The Story of Problem Solving: Looking Back and Moving Forward

NCTM 100 Days of Professional Learning

August 18, 2020

Dr. Sherri Martinie, Kansas State University

Dr. Julie Thiele, Wichita State University

Session Overview

- Consider approaches to problem solving
- Examine 6 phases in the history of math education through the lens of problem solving
- Share current practices and resources to engage your students in a balanced approach to problem solving
- Entertain ideas about where problem solving will go in the future

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What do you notice?

Multiplying Large Numbers

1) 2,456,710 × 5

2) 97,184,368 × 7

3) 368,500 × 2

4) 168,532 × 9

5) 85,363,817 × 6

6) 6,469,701 × 4

7) 41,374,902 × 1

8) 782,369 × 8

9) 5,628,125 × 3

10) An ethanol manufacturer exports 9,873,250 gallons of ethanol to markets around the world in a year. How many gallons of ethanol do they export in 6 years?

_____

11) The capacity of Emerald ship is 3,166,353 barrels. The capacity of Ruby ship is thrice as much as the Emerald ship. What is the capacity of the Ruby ship?

_____

Name _____ Date _____

Homework 6.4

Problem-Solving Strategy: Make a Table

Make a table to solve each problem.

Brice received 4 stickers on the first day of school. Each day after that, he received 2 more stickers than the day before. How many stickers will he receive on Day 5?

Make a table. Each day is 2 more than the day before.

Day 1	Day 2	Day 3	Day 4	Day 5
4	6	8	10	12

Brice will receive 12 stickers on Day 5.

1. Miss Ruiz's class collected canned goods for one week. On Monday, they collected 30 canned goods. Each day, they collected 15 more canned goods than the day before. How many canned goods did they collect on Friday?

Show your work.

2. Donovan gets 25 cents each week for his allowance. If he saves his allowance for 5 weeks, how much money will he have?

3. Greg is waiting to catch the bus that passes his bus stop at 8:30 A.M. The first bus passes at 8:13 A.M. The second bus passes ten minutes later. The third bus passes ten minutes later than the second bus. Which bus will Greg catch?

Use with text pages 159–160.

Persevere On Your Own

Read the problem. Write a solution on a separate sheet of paper.

Skate Park

Sweet T has $80 left after buying items for the team. He wants to buy at least three different items for the skate park he is making. Here are the items Sweet T is looking at, along with the prices.

• Table: $24

• Bench: $15

• L-Shaped Box: $15

• Box: $18

• Pallet: $10

• Rail: $22

What items should Sweet T buy?

SOLVE IT

Tell which items Sweet T should buy.

- Give the total cost.
- Explain why you chose the items you did.

REFLECT

Use Mathematical Practices After you complete the task, choose one of these questions to discuss with a partner.

- **Persevere** What steps did you take to get your solution?
- **Use a Model** Which operations did you use to solve the problem?

Approaches to Problem Solving

- Teaching for problem solving
- Teaching about problem solving
- Teaching via problem solving

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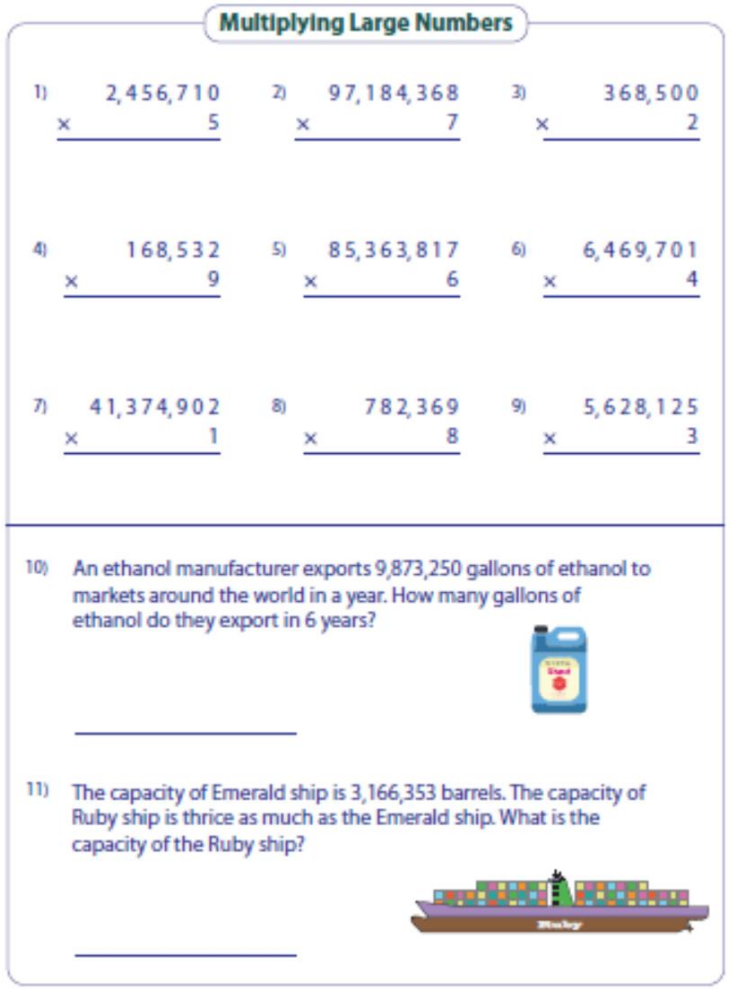
Teaching for problem solving

- Teaching a skill so that a student can later problem solve
- Often starts with learning an abstract concept and then moving to solving problems as a way to apply the learned skills



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Teaching about problem solving

PLAN IT AND SOLVE IT

Find a solution for the Fingerboard Parts problem.

- Make a list of the parts to buy. Include the numbers of the different parts and the colors.
- Tell why you chose the parts that you did.
- Find the total cost to buy the parts. Tell how much money is left.

You may want to use the Problem-Solving Tips to get started.

PROBLEM-SOLVING TIPS

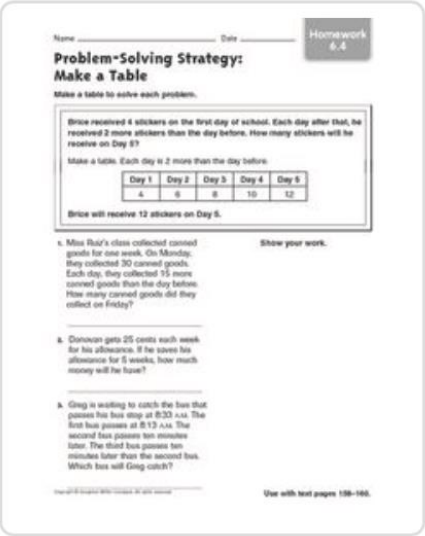
- **Questions**
  - How many of each part do you need to make one fingerboard?
  - Do you want to have more of one kind of part? Why?
- **Tools** You may want to use . . .
  - a table.
  - an organized list.
- **Sentence Starters**
  - I would like to have _____
  - I will buy _____

PROBLEM-SOLVING CHECKLIST

Make sure that you . . .

- ☐ tell what you know.
- ☐ tell what you need to do.
- ☐ show all your work.
- ☐ show that the solution works.

- Involves teaching students how to problem solve
- Often includes teaching a process (Polya's 4 step process)
- Often includes strategies for solving a problem



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Use with text pages 158–166.

Teaching via problem solving

- Students learn math through real contexts, problems, situations, models, etc.
- The context and models allow students to build meaning for the concepts so that they can move to abstract concepts.
- Might be described as “upside-down” from teaching for problem solving

Persevere On Your Own

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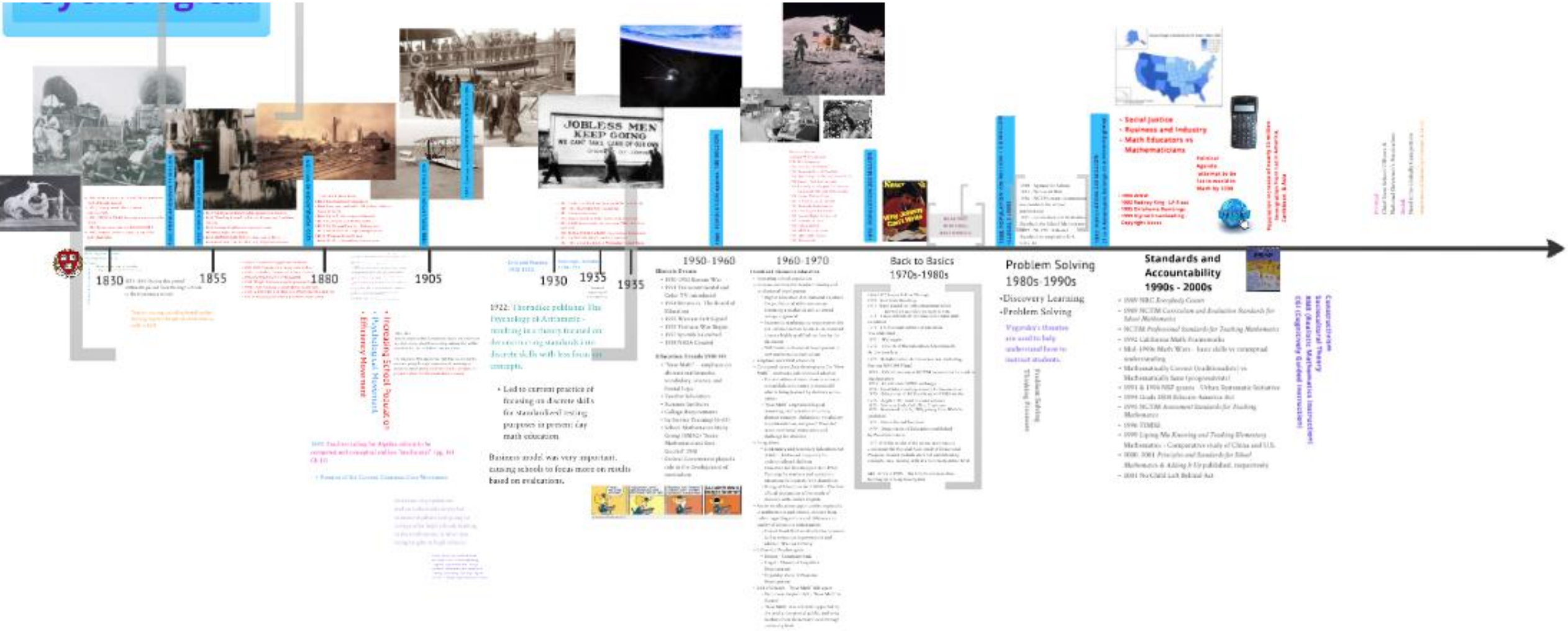
Standards for Mathematical Practice: Make sense of problems and persevere in solving them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

But how did we get to this place with problem solving in math education, and where are we going?

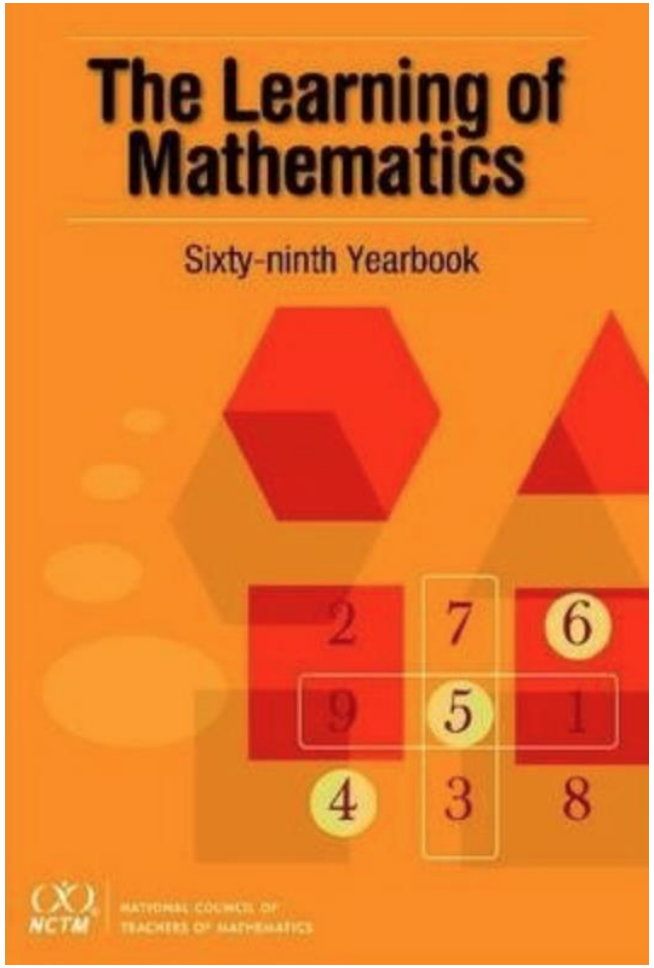
How did we get here and where are we going?



Allen, 2018

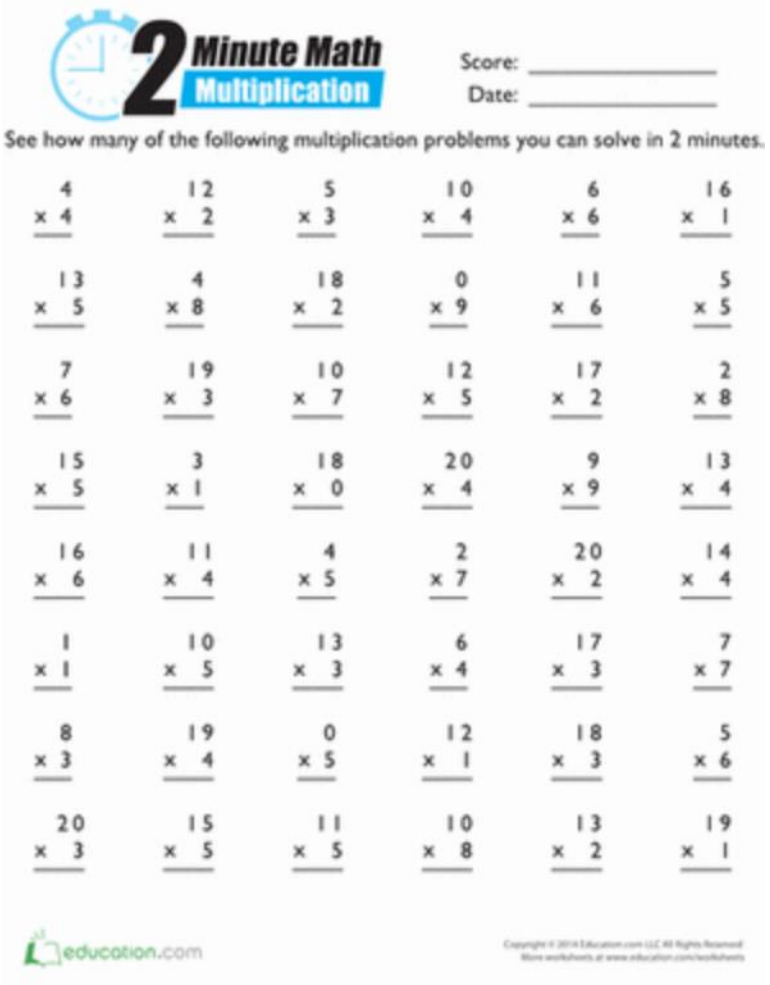
Looking back at 6 Identifiable Phases in Mathematics Education

1. Drill and practice
2. Meaningful arithmetic
3. New math
4. Back to basics (again)
5. Problem solving
6. Standards and accountability



Lambdin & Walcott (2007) *Changes through the Years: Connections between Psychological Learning Theories and the School Mathematics Curriculum.* The Learning of Mathematics: 69th NCTM Yearbook

Looking back…Drill and Practice



2 Minute Math Multiplication

Score: _____

Date: _____

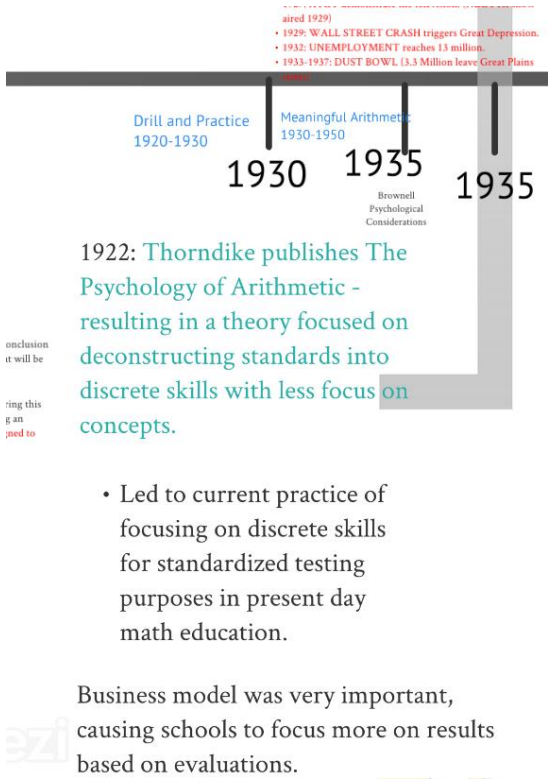
See how many of the following multiplication problems you can solve in 2 minutes.

4 × 4	12 × 2	5 × 3	10 × 4	6 × 6	16 × 1
13 × 5	4 × 8	18 × 2	0 × 9	11 × 6	5 × 5
7 × 6	19 × 3	10 × 7	12 × 5	17 × 2	2 × 8
15 × 5	3 × 1	18 × 0	20 × 4	9 × 9	13 × 4
16 × 6	11 × 4	4 × 5	2 × 7	20 × 2	14 × 4
1 × 1	10 × 5	13 × 3	6 × 4	17 × 3	7 × 7
8 × 3	19 × 4	0 × 5	12 × 1	18 × 3	5 × 6
20 × 3	15 × 5	11 × 5	10 × 8	13 × 2	19 × 1

education.com

- Approx 1920-1930
- Focus on facility with computation
- Rote memorization of facts and algorithms
- Break all work into a series of small steps

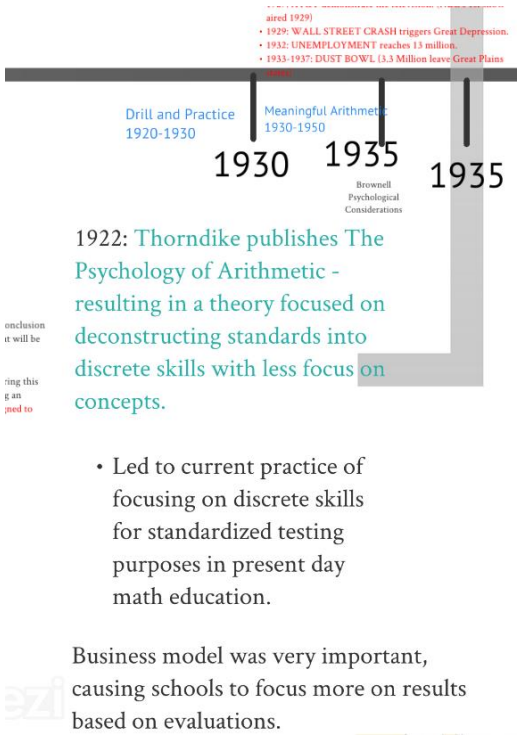
Lambdin & Walcott (2007)



Looking back.... Meaningful Arithmetic

- Approx. 1930-1950
- Understanding arithmetic ideas and skills
- Application of math to real-world problems
- Emphasis placed on mathematical relationships
- Incidental learning and activity-oriented approach

Lambdin & Walcott (2007)



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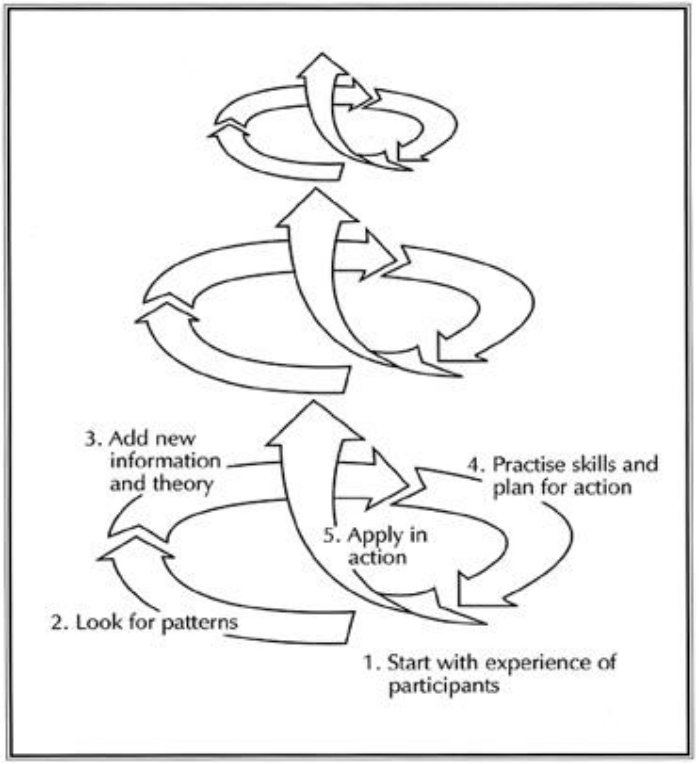
Looking back… New Math

- Approx 1960-1970
- Focus on understanding the structure of the discipline
- Spiral curriculum
- Discovery learning
- Bruner, Piaget, Dienes



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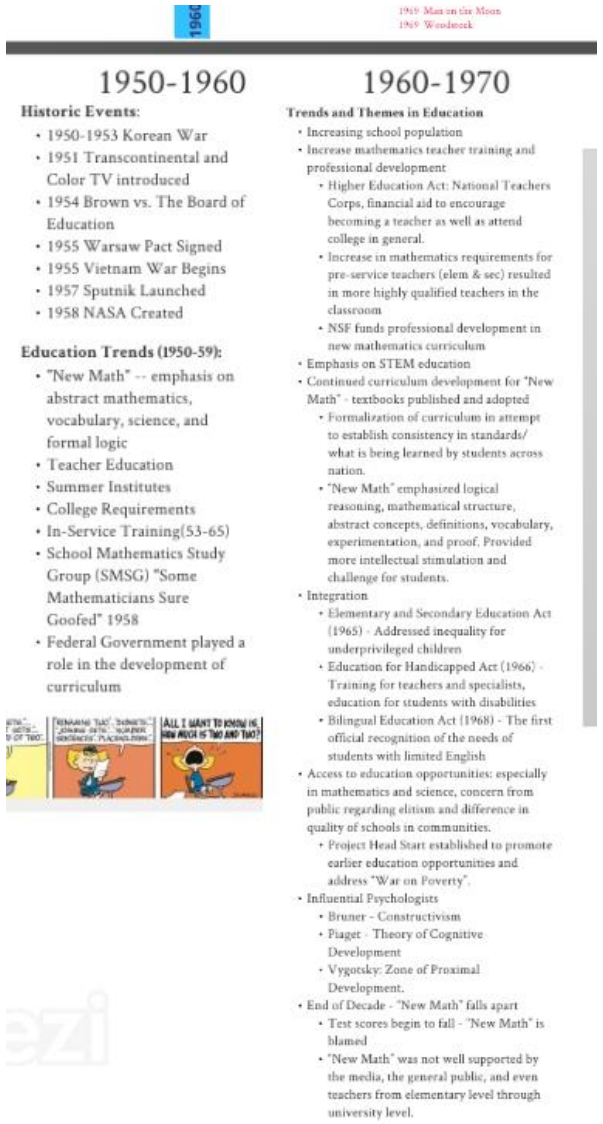


1960

1969 Man on the Moon
1969 Woodstock

1950-1960

Historic Events:

- 1950-1953 Korean War
- 1951 Transcontinental and Color TV introduced
- 1954 Brown vs. The Board of Education
- 1955 Warsaw Pact Signed
- 1955 Vietnam War Begins
- 1957 Sputnik Launched
- 1958 NASA Created

Education Trends (1950-59):

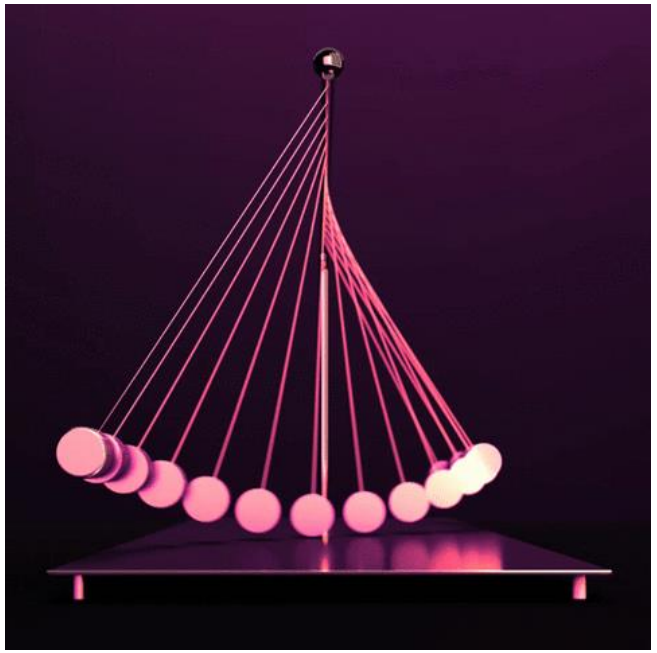
- "New Math" -- emphasis on abstract mathematics, vocabulary, science, and formal logic
- Teacher Education
- Summer Institutes
- College Requirements
- In-Service Training(53-65)
- School Mathematics Study Group (SMSG) "Some Mathematicians Sure Goofed" 1958
- Federal Government played a role in the development of curriculum

1960-1970

Trends and Themes in Education

- Increasing school population
- Increase mathematics teacher training and professional development
  - Higher Education Act: National Teachers Corps, financial aid to encourage becoming a teacher as well as attend college in general.
  - Increase in mathematics requirements for pre-service teachers (elem & sec) resulted in more highly qualified teachers in the classroom
  - NSF funds professional development in new mathematics curriculum
- Emphasis on STEM education
- Continued curriculum development for "New Math" - textbooks published and adopted
  - Formalization of curriculum in attempt to establish consistency in standards/ what is being learned by students across nation.
  - "New Math" emphasized logical reasoning, mathematical structure, abstract concepts, definitions, vocabulary, experimentation, and proof. Provided more intellectual stimulation and challenge for students.
- Integration
  - Elementary and Secondary Education Act (1965) - Addressed inequality for underprivileged children
  - Education for Handicapped Act (1966) - Training for teachers and specialists, education for students with disabilities
  - Bilingual Education Act (1968) - The first official recognition of the needs of students with limited English
- Access to education opportunities: especially in mathematics and science, concern from public regarding elitism and difference in quality of schools in communities.
  - Project Head Start established to promote earlier education opportunities and address "War on Poverty".
- Influential Psychologists
  - Bruner - Constructivism
  - Piaget - Theory of Cognitive Development
  - Vygotsky: Zone of Proximal Development.
- End of Decade - "New Math" falls apart
  - Test scores begin to fall - "New Math" is blamed
  - "New Math" was not well supported by the media, the general public, and even teachers from elementary level through university level.

Looking back… Back to the Basics



- Approx 1970s
- Return to concern for knowledge and skill development
- Return to learning facts by drill and practice

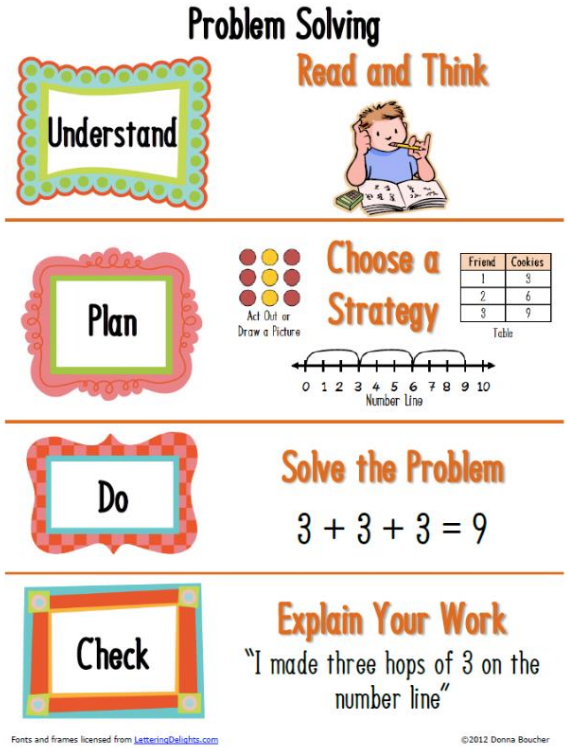
Back to Basics
1970s-1980s

1968-1977 Project Follow Through
1970 - Kent State Shootings
1971- Nixon passed the 26th amendment which allowed 18 year old's the right to vote.
1972 - Texas Instruments introduces first hand-held calculators
1972 - The National Institute of Education was established.
1972 - Watergate
1972 - Title IX of the Education Amendments Act became law
1973 - Rehabilitation Act becomes law, including Section 504 (504 Plans)
1973 - 25% of sessions at NCTM convention focused on manipulative.
1973 - 1st oil crisis/OPEC embargo
1974 - Equal Education Opportunity Act becomes law
1975 - Education of All Handicapped Children Act
1975 - Apple 1 PC used in some schools
1975 - Vietnam Ends, Cold War Continues
1975 - Newsweek article, "*Why Johnny Can't Write*" is published
1976 - Carter Elected President
1979 - Department of Education established by President Carter
1977-1978 the results of the second mathematics assessment, the National Assessment of Educational Progress, showed students were not understanding concepts, only learning skills at a rote manipulation level.

Mid-1970's to 1980's- The benefits of cooperative learning were being investigated.

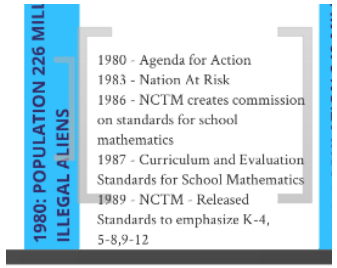
Lambdin & Walcott (2007)

Looking back… Problem Solving



- Approx 1980s
- Focus on mathematical thinking processes and problem solving strategies (about problem solving)
- Cooperative groups
- Shift toward learning THROUGH problem solving

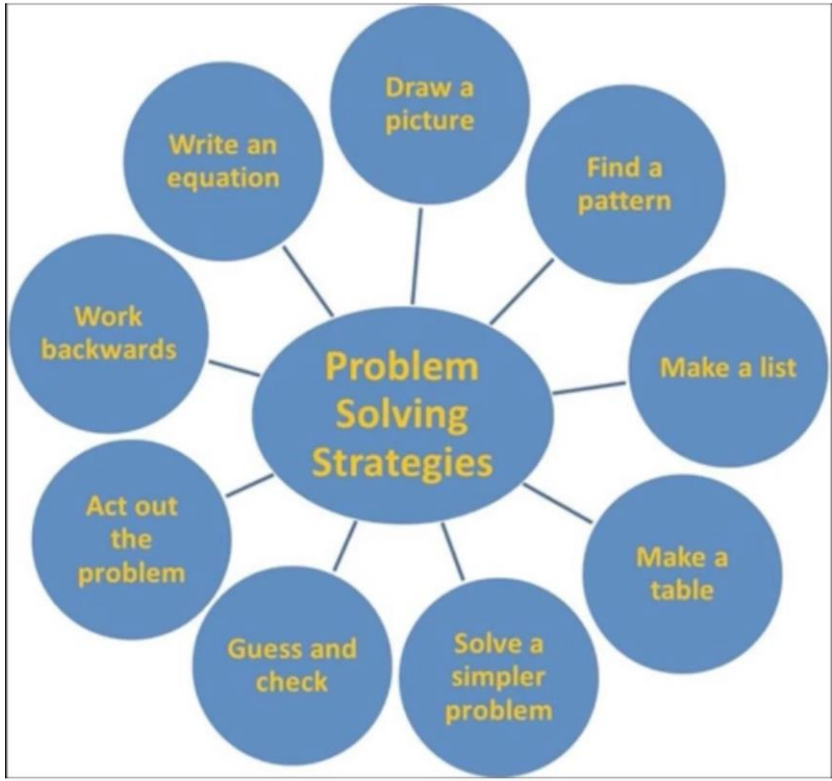
Lambdin & Walcott (2007)



Problem Solving 1980s-1990s

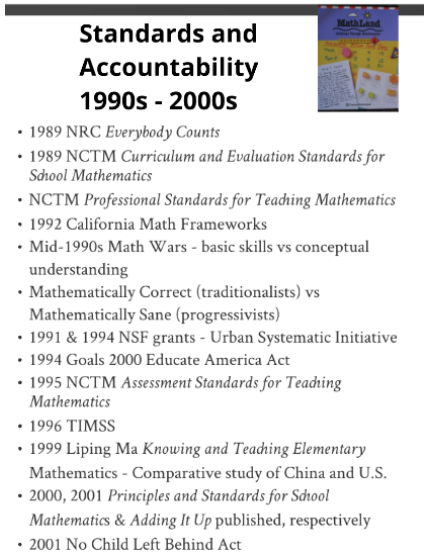
- Discovery Learning
- Problem Solving

Vygotsky's theories are used to help understand how to instruct students.



Current phase… Standards and accountability

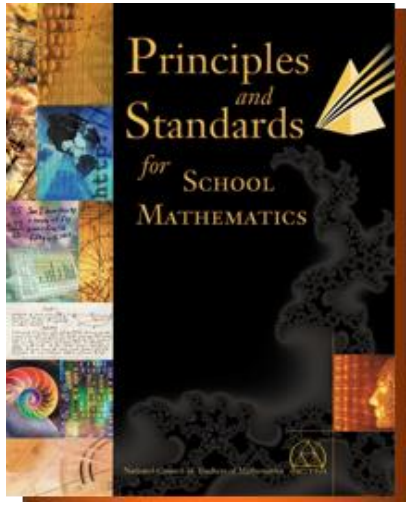
- NCTM 1989 *Curriculum and Evaluation Standards for School Mathematics*
- A new kind of math war
- Focus on math vs. focus on the learner
- National Science Foundation funded curriculum development projects to make the vision of the NCTM Standards a reality
- Student oriented, standards-based curricula vs. focus on test preparation



NCTM Process Standards & NRC Strands of Mathematical Proficiency

5 **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

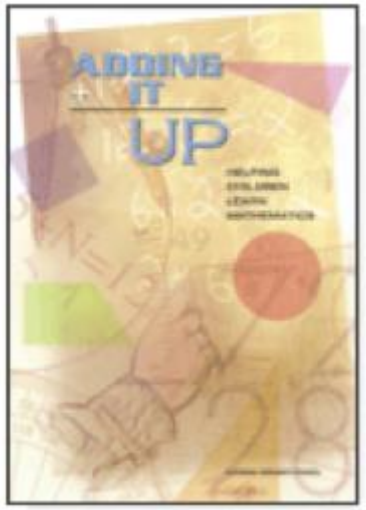


NCTM(2000). Principles and Standards for School Mathematics. Reston, VA: NCTM

Strands of Mathematical

Proficiency

- Strategic competence
- Adaptive reasoning
- Conceptual understanding
- Productive disposition
- Procedural fluency



NRC (2001). Adding It Up. Washington, DC: National Academies Press

Memorizing Basics Skills versus Developing Procedural Fluency

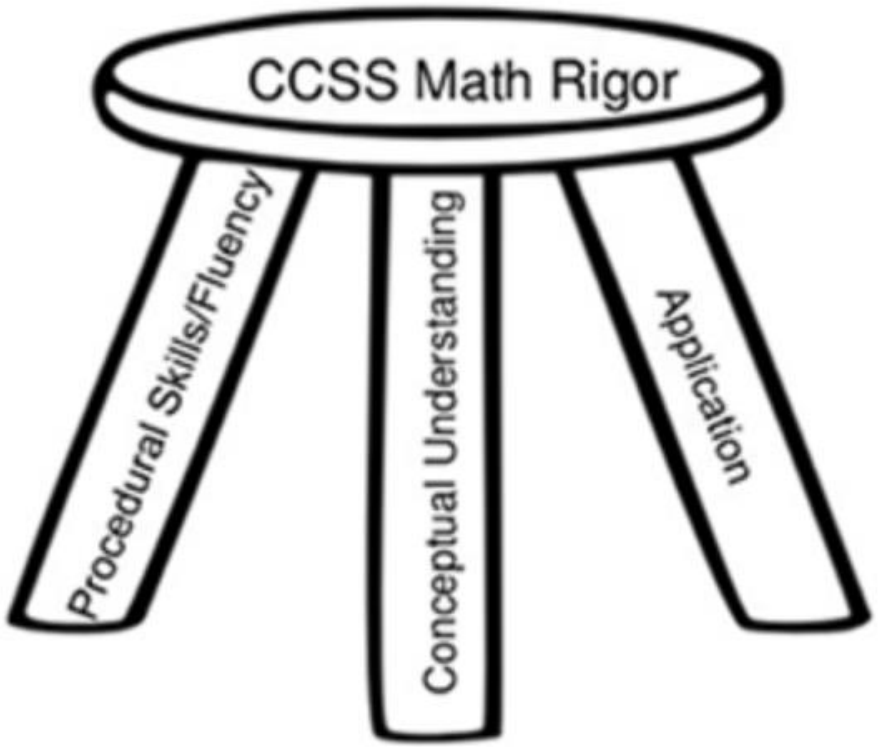
Procedural Fluency is "skill in carrying out procedures flexibly, accurately, efficiently and appropriately"

(CCSSO, 2010, p. 6)

efficiency

Current focus on rigor

The Common Core State Standards for math (CCSSM) clearly states that rigor in math includes a balance of *procedural skills and fluency*, *conceptual understanding*, and *application*. It also states that all three aspects should be pursued with “equal intensity.” We want students to know ***how*** (procedural skill and fluency), know ***why*** (conceptual understanding), and know ***when*** (application).



Burnett (2019) Origo Education

What is our goal?

Problem Performers

- Emphasis on developing students that focus on simply completing the problem
- Instruction focuses on a particular way of thinking

Problem Solvers

- Emphasis on developing students that are flexible and fluent mathematical thinkers
- Instruction focuses on mathematical knowledge and processes rather than specific strategies based on problem types

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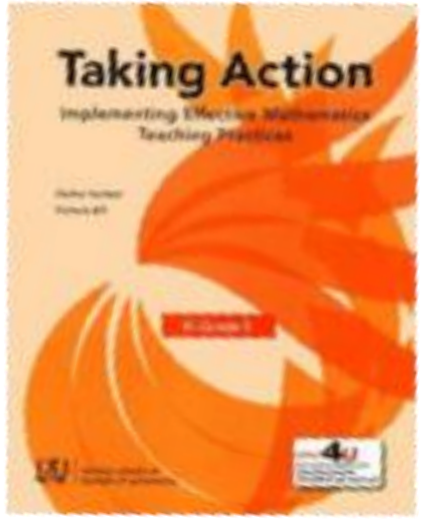
Effective Mathematics Teaching Practices



Implement tasks that promote reasoning and problem solving

- Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Support productive struggle in learning mathematics

- Effective teaching of mathematics consistently provide students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Strategies to enable and support problem SOLVERS

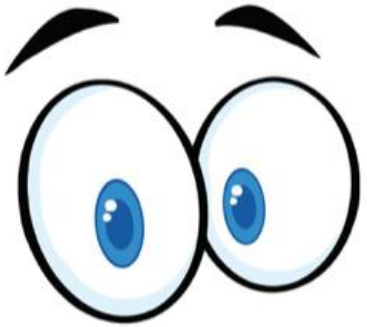
1. **Introduce a simpler problem**
2. **Elicit prior knowledge**
3. **Delay the question**
4. Give students the answer first
5. Challenge students to create the context

FIGURE 5

When working on the Fruit Scales problem (Lappan and Smith 2012) with problem performers, the authors present only the first two of the three pictures.

Barlow, A.T, Duncan, M., Lischka, A.E., Hartland, K.S. & Willingham J.C. (2017) Are your students problem performers or problem solvers? *Teaching Children Mathematics, v23 n9* p550-559

Notice and Wonder Routines Problem-Posing



- Students observe an image, video, or graphic and they are asked “What do you notice? What do you wonder?”
- They are asked to share things they notice and the teacher records suggestions
- Repeat the process with wondering
- The wondering then leads to a mathematical question to answer

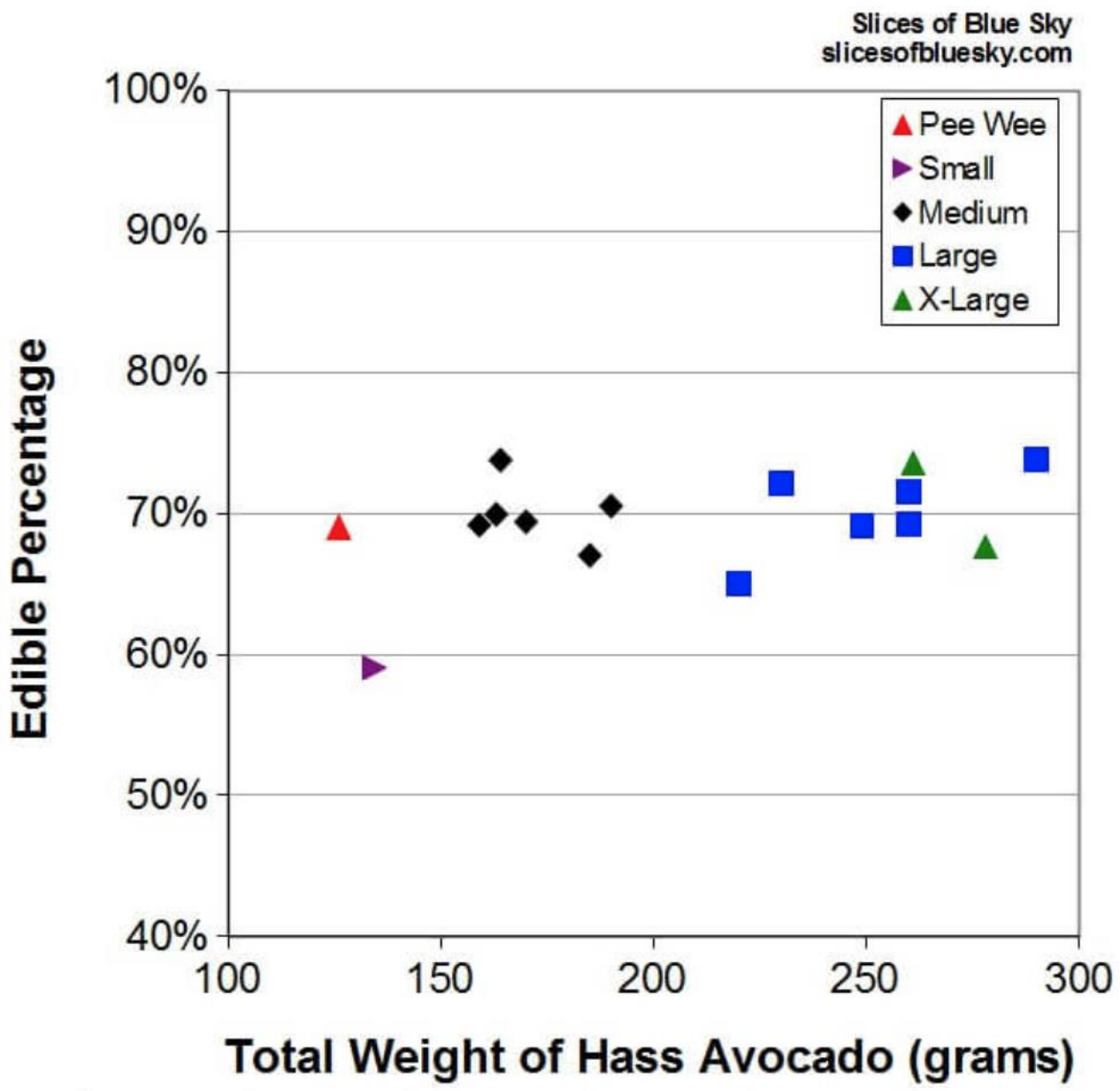
What do you notice? What do you wonder?

How much of an avocado is edible?

After seeing an array of avocados, they wonder: Does the edible fraction depend on the size of the avocado?

To answer these question, they bought almost 20 avocados of various sizes and weighed them with and without skin and pit.

https://slicesofbluesky.com/does-edible-percentage-of-avocado/



Three-Act Tasks

Three parts:

1. An engaging and perplexing "Act One"
2. An information and solution seeking "Act Two"
3. A solution discussion and solution revealing "Act Three"

Resources:

- Graham Fletcher
- Robert Kaplinsky
- Dan Meyer
- John Orr
- Kyle Pearce
- Andrew Stradel

Numberless Word Problems

There were 36 kids eating pizza. 20 of them were eating cheese pizza. The rest were eating pepperoni. How many were eating pepperoni?

How could you change this to a numberless word problem?

Numberless Word Problems

What becomes the focus when you rewrite the task as a numberless work problem? What do you envision?

There were some kids were eating pizza. Some of them were eating cheese pizza. The rest were eating pepperoni. How many were eating pepperoni?

Img Source: theamazingrecipes.com

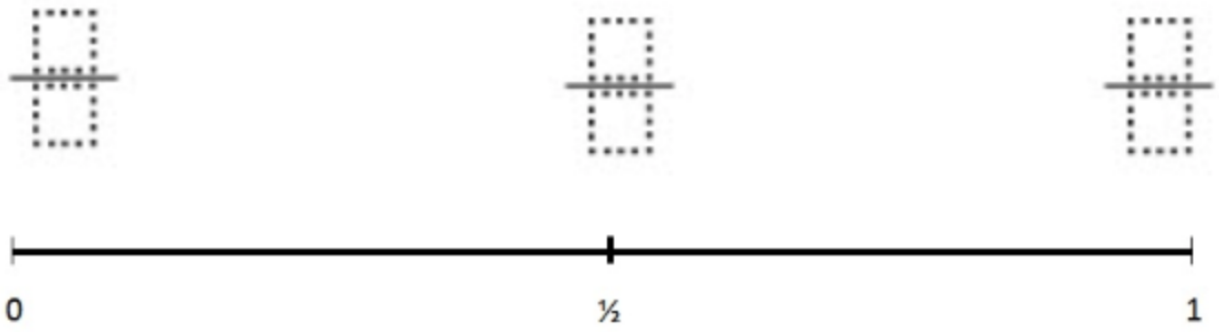
Open Middle Tasks

A specific type of task with

- a “closed beginning”
- a “closed end”
- an “open middle”

BENCHMARK FRACTIONS

Directions: Use the digits 1 to 9, no more than once, to create three fractions that are as close to zero, one half and one as possible. NOTE: Close as possible is measured by adding up all the differences and making it the least possible value.



Characteristics include:

- Multiple ways of solving them
- Involve optimization
- Appear simple and procedural in nature but turn out to be more challenging and complex
- Generally not as complex as performance tasks

https://www.openmiddle.com

Looking forward…

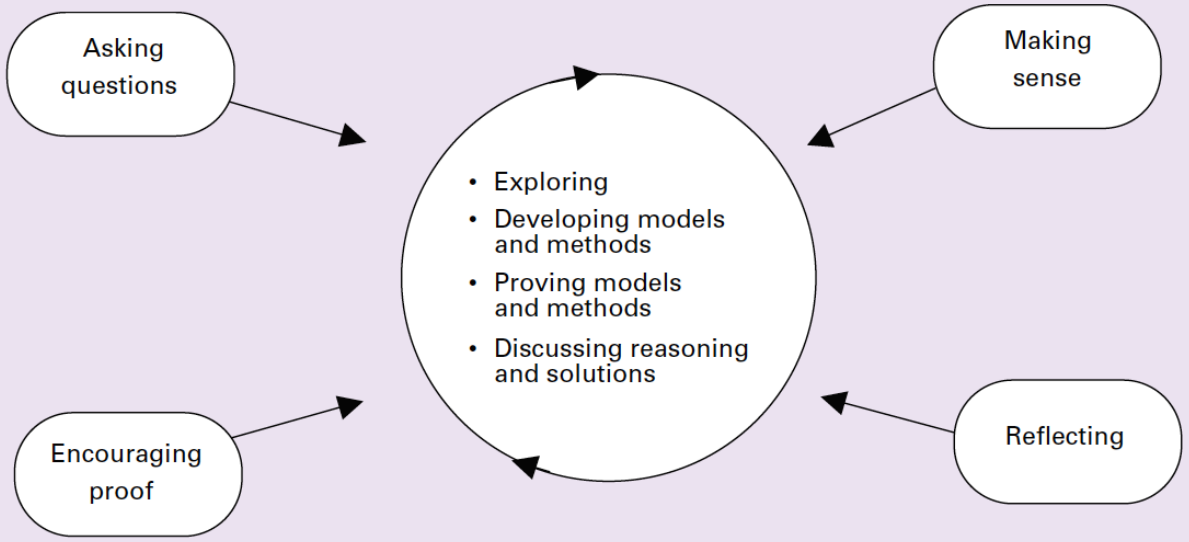
Problem-Based Learning and Project-Based Learning
Similarities
Focus on an open-ended question or task Provide authentic applications of content and skills Build inter- and intra-personal student success skills Emphasize student independence and inquiry Are longer, more complex and multifaceted than traditional lessons or assignments

Typical Differences	
Problem-Based Learning	Project-Based Learning
Single content area	Multiple content areas
Follows specific, traditionally prescribed steps	Follows general, variously named steps
Product may be tangible or proposed, a solution to the problem	Includes the creation of a product or performance
Case studies or fictitious scenarios	Real-world, fully authentic tasks and settings
Shorter time frame (minutes or days)	Lengthy time frame (weeks or months)
Primarily teacher generated and led	Primarily student generated and led

A new way to think about the problem-solving process

Figure 3

Mathematical problem-solving process



Adapted from Rigelman (2002, p. 184)

Rigelman, Nicole. (2007). Fostering mathematical thinking and problem solving: The teacher's role. Teaching Children Mathematics. 13. 308-314.

Next Steps...

- Reflect on the history of math education and consider how it has influenced teaching and learning in your classroom
- Find a balanced approach to problem solving
- Explore math resources for Problem and Project Based Learning Opportunities
- Look for across and within content connections