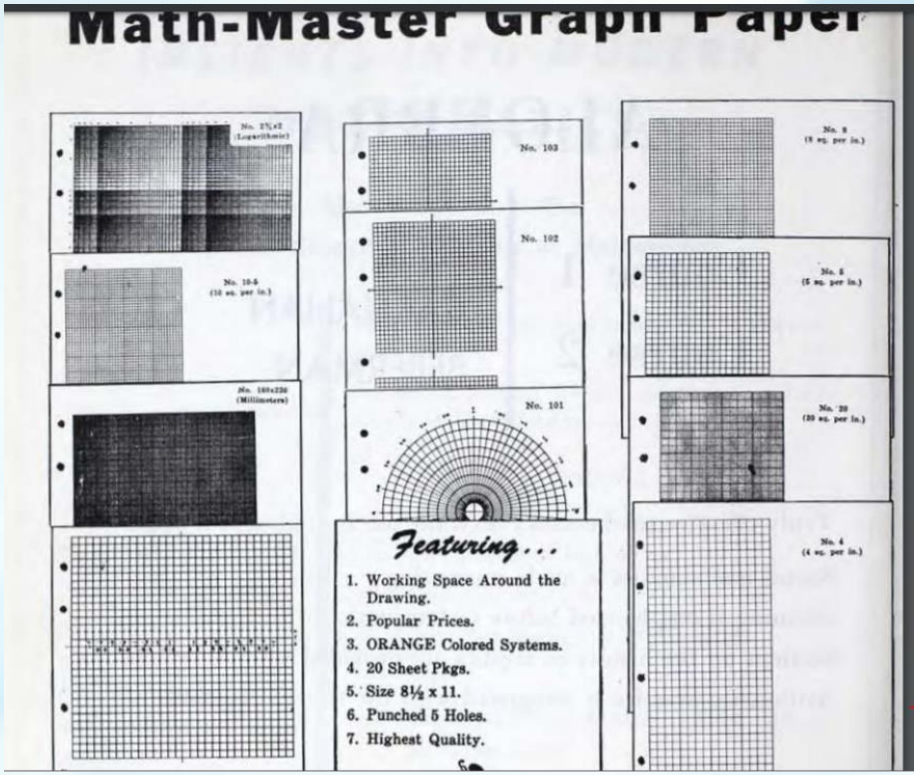
Take 3 or 4 minutes to work through a slide or 2 or 3…



1960 – Algebra or General Mathematics?

- An appreciation of mathematics is a cultural necessity and a command of its techniques is a practical occupational need
- One phenomenon associated with society is the sharply dwindling market for unskilled labor and the consequently enhanced value of scientific and technical education.
- High-speed computers
- 1955 was “Teaching Arithmetic With Calculators” in Elementary School
- March toward almost every American completing high school
- Students with every type of motivation of lack of it





Plato on the ILLIAC

1930s Flashback! From 1960.

From *The Mathematics Teacher* of thirty years ago

" . . . The fact remains that a large number of pupils cannot handle fractions either common or decimal, and that lack makes their algebra work very difficult. There is evidence also of weakness in the very fundamentals of arithmetic. . . .—*Joseph B. Orleans and Jacob S. Orleans*, "A Study of Prognosis in High School Algebra."

Algebra 1 in the 8th grade. 1960

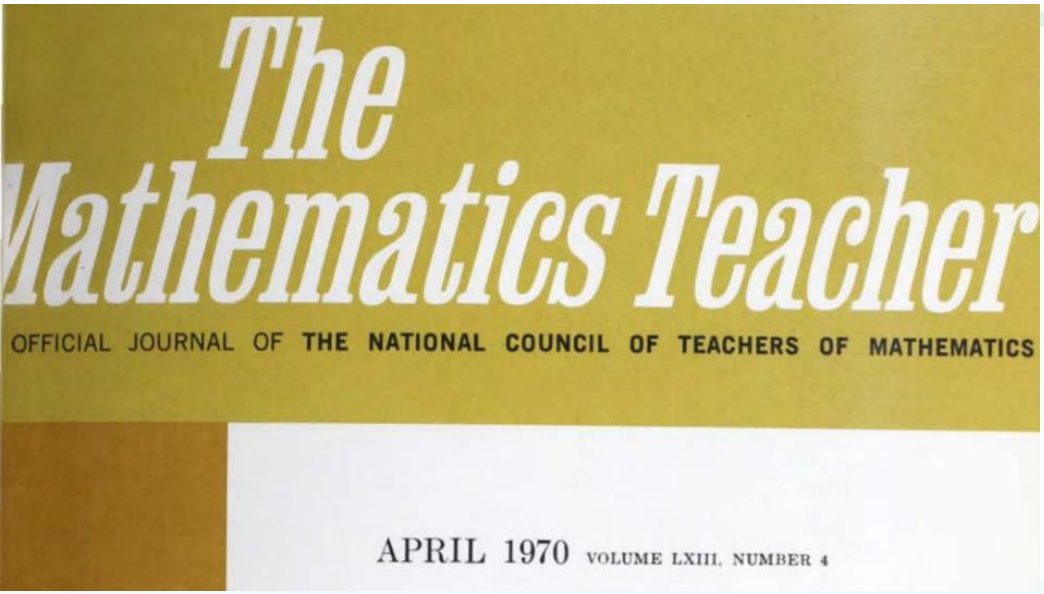
- 1960 Recommendation: Combine 7&*8th grade mathematics and take Algebra 1 in the 8th grade
- In 1990, only 16% enrolled in an algebra course, compared to 20% in pre-algebra and 61% in 8th grade math
- By 2011, nearly half (47%) of all eighth graders took algebra or a more advanced course. Only 48% were in a basic math course, down from 81% in 1990.

Catalyzing Change, 2018

High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

April 1970



Teaching that:

- Anticipates student inquiry
- Opportunities for student conjecture and exploration
- Students learning from discussion

2014, NCTM, *Principles to Actions,*

Effective Mathematical Teaching Practices

Pose Purposeful Questions

Facilitate Meaningful Mathematical Discourse

LIVELY FUNCTIONS for ALGEBRA ONE

By LOUIS R. McCREERY
San Marino High School
San Marino, California

THE abstract ideas making up the con-
cept of function can be made to come alive
n the everyday experiences of a begin-
ning algebra student. For instance, the gas
pump at the corner service station pro-
duces a realistic example. The gallon indi-

it with the student's experience, to show that this "stuff" has some immediate relevance. Two, applications give the idea itself meaning and reinforce the student's understanding. Some pupils may be able to learn in the abstract, but most need a concrete example before they really *know* an idea. Mathematics taught for itself alone is not enough for many of our students in Algebra I.

May, 1969, *Mathematics Teacher*

y =mx + b

Can you imagine a rate problem where b is not zero? (Ans. When a car is b miles away from an arbitrary starting point at the zero hour.)

Where is m in an interest problem? (Two or perhaps three answers here.)

What might b stand for in an interest problem?

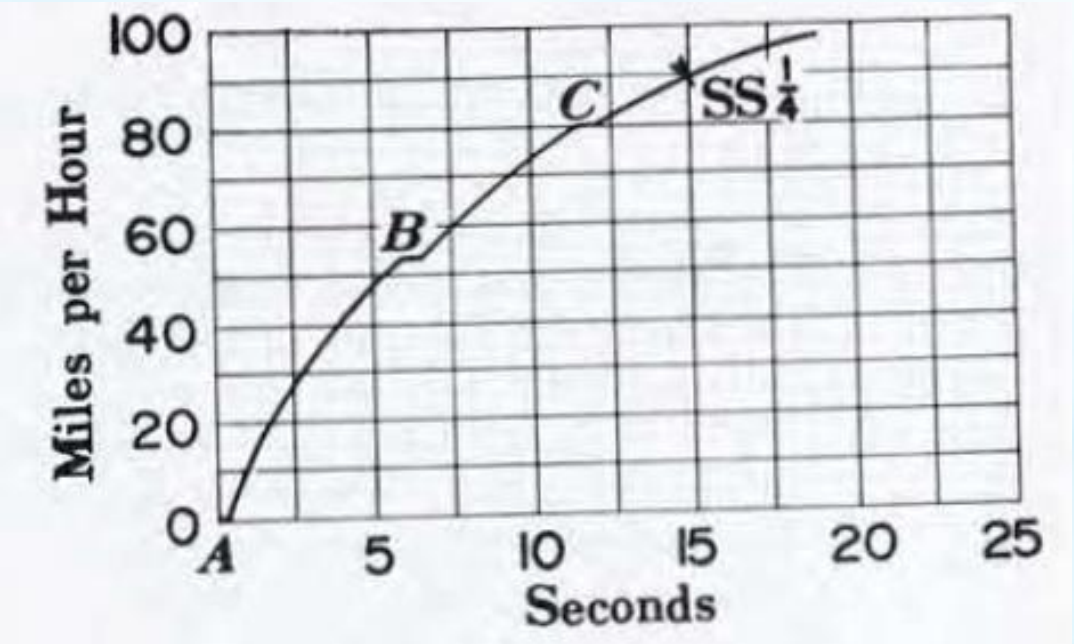
Can you make an equation of the form $y = mx + b$ for a printer's bid of fifty dollars plus three cents a copy, where x is the number of copies? (Total cost $= .03x + \$50.00$)

If it costs two dollars for the initiation fee to a club, and the dues are fifty cents a month, what is the amount paid in x months?

What are the slope and the intercept in each of the last two questions?

Through such questions the students understand that slope and intercept are powerful and immediately useful concepts.

Mustang 350 GT: Acceleration From a Standing Start.



From the "SS $\frac{1}{4}$" point can be read the elapsed time to cover a quarter-mile and the rate of speed at that time. SS $\frac{1}{4}$ means "from a standing start to a quarter-mile."

Now pick a fairly straight piece of the curve and continue questioning:

Just what does the slope mean here?

What does a change in y divided by the change in x mean?

What is a change in miles per hour with respect to time?

What questions could we ask students?

Use the chat box.

Excellence in Mathematics Education—For All

EXCELLENCE IN MATHEMATICS EDUCATION—FOR ALL is the goal to which the NCTM looks forward. As the great accomplishments of the last fifty years toward this goal are recounted, they challenge us to meet the dawning fifty with even greater vigor. What are some of these challenges?

Students are still dropping out of school because their interest in mathematics has not been "turned on." How can we make mathematics more relevant to them? Students are progressing at ever-varying rates. How can we provide the best individualized programs? Computers are assuming an overwhelming place in modern life. What is the responsibility of mathematics teachers and NCTM in this field? School organizations are changing to adjust to research findings. What are the responsibilities of the NCTM to administrators and curriculum directors? Evaluation and school marks are a constant challenge to teachers and a source of frustration to students and parents. How can the NCTM

Our astronauts, thanks to the mathematics of celestial navigation and the technology of the twentieth century, have conquered the moon. How can mathematics help in the next steps to Mars and to other universes? Let us all pitch in and help the NCTM meet these challenges!

Sir Isaac Newton said that the reason he could see so far was that he stood on the shoulders of giants. Similarly, are we in the NCTM not standing on the shoulders of the dedicated and inspired workers of the past fifty years? Let us all rededicate ourselves to carry on the great work of the NCTM, to which we say on its birthday, "May the next fifty years be even more glorious than the first fifty!"

Veryl Schult, the author of this article, was formerly supervising director of mathematics in the District of Columbia Public Schools. She is at present with the U.S. Office of Education, Washington, D.C., and is chairman of the NCTM Fiftieth Anniversary Committee.

1969-1970

Get your whole class involved with a computer for teenagers.

$142,000

Now you can speed up the learning process even more—with a computer teaching aid called the HP 2007A Educational System. The whole class gets into the act since the system handles up to 50 programs in a single classroom period.

For $21,400, your school gets a computer, card reader and teletype. That's only about 18 cents per student hour. Not a high price to pay for helping even the slowest student tackle advanced math, chemistry and physics problems.

Simplicity is the key to our system's success. The computer uses the easiest language around: HP BASIC. It's the closest thing you'll find to conversational English. So most students are conversing freely in 3 to 4 hours.

But there's an even bigger benefit: Our system gives students more time for learning and teachers more time for teaching. The secret is our unique Optical Mark Reader. With it, students can run programs by marking computer cards with ordinary pencils. They can even mark the cards at home, then check their homework answers the next day on the computer.

The teen age is fast becoming the computer age. So don't short change your students. Our computer can help bridge the generation gap—for an individual school or an entire district. (Our 2000A Time Share System hooks up 16 schools to a single computer.) Get the full story on both. Write: Hewlett-Packard, Palo Alto, Calif. 94304; Europe: 1217 Meyrin-Geneva, Switzerland.

HEWLETT hp PACKARD

DIGITAL COMPUTERS

1972 – TI Handhelds

1980

- Teachers and Sex Bias In Mathematics

By ELIZABETH FENNEMA
University of Wisconsin—Madison
Madison, WI 53706

Females are receiving inadequate mathematical education in high schools.

Teachers are important! Teachers can influence females and they should! As Grace Burton (1979) so aptly said, it is "the gentle persuasion of secondary school teachers of mathematics" that will enforce a change in the mathematics education of women.

- Low Achievers and High Achievers

January, 1983, *Mathematics Teacher*

EQUITY IN MATHEMATICS: A CASE STUDY

By B. ROSS TAYLOR
Minneapolis Public Schools
Minneapolis, MN 55413

In recent years, underrepresentation in enrollments in mathematics courses by females and minorities has started to receive considerable attention. For example, in 1979 NCTM established a task force on "Problems in the Mathematics Education of Girls and Young Women." NCTM also received a grant from the National Science Foundation to conduct a series of conferences on equity issues in 1982 and 1983. In view of the increasing

sexism. At the conclusion of these workshops, each school and department was asked to identify examples of institutional racism and sexism and to take corrective measures. The underrepresentation of minorities and females in college-preparatory mathematics classes appeared as a potential consequence of institutional racism and sexism.

Assessing the status quo

Before the implementation of intervention strategies, an assessment was conducted to determine the scope and magnitude of the problem. The Minneapolis

1986 Benjamin Banneker Association

Summary and Conclusion

The experience in Minneapolis supports the concept that the first step in correcting inequities is the development of an awareness that inequities exist. The Minneapolis experience also supports the notion that once awareness is established, the problems should be clearly identified by means of an assessment of patterns of participation (and also, if possible, achievement). Then the solutions to the problems can be developed and implemented in the form of intervention programs. Mathematicians should have no difficulty avoiding the often counterproductive approach of starting with a solution and then trying to identify the problem.

0 = 1 = 8/12?

One side of the label of a Pepsi Light eight-ounce can reports that it contains 1 calorie. Another portion of the label reports that it contains no calories. The truth is that it contains 8/12 of a calorie.

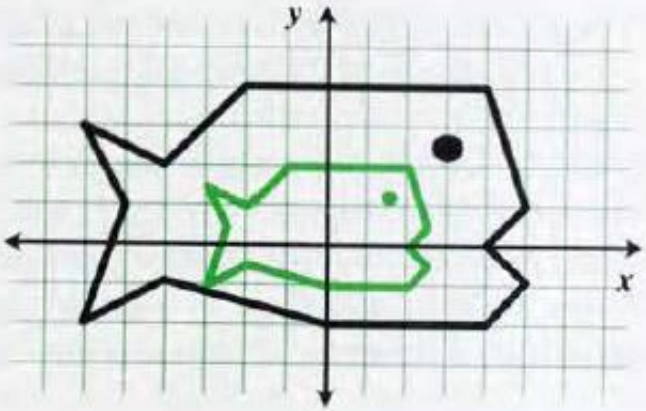
One label rounds the 8/12 to the nearest integral value of 1. For the other label, the Food and Drug Administration regulations require that products must be labeled with caloric content per serving and that the calories per serving must be rounded to the nearest multiple of 2. Since an eight-ounce can is one serving, the 8/12 is rounded to 0, the nearest multiple of 2.

1990

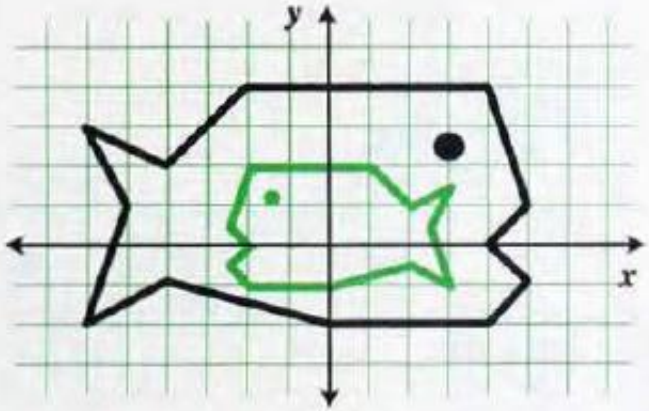
Are Teachers Prepared to Implement the Standards?

Iris M. Carl is elected NCTM's first African American president. During her tenure as President (1990–1992), Carl helps catapult NCTM into the forefront of the public debate on establishing and adopting national academic standards.

For many students, working with such complex figures as nonconvex polygons, animals, or household objects is easier than working with triangles or rectangles. In figure 3a, the image of each point (x, y) of the larger fish is the point $(1/2\ x,\ 1/2\ y)$, resulting in the smaller fish in its interior.



(a)



(b)

What rule can we use to transform the big fish in **figure b** to the small fish in **figure b**?

What rule can we use to transform small fish in **figure a** to the small fish in **figure b**?

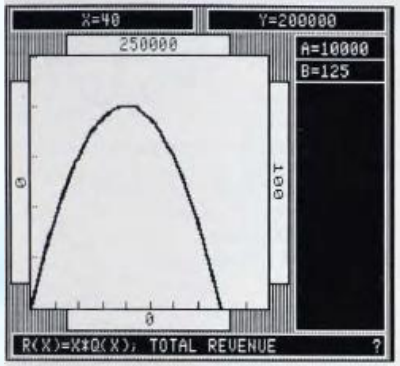
(b)

Fig. 3. Some students enjoy working with amusing images.

Graphics Calculator

Math software for the '90s

Free Demo Disk



In its *Curriculum and Evaluation Standards for School Mathematics*, the NCTM recommends that high school mathematics emphasize:

- the use of computer utilities to develop conceptual understanding
- the use of graphing utilities to solve equations and inequalities
- the connections among a problem situation, its model as a function, and the graph of that function

(at left) GRAPHICS MODE: The revenue R(x) has a maximum at a price of 40 dollars.

Graphics Calculator supports all of these objectives.

It is a multi-featured exploratory tool with three fully integrated modes:

- CALCULATOR Mode: a multi-parameter, multi-function calculator
- ARRAY Mode: a scrolling table of function values
- GRAPHICS Mode: a versatile, quick graphics display with additional features

```
F(X)=0
G(X)=0
H(X)=0
R(X)=X*Q(X) ; TOTAL REVENUE
S(X)=0
Q(X)=A-B*X ; DEMAND (QNTY SOLD AT X)

A=10000          L=0
B=125            M=0
C=0              N=0
D=0              O=0
E=0              P=3.14159266
I=0              U=0
J=0              V=0
K=0              W=0

B=125 ; (QUANTITY/PRICE)
```

```
F(X)=0
G(X)=0
H(X)=0
R(X)=X*Q(X) ; TOTAL REVENUE
S(X)=0
Q(X)=A-B*X ; DEMAND (QNTY SOLD AT X)

 X    R(X)
-----------------------------------
-10   -112500
0     0
10    87500
20    150000
30    187500
40    200000
50    187500
60    150000
70    87500
80    0
90    -112500

PRESS ARROWS TO SCROLL
```

(far left) CALCULATOR MODE: Q(x) is the demand function for the price x.

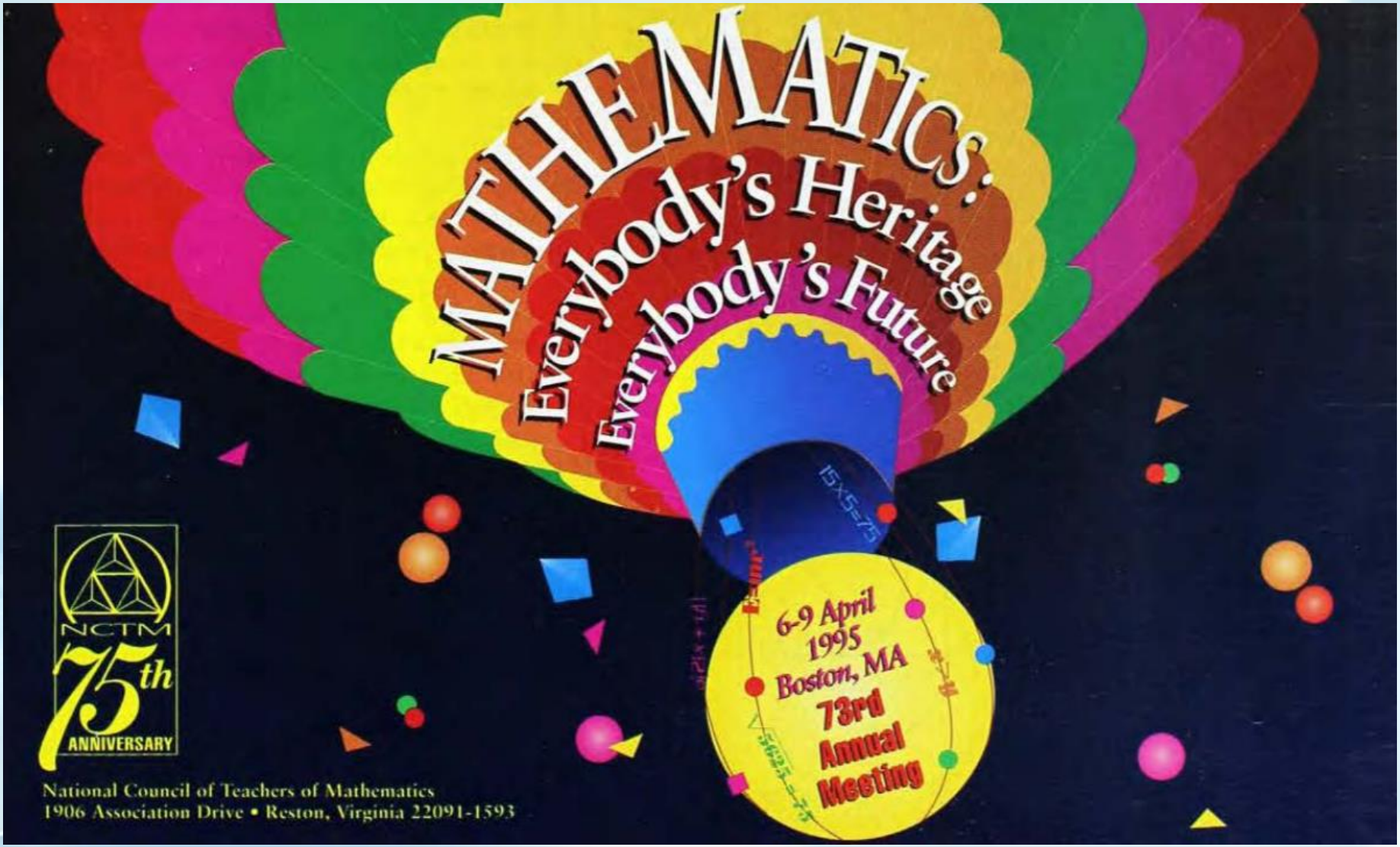
(at left) ARRAY MODE: Table of values for R(x) corresponding to the graph above.

Available for the Apple II and Apple IIgs. Single package, $75. EdPack6, $150.

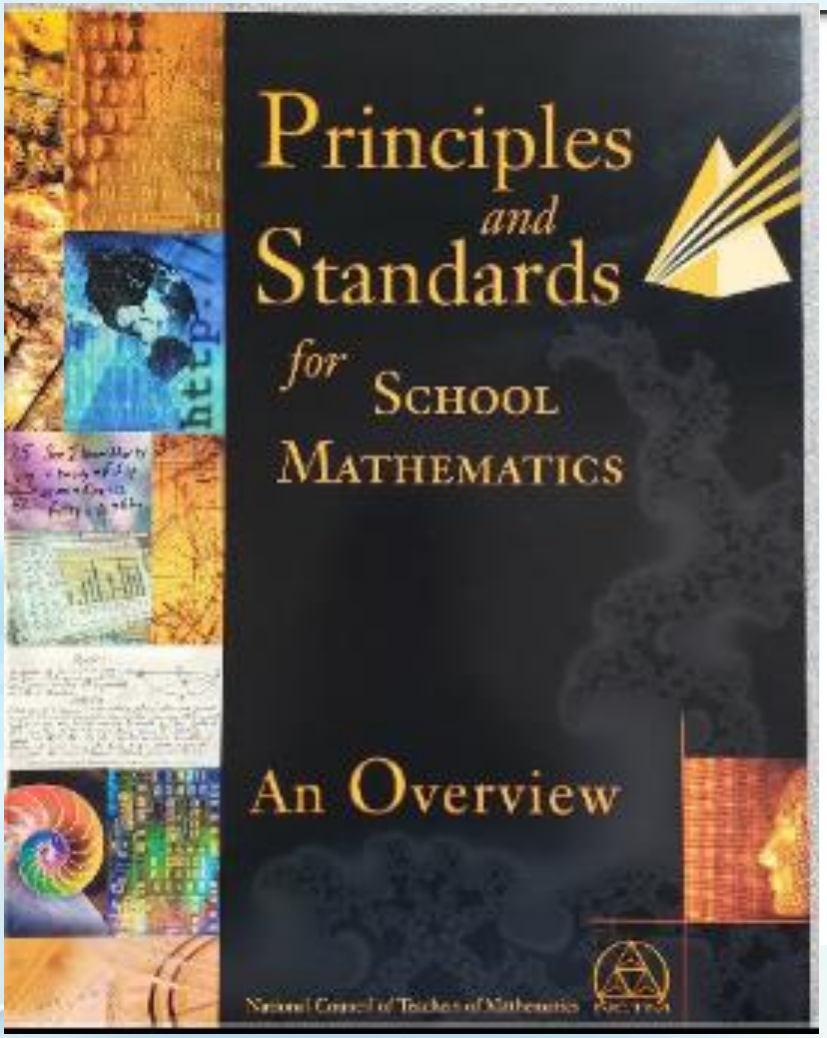
Call 1-800-365-9774/Dept. W for a free 5.25" demo disk.

CONDUIT Educational Software

CONDUIT / The University of Iowa / Oakdale Campus / Iowa City, IA 52242


MATHEMATICS:
Everybody's Heritage
Everybody's Future
6-9 April
1995
Boston, MA
73rd
Annual
Meeting
NCTM
75th
ANNIVERSARY
National Council of Teachers of Mathematics
1906 Association Drive • Reston, Virginia 22091-1593

2000



Six Principles for

Equity. *Excellence in mathematics education requires equity—high expectations and strong support for all students.*

All students, regardless of their personal characteristics, backgrounds, or physical challenges, can learn mathematics when they have access to high-quality mathematics instruction. Equity does not mean that every student should receive identical instruction. Rather, it demands that reasonable and appropriate accommodations be made and appropriately challenging content be included to promote access and attainment for *all* students.

ALGEBRA for All

Ruth Feigenbaum

Algebra for Students with Learning Disabilities

I use what my students have dubbed the "imaginary 1" as a reading aid in simplifying algebraic expressions, for example,

$$2x + x = 2x + \mathbf{1}x,$$
$$a^3 \bullet a = a^3 \bullet a^1.$$

THE FOUNDING OF TODOS

TODOS: Mathematics for ALL is a mathematics equity organization established through initial efforts from the Equity and Diversity Advisory Committee of NCTM. The mission of TODOS is to advocate for equity and high quality mathematics education for all students—in particular, Latinx students.

Find out more

DELVING deeper

Benjamin Sinwell

The Chebyshev Polynomials: Patterns and Derivation

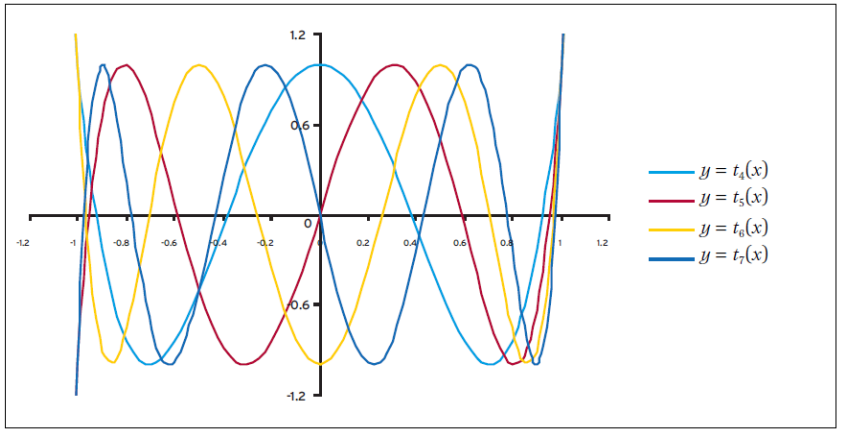


Fig. 2 The graphs of the second four Chebyshev polynomials

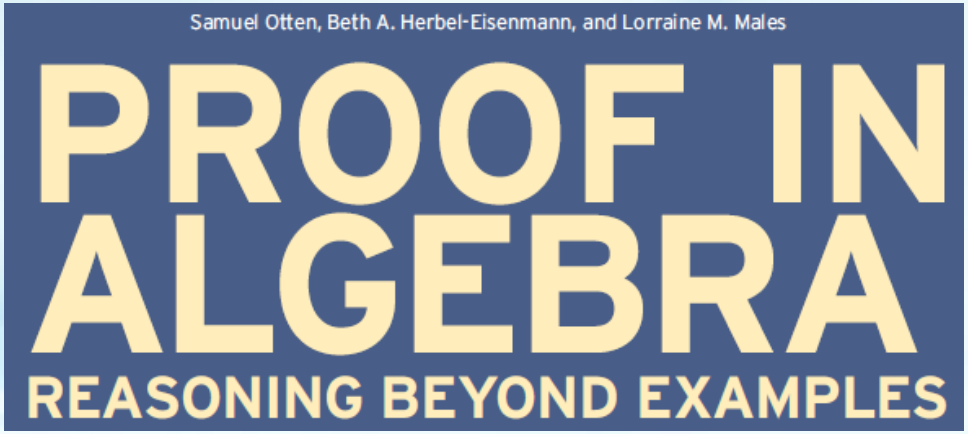
$$
\begin{aligned}
t_0(x) &= 1 &&= t(0, 0)x^0 \\
t_1(x) &= x &&= t(1, 0)x^1 \\
t_2(x) &= 2x^2 - 1 &&= t(2, 0)x^2 + t(2, 1)x^0 \\
t_3(x) &= 4x^3 - 3x &&= t(3, 0)x^3 + t(3, 1)x \\
t_4(x) &= 8x^4 - 8x^2 + 1 &&= t(4, 0)x^4 + t(4, 1)x^2 + t(4, 2)x^0 \\
t_5(x) &= 16x^5 - 20x^3 + 5x &&= t(5, 0)x^5 + t(5, 1)x^3 + t(5, 2)x \\
t_6(x) &= 32x^6 - 48x^4 + 18x^2 - 1 &&= t(6, 0)x^6 + t(6, 1)x^4 + t(6, 2)x^2 + t(6, 3)x^0 \\
t_7(x) &= 64x^7 - 112x^5 + 56x^3 - 7x &&= t(7, 0)x^7 + t(7, 1)x^5 + t(7, 2)x^3 + t(7, 3)x
\end{aligned}
$$

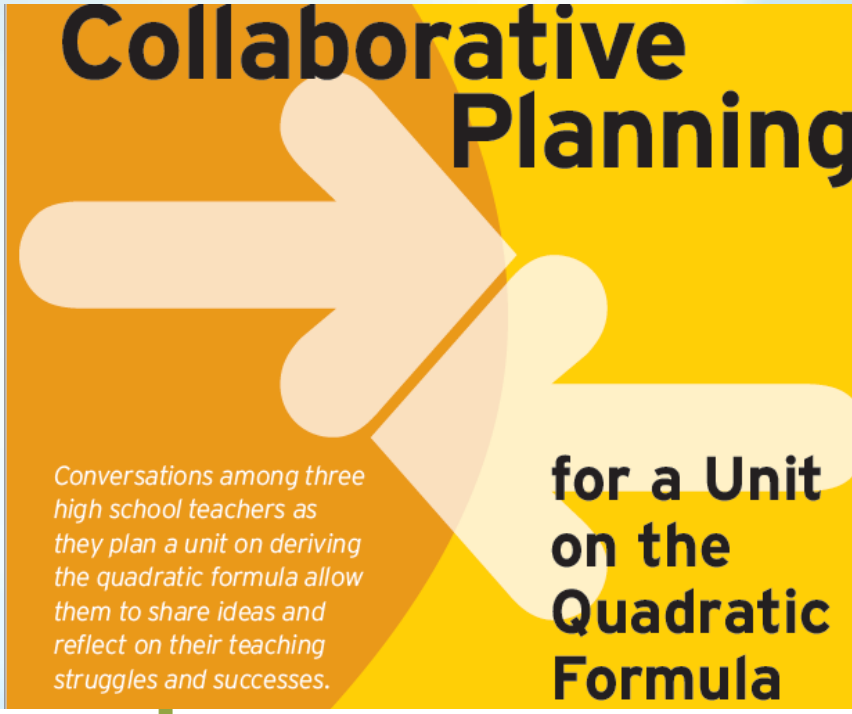
Fig. 3 Looking for patterns in the coefficients of the Chebyshev polynomials

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

2010

Common Core State Standards





CONTEMPORARY curriculum issues

Organizing a Curriculum around Mathematical Habits of Mind

Al Cuoco, E. Paul Goldenberg, and June Mark

Common Core State Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice (NGO Center and CCSSO 2010, pp. 6–8)

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

Principles to Actions, NCTM, 2014

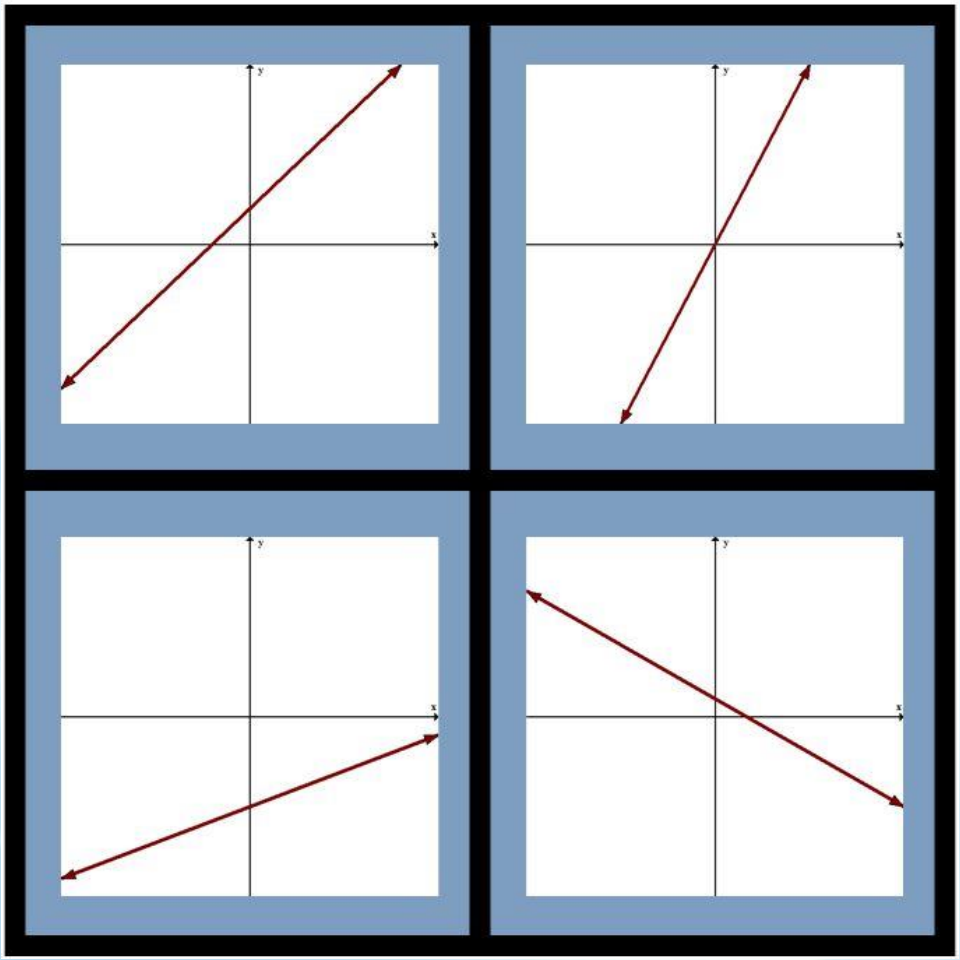
Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

$x + 7 = 3$

$y = 6x - 4$

Which One Doesn't Belong?

http://wodb.ca/



2019

THE BACK PAGE

MY FAVORITE lesson

Nicole Bannister and Benjamin J. Sinwell

Connecting Hexagon Constructions and Similarity

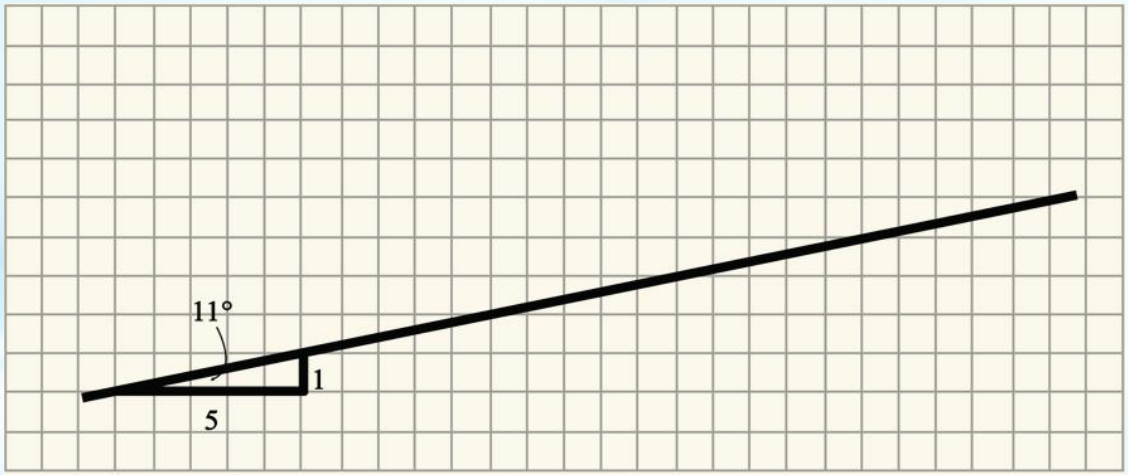
Our favorite lesson helps geometry students learn to use diagrams for thinking and communicating (Sinclair, Pimm and Skelin 2012) and motivates a discussion about similarity. The lesson connects sidewalk chalk-and-string (C&S) constructions of regular hexagons with a dynamic geometry software (DGS) approach.

9. Mark the two points where the second and third circles intersect.
10. The center of the second circle is the center of the hexagon. There are six marked points on the second circle that determine the hexagon. See **figure 1**.

We now ask, "Is the hexagon you con-

Fig. 1 Students use chalk and string to construct a regular hexagon.

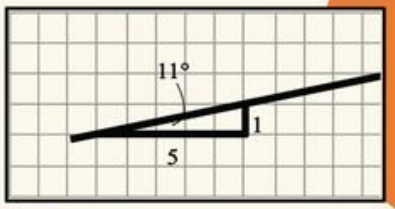
April, 2020, Noticing Before Responding



a. Draw three new slope triangles on the line. Each should be a different size. Label each triangle with as much information as you can, such as its horizontal and vertical lengths and its angle measures.

b. Explain why all of the slope triangles on this line must be similar.

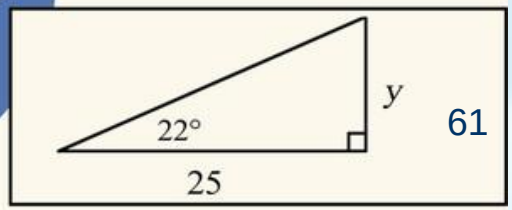
c. Since the triangles are similar, what does that tell you about the slope ratios?



Initial Task: Based on Decision To Respond

Evidence Generation: Teacher asks questions for formative assessment, attending, and interpreting student thinking. Class ends.

Evidence Based: Teacher makes the decision that students need experience with different slope triangles; teacher implements task Day Two.



Evidence Generation: Task from decision to respond is implemented; teacher gathers evidence of student thinking.

Tonya: It's twice 11. So, $y =$ _ 10? And then, this is a two-fifths ratio, so

Evidence Based: Teacher realizes students are applying multiplicative reasoning so teacher decides to respond with a new task

Teacher: Let's take a moment to think about this. Go ahead and create a slope triangle and calculate the slope ratio for an angle with a measure of 55°.

New Initial Task

Moving Forward, advocate for—

- providing and protecting access to rigorous and engaging instruction, quality learning experiences and instructional time, and supporting resources for each and every student;
- building and sustaining a positive identity and disposition toward mathematics for all teachers and students;
- ensuring that we are increasing, not decreasing, opportunities for each and every student; and
- communicating with, engaging with, and supporting our families and community.

https://www.nctm.org/uploadedFiles/Research_and_Advocacy/NCTM_NCSM_Moving_Forward.pdf

2020 – Robert Berry, NCTM, Past-President

- Our love for mathematics, for students, and for communities is a common thread that binds teachers of mathematics together. It explains the willingness of teachers to engage in activities supportive of building community; to engage in professional networks, observing and providing feedback to one another, deepening knowledge of mathematics content and pedagogy, and taking time to learn about our students and their communities; and to engage in critical conversations on issues impacting mathematics teaching and learning.

- Thank you for attending
- Thank you Cindy Bryant and the members of the Centennial Annual Conference Committee
- Thank you to the people at NCTM who made tonight possible.
- Good luck meeting the unique challenges this school year will bring to our teaching this year. Let’s do the best we can to move our students forward. Remember, our emotional and mental health as a teacher is invaluable to our instruction. It is also in limited supply.

Questions

bsinwell@anderson4.org

- Thank you for attending
- Thank you Cindy Bryant and the members of the Centennial Annual Conference Committee
- Thank you to the people at NCTM who made tonight possible.