Questioning Practices to Support Mathematical Practices

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<table>
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<th>Tips for Posing Effective Questions</th>
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<tr>
<td>1. Plural forms</td>
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<td>2. Tentative language</td>
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<td>3. Open-ended</td>
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<td>4. Positive presuppositions</td>
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<td>5. Higher-order thinking</td>
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<td>6. Approachable voice</td>
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Notes

**Noticing the Tips in Coaching Questions**

**Questions for Teachers**
1. What might be some strategies you have tried before that were successful?
2. What are some connections between this goal and the standards?
3. What seems most useful in this situation?
4. What might be some of your choices?
5. In what ways might you sequence those ideas?
6. What are some specific patterns or trends that you are noticing?
7. How did the lesson compare to how you planned it?
8. What do you think might have been going on for students during the lesson?
9. What are some criteria you used to decide about using manipulatives in this lesson?
10. How might you summarize that student’s thinking on this task?
Using the Tips in Questions with Teachers and Students

As you support the teacher or student, what questions might you ask?

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<th>Statement a Teacher Might Make</th>
<th>Statement a Student Might Make</th>
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<td>A. “My students can’t work together in groups. I give them topics to discuss, but they are off task when I listen to their conversations. I feel like I am losing control of the room when the students talk with each other.”</td>
<td>B. “I don’t understand how to do this.”</td>
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<td>C. “I have really been thinking about Mathematical Practice #4, Model with mathematics. I am not quite sure how it is different from Mathematical Practice #5, Use appropriate tools strategically.”</td>
<td>D. “When I read a word problem, I don’t know which operation to use.”</td>
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<td>E. “I want to try number talks in my classroom but am not sure how to get started.”</td>
<td>F. “I have the answer, but I don’t know how I got it.”</td>
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Who is Winning Red Light, Green Light?

Have you played this game? Everyone except the caller lines up at the start (0) and try to get to the caller at the other end (1). Facing the runners, the caller says “Green light” and turns his/her back to the runners. The runners go quickly towards the caller, ready to freeze in place when caller turns back around saying, “Red Light.” If caller sees anyone still moving, they have to go back to the start. Winner is first one to reach the caller.

Who is winning Red-light, Green-light? Here is the fraction of the distance covered from the start to the caller by the runners:

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<tr>
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<th>Mary: ( \frac{3}{4} )</th>
<th>Larry: ( \frac{1}{2} )</th>
<th>Carrie: ( \frac{5}{6} )</th>
<th>Han: ( \frac{5}{8} )</th>
<th>Shawn: ( \frac{5}{9} )</th>
<th>Juan: ( \frac{2}{3} )</th>
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Part 1: Who is Winning the Race?

1. Predict.
   a. Who do you think might be winning?

   b. Who can you rule out?

2. Solve. Use any strategy to figure out who is winning. Show or explain how you figured it out.

Adapted from Elementary and Middle School Mathematics: Teaching Developmentally (10th Edition) (Van de Walle, Karp, & Bay-Williams, 2019), Activity 14.2. (p. 1 of 2)
Part 2: How far has each person gone?

3. Place each person in their approximate place between start and end:

![Distance Diagram]

Part 3: What fraction of the distance have others traveled?

4. More people arrive to play. Assign a fractional distance to how far they have traveled based on this information:
   a. Alicia is between Harry and Han. _____________
   b. Benjamin is between Larry and Angela __________
   c. Corey is between Han and Miguel. ______________

Adapted from *Elementary and Middle School Mathematics: Teaching Developmentally (10th Edition)* (Van de Walle, Karp, & Bay-Williams, 2019), Activity 14.2. (p. 2 of 2)
## Focusing Questions on Student Reasoning

### Mathematical Practices Placemat

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<td><strong>2.</strong> Reason abstractly and quantitatively.</td>
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<td><strong>6.</strong> Attend to precision.</td>
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<td><strong>7.</strong> Look for and make use of structure.</td>
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<td><strong>8.</strong> Look for and express regularity in repeated reasoning.</td>
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Cutting Ribbon

*Use paper strips, Cuisenaire Rods, or drawings to solve these problems.*

1. How many $\frac{1}{4}$ ft. ribbon strips can you get from $\frac{3}{2}$ feet of ribbon?

2. How many $\frac{1}{8}$ ft. ribbon strips can you get from $3\frac{1}{4}$ feet of ribbon?

3. How many $\frac{3}{4}$ ft. ribbon strips can you get from 6 feet of ribbon?

4. How many $\frac{5}{4}$ yard ribbon strips can you get from 10 yards of ribbon?

5. How many $\frac{3}{8}$ yards of ribbon strips can you get from 6 yards of ribbon?

Based on *Teaching Student Centered Mathematics, Volume 3 (3rd Edition)* (Van de Walle, Bay-Williams, Lovin, & Karp, 2018).
2.1 Shifts in Classroom Practice Self-Assessment

Instructions: Place an X along each continuum that best represents your classroom practice.

**Shift 1: From stating a standard toward communicating expectations for learning**
- Teacher states broad performance goals and/or those provided in standards or curriculum documents.
- Teacher creates lesson-specific learning goals and communicates these goals at critical times within the lesson to ensure students understand the lesson’s purpose and what is expected of them.

**Shift 2: From routine tasks toward reasoning tasks**
- Teacher uses tasks involving recall of previously learned facts, rules, or definitions and provides students with specific strategies to follow.
- Teacher uses tasks that lend themselves to multiple representations, strategies, or pathways encouraging student explanation (how) and justification (why/when) of solution strategies.

**Shift 3: From teaching about representations toward teaching through representations**
- Teacher shows students how to create a representation (e.g., a graph or picture).
- Teacher uses lesson goals to determine whether to highlight particular representations or to have students select a representation; in both cases, teacher provides opportunities for students to compare different representations and how they connect to key mathematical concepts.

**Shift 4: From show-and-tell toward share-and-compare**
- Teacher has students share their answers.
- Teacher creates a dynamic forum where students share, listen, honor, and critique each other’s ideas to clarify and deepen mathematical understandings and language; teacher strategically invites participation in ways that facilitate mathematical connections.

**Shift 5: From questions that seek expected answers toward questions that illuminate and deepen student understanding**
- Teacher poses closed and/or low-level questions, confirms correctness of responses, and provides little or no opportunity for students to explain their thinking.
- Teacher poses questions that advance student thinking, deepen students’ understanding, make the mathematics more visible, provide insights into student reasoning, and promote meaningful reflection.

**Shift 6: From teaching so that students replicate procedures toward teaching so that students select efficient strategies**
- Teacher approaches facts and procedures with the goal of speed and accuracy.
- Teacher provides time for students to engage with mathematical problems, developing flexibility by encouraging student selection and use of efficient strategies; teacher provides opportunities for students to evaluate when a strategy is best suited for the problem at hand.

**Shift 7: From mathematics-made-easy toward mathematics-takes-time**
- Teacher presents mathematics in small chunks so that students reach solutions quickly.
- Teacher questions, encourages, provides time, and explicitly states the value of grappling with mathematical tasks, making multiple attempts, and learning from mistakes.

**Shift 8: From looking at correct answers toward looking for students’ thinking**
- Teacher attends to whether an answer or procedure is (or is not) correct.
- Teacher identifies specific strategies or representations that are important to notice; strategically uses observations, student responses to questions, and written work to determine what students understand; and uses these data to inform in-the-moment discourse and future lessons.

### Mathematical Practices & Student Look Fors

1. **Make sense of problems and persevere in solving them.**
   - Analyze information (givens, constraints, relationships).
   - Make conjectures and plan a solution pathway.
   - Use objects, drawings, and diagrams to solve problems.
   - Monitor progress and change course as necessary.
   - Check answers to problems and ask, "Does this make sense?"

2. **Reason abstractly and quantitatively.**
   - Make sense of quantities and relationships in problem situations.
   - Create a coherent representation of a problem.
   - Translate from contextualized to generalized or vice versa.
   - Flexibly use properties of operations.

3. **Construct viable arguments and critique the reasoning of others.**
   - Make conjectures and use counterexamples to build a logical progression of statements to support ideas.
   - Use definitions and previously established results.
   - Listen to or read the arguments of others.
   - Ask probing questions to other students.

4. **Model with mathematics.**
   - Determine equation that represents a situation.
   - Illustrate mathematical relationships using diagrams, two-way tables, graphs, flowcharts, and formulas.
   - Check to see whether an answer makes sense within the context of a situation and change a model when necessary.

5. **Use appropriate tools strategically.**
   - Choose tools that are appropriate for the task (e.g., manipulative, calculator, digital technology, ruler).
   - Use technological tools to visualize the results of assumptions, explore consequences, and compare predictions with data.
   - Identify relevant external math resources (digital content on a website) and use them to pose or solve problems.

6. **Attend to precision.**
   - Communicate precisely, using appropriate terminology.
   - Specify units of measure and provide accurate labels on graphs.
   - Express numerical answers with appropriate degree of precision.
   - Provide carefully formulated explanations.

7. **Look for and make use of structure.**
   - Notice patterns or structure, recognizing that quantities can be represented in different ways.
   - Use knowledge of properties to efficiently solve problems.
   - View complicated quantities both as single objects and as compositions of several objects.

8. **Look for and express regularity in repeated reasoning.**
   - Notice repeated calculations and look for general methods and shortcuts.
   - Maintain oversight of the process while attending to the details.
   - Evaluate reasonableness of intermediate and final results.