Questioning Practices to Support Mathematical Practices

Maggie B. McGatha
Jennifer Bay-Williams

NCTM 100 Days of Professional Learning
August 13, 2020
Joint publication of Corwin, NCTM & NCSM
Coach

Teacher
Tips for Posing Questions
Tips for Posing Questions, p. 1

• Plural forms
• Tentative language
• Open-ended
• Positive presuppositions
• Higher-order thinking
• Approachable voice
Tips for Posing Questions, p. 1

• Plural forms

What strategy will you use? vs. What strategies will you use?
Tips for Posing Questions, p. 1

• Plural forms

How will you solve this problem?

vs.

What are some ways you might solve this problem?
Tips for Posing Questions, p. 1

• Tentative language: might, seem, some

Why is Ryan struggling? vs. What might be some reasons Ryan is struggling?
Tips for Posing Questions, p. 1

• Open-ended

Have you thought about... vs. What is your thinking about...
Tips for Posing Questions, p. 1

• Open-ended

Have you thought about... vs. What is your thinking about...
Tips for Posing Questions, p. 1

- Positive presuppositions

As you focus on ELLs in this lesson, what strategies might you use?

What are you learning that you want to remember?
Higher-order thinking

What else can you tell me about that idea?

How does that idea compare with others you generated?
Tips for Posing Questions, p. 1

• Approachable voice

Approachable Voice vs. Credible Voice
Posing Questions: Teachers, p. 1

- Plurals
- Tentative
- Positive presuppositions
- Higher-order thinking
- Open-ended
1. What might be some strategies you have tried before that were successful?

2. What are some connections between this goal and the standards?

3. What seems most useful in this situation?

4. What might be some of your choices?

5. In what ways might you sequence those ideas?
1. What might be some strategies you have tried before that were successful? * ✓

2. What are some connections between this goal and the standards? * X ✓

3. What seems most useful in this situation? * ✓

4. What might be some of your choices? * ✓

5. In what ways might you sequence those ideas? * X ✓
PRACTICE
<table>
<thead>
<tr>
<th>Statement a Teacher Might Make</th>
<th>Statement a Student Might Make</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. My students can’t work together in groups. I give them topics to discuss, but they are off task when I listen to their conversations. I feel like I am losing control of the room when the students talk with each other.”</td>
<td>B. “I don’t understand how to do this.”</td>
</tr>
<tr>
<td><strong>Coaches, first name beginning with A - H</strong></td>
<td><strong>Teachers, first name beginning with A - H</strong></td>
</tr>
<tr>
<td>C. “I have really been thinking about Mathematical Practice #4, Model with mathematics. I am not quite sure how it is different from Mathematical Practice #5, Use appropriate tools strategically.”</td>
<td>D. “When I read a word problem, I don’t know which operation to use.”</td>
</tr>
<tr>
<td><strong>Coaches, first name beginning with I-P</strong></td>
<td><strong>Teachers, first name beginning with I - P</strong></td>
</tr>
<tr>
<td>E. “I’ve tried think-pair share. I can get my students in groups and I give them prompts to discuss but I am not sure if they are staying on task.”</td>
<td>F. “I have the answer, but I don’t know how I got it.”</td>
</tr>
<tr>
<td><strong>Coaches, first name beginning with Q-Z</strong></td>
<td><strong>Teachers, first name beginning with I - P</strong></td>
</tr>
</tbody>
</table>
Write a question for your assigned statement that includes as many of the tips for posing questions as appropriate.
A. “My students can’t work together in groups. I give them topics to discuss, but they are off task when I listen to their conversations. I feel like I am losing control of the room when the students talk with each other.”

If you were assigned statement A, please hit submit in the chat box.

Everyone else, take a moment to read a few of the questions, looking for the tips we discussed.
A. “My students can’t work together in groups. I give them topics to discuss, but they are off task when I listen to their conversations. I feel like I am losing control of the room when the students talk with each other.”

“How does their off-task behavior in groups compare to when they work independently?”
Using the Tips for Posing Questions, p. 2

B. “I don’t understand how to do this.”

If you were assigned statement B, please hit submit in the chat box.

Everyone else, take a moment to read a few of the questions, looking for the tips we discussed.
B. “I don’t understand how to do this.”

“If you did know what to do, what might you try first?”
C. “I have really been thinking about Mathematical Practice #4, Model with mathematics. I am not quite sure how it is different from Mathematical Practice #5, Use appropriate tools strategically.”

If you were assigned statement C, please hit submit in the chat box.

Everyone else, take a moment to read a few of the questions, looking for the tips we discussed.
C. “I have really been thinking about Mathematical Practice #4, Model with mathematics. I am not quite sure how it is different from Mathematical Practice #5, Use appropriate tools strategically.”

“Which aspects of these two mathematical practices seem similar to you?”
Using the Tips for Posing Questions, p. 2

D. “When I read a word problem “I don’t know which operation to use.”

If you were assigned statement D, please hit submit in the chat box.

Everyone else, take a moment to read a few of the questions, looking for the tips we discussed.
D. “When I read a word problem “I don’t know which operation to use.””

“When you read a word problem, how might you make decisions about what to do first?”
Using the Tips for Posing Questions, p. 2

E. “I want to try number talks in my classroom but am not sure how to get started.”

If you were assigned statement E, please hit submit in the chat box.

Everyone else, take a moment to read a few of the questions, looking for the tips we discussed.
E. “I want to try number talks in my classroom but am not sure how to get started.”

“What resources might you have available to help you get started?”
F. “I have the answer, but I don’t know how I got it.”

If you were assigned statement F, please hit submit in the chat box.

Everyone else, take a moment to read a few of the questions, looking for the tips we discussed.
Using the Tips for Posing Questions, p. 2

F. “I have the answer, but I don’t know how I got it.”

“What might be the first thing you did in solving the task? Second?”
Posing Questions about Mathematical Reasoning
Who is winning the race?

<table>
<thead>
<tr>
<th></th>
<th>Mary: $\frac{3}{4}$</th>
<th>Larry: $\frac{1}{2}$</th>
<th>Carrie: $\frac{5}{6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Han:</td>
<td>$\frac{5}{8}$</td>
<td>Shawn: $\frac{5}{9}$</td>
<td>Juan: $\frac{2}{3}$</td>
</tr>
</tbody>
</table>
Red Light, Green Light
Who is winning the race?

<table>
<thead>
<tr>
<th></th>
<th>Mary</th>
<th>Larry</th>
<th>Carrie</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{5}{6}$</td>
</tr>
<tr>
<td>Han</td>
<td>$\frac{5}{8}$</td>
<td>Shawn</td>
<td>$\frac{5}{9}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Juan</td>
<td>$\frac{2}{3}$</td>
</tr>
</tbody>
</table>
# Red Light, Green Light

<table>
<thead>
<tr>
<th></th>
<th>Mary: $\frac{3}{4}$</th>
<th>Larry: $\frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrie:</td>
<td>$\frac{5}{6}$</td>
<td>Han: $\frac{5}{8}$</td>
</tr>
<tr>
<td>Shawn:</td>
<td>$\frac{5}{9}$</td>
<td>Juan: $\frac{2}{3}$</td>
</tr>
</tbody>
</table>

What questions might you ask students about this task?

Record at the bottom on p.4
Red Light, Green Light – Possible Questions for Students

• What fraction comparison strategies do you know that might be useful in solving this problem?
• What are some ways you might rule out some of the runners?
• Which fractions are easy to compare and which are difficult (and why)?
• How does using common denominators compare to using benchmark fractions for figuring out this task?
Red Light, Green Light

What questions might you ask teachers about this task?

Mary: \( \frac{3}{4} \)  
Larry: \( \frac{1}{2} \)

Carrie: \( \frac{5}{6} \)  
Han: \( \frac{5}{8} \)

Shawn: \( \frac{5}{9} \)  
Juan: \( \frac{2}{3} \)

Record at the bottom on p.4
Red Light, Green Light – Possible Questions for Teachers

- As you make sure the task keeps it rigor, how might you make it accessible to all students?
- What are some questions you might pose as students are working to deepen their reasoning?
- What strategies are you considering for ensuring all students are doing their own thinking?
Connecting Questioning to Mathematical Practices
Mathematical Practices & Student Look Fors

1. Make sense of problems and persevere in solving them.
   - Analyze information (givens, constraints, relationships).
   - Make conjectures and plan a solution pathway.
   - Use objects, drawings, and diagrams to solve problems.
   - Monitor progress and change course as necessary.
   - Check answers to problems and ask, “Does this make sense?”

2. Reason abstractly and quantitatively.
   - Make sense of quantities and relationships in problem situations.
   - Create a coherent representation of a problem.
   - Translate from contextualized to generalized or vice versa.
   - Flexibly use properties of operations.

3. Construct viable arguments and critique the reasoning of others.
   - Make conjectures and use counterexamples to build a logical progression of statements to support ideas.
### Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>MP.1</th>
<th>Make sense of problems and persevere in solving them.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP.2</td>
<td>Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>MP.3</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>MP.4</td>
<td>Model with mathematics.</td>
</tr>
<tr>
<td>MP.5</td>
<td>Use appropriate tools strategically.</td>
</tr>
<tr>
<td>MP.6</td>
<td>Attend to precision.</td>
</tr>
<tr>
<td>MP.7</td>
<td>Look for and make use of structure.</td>
</tr>
<tr>
<td>MP.8</td>
<td>Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>

### Cluster: Extend understanding of fraction equivalence and ordering.

#### Standards

<table>
<thead>
<tr>
<th>KY.4.NF.1</th>
<th>Understand and generate equivalent fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Use visual fraction models to recognize and generate equivalent fractions that have different numerators/denominators even though they are the same size.</td>
</tr>
<tr>
<td>b.</td>
<td>Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$.</td>
</tr>
</tbody>
</table>

**MP.4, MP.7, MP.8**

<table>
<thead>
<tr>
<th>KY.4.NF.2</th>
<th>Compare two fractions with different numerators and different denominators using the symbols $&lt;$, $=$, or $&gt;$. Recognize comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions.</th>
</tr>
</thead>
</table>

**MP.2, MP.3**

#### Clarifications

Students use a variety of representations to compare fractions including concrete models, benchmarks, common denominators and common numerators.

*Note: Students determine which strategy makes the most sense to them, realizing they use different strategies for different situations.*

**Coherence KY.3.NF.3d → KY.4.NF.2 → KY.5.NF.2**
LESSON 1
One of These Things Is Not Like the Others

1.1: Remembering Double Number Lines (5 minutes)

CCSS Standards

Building On
6.RP.A

Required Preparation

Instructional Routines
Think Pair Share

Source: https://www.illustrativemathematics.org/curriculum/im-6-8-math-curriculum/
SCAFFOLDING TASK: ARRAYS ON THE FARM

In this task, the students use arrays to solve multiplication problems. Farmers grow their crops in arrays to make them easier to look after and to harvest. Additionally, students are asked to be involved in guessing and estimating. These are both useful skills that take time to develop. This task provides some practice for these skills.

APPROXIMATE TIME: 3-4 days

CONTENT STANDARDS

MGSE3.OA.5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) Use arrays, area models, and manipulatives to develop understanding of properties.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

**Domino Addition: Understanding the Part/Part/Whole Relationship**

2nd Grade Math » Unit: Addition and Subtraction Basic Training

**Big Idea:** The big idea of this lesson is the understanding that addition can be represented as parts of a whole and that we can use addition sentences to represent those parts.

**Standards**

2.NBT.B.5  2.NBT.B.6  2.NBT.B.9  MP1  MP4

Kristen O'Connor
Rural Env.

Source: https://betterlesson.com/
Shift #5: From questions that seek expected answers toward questions that illuminate and deepen student understanding.
Shift #5: From questions that seek expected answers toward questions that illuminate and deepen student understanding.

Teacher poses closed and/or low-level questions, confirms correctness of responses, and provides little or no opportunity for students to explain their thinking.

Teacher poses questions that advance student thinking, deepen students’ understanding, make the mathematics more visible, provide insights into student reasoning, and promote meaningful reflection.

See Tool 2.1Shifts Self-Assessment in Handout
Mathematical Practices & Student Look Fors

1. Make sense of problems and persevere in solving them.
   - Analyze information (givens, constrains, relationships).
   - Make conjectures and plan a solution pathway.
   - Use objects, drawings, and diagrams to solve problems.
   - Monitor progress and change course as necessary.
   - Check answers to problems and ask, “Does this make sense?”

2. Reason abstractly and quantitatively.
   - Make sense of quantities and relationships in problem situations.
   - Create a coherent representation of a problem.
   - Translate from contextualized to generalized or vice versa.
   - Flexibly use properties of operations.

3. Construct viable arguments and critique the reasoning of others.
   - Make conjectures and use counterexamples to build a logical progression of statements to support ideas.
<table>
<thead>
<tr>
<th>Mathematical Practices Placemat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
</tr>
</tbody>
</table>
Decide where each question might go on the placemat.

1. How is this task similar to a previous task you have completed?
2. What other resources might help you with this problem?
3. When will this strategy work?
4. How might you use **break-apart** to solve this problem?
5. How are these problems the same? Different?
6. What manipulative or picture might you use to solve the problem?
7. What do the rest of you think about Anna’s strategy?
One possible set of answers!
Relationship of the Mathematical Practices

http://www.fosteringmathpractices.com/routinesforreasoning/
Red Light, Green Light

Take a moment and look at the questions for students you recorded at the bottom on p. 4.

Which mathematical practices might students have the opportunity to demonstrate when answering your questions?
Red Light, Green Light – Possible Questions for Students

1. What fraction comparison strategies do you know that might be useful in solving this problem?
2. What are some ways you might rule out some of the runners?
3. Which fractions are easy to compare and which are difficult (and why)?
4. How does using common denominators compare to using benchmark fractions for figuring out this task?
Let’s be Intentional!

1. Identify the MP
2. Identify questions
3. Remember questioning tips
Cutting Ribbon
1. How many $\frac{1}{4}$ ft. ribbon strips can you make from $2\frac{1}{2}$ feet of ribbon?
2. How many $\frac{1}{8}$ ft. ribbon strips can you make from $3\frac{1}{4}$ feet of ribbon?
3. How many $\frac{3}{4}$ ft. ribbon strips can you make from 6 feet of ribbon?

Mathematical Practices & Student Look Fors

1. Make sense of problems and persevere in solving them.
   - Analyze information (givens, constraints, relationships).
   - Make conjectures and plan a solution pathway.
   - Use objects, drawings, and diagrams to solve problems.
   - Monitor progress and change course as necessary.
   - Check answers to problems and ask, "Does this make sense?"

2. Reason abstractly and quantitatively.
   - Make sense of quantities and relationships in problem situations.
   - Create a coherent representation of a problem.
   - Convert from contextualized to generalized perspectives, and vice versa.

3. Construct viable arguments and critique the reasoning of others.
   - Make conjectures and use counterexamples to build a logical progression of statements to support ideas.
   - Use definitions and previously established results.
   - Listen to or read the arguments of others.
   - Ask probing questions to other students.
Enhance a Task

Cutting Ribbon

1. How many \( \frac{1}{4} \) ft. ribbon strips can you make from 2\( \frac{1}{2} \) feet of ribbon?

2. How many \( \frac{1}{8} \) ft. ribbon strips can you make from 3\( \frac{1}{4} \) feet of ribbon?

3. How many \( \frac{3}{4} \) ft. ribbon strips can you make from 6 feet of ribbon?

- Which Math Practice might you pick?
- How might you enhance the task to press on selected MP?
Enhance a Task

**Cutting Ribbon**

1. How many \( \frac{1}{4} \) ft. ribbon strips can you make from \( 2 \frac{1}{2} \) feet of ribbon?

2. How many \( \frac{1}{8} \) ft. ribbon strips can you make from \( 3 \frac{1}{4} \) feet of ribbon?

3. How many \( \frac{3}{4} \) ft. ribbon strips can you make from 6 feet of ribbon?

- **SMP#5:**
- Provide ribbon, c-rods, paper strips
- Ask
- What materials or tools might be helpful in solving this task?

p. 6
Enhance a Task

**Cutting Ribbon**

1. How many $\frac{1}{4}$ ft. ribbon strips can you make from $2\frac{1}{2}$ feet of ribbon?

2. How many $\frac{1}{8}$ ft. ribbon strips can you make from $3\frac{1}{4}$ feet of ribbon?

3. How many $\frac{3}{4}$ ft. ribbon strips can you make from 6 feet of ribbon?

• **SMP#8:**

• **Ask**
  - What patterns do you notice across these problems?
  - In what ways are these problems the same? Different?

p. 6
Enhance a Task

Cutting Ribbon

1. How many $\frac{1}{4}$ ft. ribbon strips can you make from $2\frac{1}{2}$ feet of ribbon?

2. How many $\frac{1}{8}$ ft. ribbon strips can you make from $3\frac{1}{4}$ feet of ribbon?

3. How many $\frac{3}{4}$ ft. ribbon strips can you make from 6 feet of ribbon?

SMP#1:

Ask

- What is happening in these stories?
- What do you notice about these situations?
- In what ways are these problems like other problems you know?
Protocol for Enhance-a-Task

1. Explore the task.
2. Discuss the mathematics with a partner.
3. Decide which MP you will emphasize.
4. Discuss 1-2 adaptations for the task and/or questions to emphasize the MP.
Shifts in Classroom Practice

How might the ideas from this webinar lead to shifting our classroom question posing?

*Shift 5: From questions that seek expected answers toward questions that illuminate and deepen student understanding*

Teacher poses closed and/or low-level questions, confirms correctness of responses, and provides little or no opportunity for students to explain their thinking.

Teacher poses questions that advance student thinking, deepen students’ understanding, make the mathematics more visible, provide insights into student reasoning, and promote meaningful reflection.
WHAT QUESTIONS MIGHT YOU HAVE AS YOU PLAN FOR SUPPORTING EFFECTIVE QUESTIONING?
Let’s stay connected!

maggiemcgatha87@gmail.com
j.baywilliams@louisville.edu