The Story of Problem Solving: Looking Back and Moving Forward

NCTM 100 Days of Professional Learning
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Session Overview

- Consider approaches to problem solving
- Examine 6 phases in the history of math education through the lens of problem solving
- Share current practices and resources to engage your students in a balanced approach to problem solving
- Entertain ideas about where problem solving will go in the future
What do you notice?

**Problem-Solving Strategy: Make a Table**

Make a table to solve each problem.

**Day 1**
- Day 1: 4 stickers
- Day 2: 8 stickers
- Day 3: 12 stickers
- Day 4: 16 stickers
- Day 5: 20 stickers
- Day 6: 24 stickers
- Day 7: 28 stickers

**Day 2**
- Day 1: 6 stickers
- Day 2: 12 stickers
- Day 3: 18 stickers
- Day 4: 24 stickers
- Day 5: 30 stickers
- Day 6: 36 stickers
- Day 7: 42 stickers

**Day 3**
- Day 1: 8 stickers
- Day 2: 16 stickers
- Day 3: 24 stickers
- Day 4: 32 stickers
- Day 5: 40 stickers
- Day 6: 48 stickers
- Day 7: 56 stickers

**Day 4**
- Day 1: 10 stickers
- Day 2: 20 stickers
- Day 3: 30 stickers
- Day 4: 40 stickers
- Day 5: 50 stickers
- Day 6: 60 stickers
- Day 7: 70 stickers

**Day 5**
- Day 1: 12 stickers
- Day 2: 24 stickers
- Day 3: 36 stickers
- Day 4: 48 stickers
- Day 5: 60 stickers
- Day 6: 72 stickers
- Day 7: 84 stickers

**Day 6**
- Day 1: 14 stickers
- Day 2: 28 stickers
- Day 3: 42 stickers
- Day 4: 56 stickers
- Day 5: 70 stickers
- Day 6: 84 stickers
- Day 7: 98 stickers

**Day 7**
- Day 1: 16 stickers
- Day 2: 32 stickers
- Day 3: 48 stickers
- Day 4: 64 stickers
- Day 5: 80 stickers
- Day 6: 96 stickers
- Day 7: 112 stickers

**Persevere On Your Own**

Read the problem. Write a solution on a separate sheet of paper.

**Skate Park**

Sweet T has $80 left after buying items for the team. He wants to buy at least three different items for the skate park he is making. Here are the items Sweet T is looking at, along with the prices.

- Table: $24
- Bench: $15
- L-Shaped Box: $15
- Box: $18
- Pallet: $10
- Rail: $22

What items should Sweet T buy?

**SOLVE IT**

Tell which items Sweet T should buy.

- Give the total cost.
- Explain why you chose the items you did.

**REFLECT**

Use Mathematical Practices. After you complete the task, choose one of these questions to discuss with a partner.

- Persevere: What steps did you take to get your solution?
- Use a Model: Which operations did you use to solve the problem?
Approaches to Problem Solving

● Teaching for problem solving
● Teaching about problem solving
● Teaching via problem solving
Teaching for problem solving

- Teaching a skill so that a student can later problem solve
- Often starts with learning an abstract concept and then moving to solving problems as a way to apply the learned skills
Teaching about problem solving

- Involves teaching students how to problem solve
- Often includes teaching a process (Polya’s 4 step process)
- Often includes strategies for solving a problem

**PLAN IT AND SOLVE IT**
Find a solution for the Fingerboard Parts problem.

- Make a list of the parts to buy. Include the numbers of the different parts and the colors.
- Tell why you chose the parts that you did.
- Find the total cost to buy the parts. Tell how much money is left.

You may want to use the Problem-Solving Tips to get started.

**PROBLEM-SOLVING TIPS**

- **Questions**
  - How many of each part do you need to make one fingerboard?
  - Do you want to have more of one kind of part? Why?

- **Tools**
  - You may want to use . . .
    - a table.
    - an organized list.

- **Sentence Starters**
  - I would like to have ____________________________
  - I will buy ____________________________

**PROBLEM-SOLVING CHECKLIST**
Make sure that you . . .
- tell what you know.
- tell what you need to do.
- show all your work.
- show that the solution works.
Teaching via problem solving

- Students learn math through real contexts, problems, situations, models, etc.
- The context and models allow students to build meaning for the concepts so that they can move to abstract concepts.
- Might be described as “upside-down” from teaching for problem solving
Standards for Mathematical Practice: Make sense of problems and persevere in solving them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

But how did we get to this place with problem solving in math education, and where are we going?
How did we get here and where are we going?
Looking back at 6 Identifiable Phases in Mathematics Education

1. Drill and practice
2. Meaningful arithmetic
3. New math
4. Back to basics (again)
5. Problem solving
6. Standards and accountability

Looking back... Drill and Practice

- Approx 1920-1930
- Focus on facility with computation
- Rote memorization of facts and algorithms
- Break all work into a series of small steps

Lambdin & Walcott (2007)
Looking back.... Meaningful Arithmetic

- Approx. 1930-1950
- Understanding arithmetic ideas and skills
- Application of math to real-world problems
- Emphasis placed on mathematical relationships
- Incidental learning and activity-oriented approach

Lambdin & Walcott (2007)
Looking back... New Math

- Approx 1960-1970
- Focus on understanding the structure of the discipline
- Spiral curriculum
- Discovery learning
- Bruner, Piaget, Dienes

Lambdin & Walcott (2007)
Looking back… Back to the Basics

- Approx 1970s
- Return to concern for knowledge and skill development
- Return to learning facts by drill and practice

Lambdin & Walcott (2007)
Looking back... Problem Solving

- Approx 1980s
- Focus on mathematical thinking processes and problem solving strategies (about problem solving)
- Cooperative groups
- Shift toward learning THROUGH problem solving

Lambdin & Walcott (2007)
Current phase... Standards and accountability

- NCTM 1989 Curriculum and Evaluation Standards for School Mathematics
- A new kind of math war
- Focus on math vs. focus on the learner
- National Science Foundation funded curriculum development projects to make the vision of the NCTM Standards a reality
- Student oriented, standards-based curricula vs. focus on test preparation
NCTM Process Standards & NRC Strands of Mathematical Proficiency

5 Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

Strands of Mathematical Proficiency

- Strategic competence
- Adaptive reasoning
- Conceptual understanding
- Productive disposition
- Procedural fluency


Memorizing Basics Skills versus Developing Procedural Fluency

Procedural Fluency is “skill in carrying out procedures flexibly, accurately, efficiently and appropriately”

(CCSSO, 2010, p. 6)
Current focus on rigor

The Common Core State Standards for math (CCSSM) clearly states that rigor in math includes a balance of procedural skills and fluency, conceptual understanding, and application. It also states that all three aspects should be pursued with “equal intensity.” We want students to know how (procedural skill and fluency), know why (conceptual understanding), and know when (application).

Burnett (2019) Origo Education
What is our goal?

**Problem Performers**
- Emphasis on developing students that focus on simply completing the problem
- Instruction focuses on a particular way of thinking

**Problem Solvers**
- Emphasis on developing students that are flexible and fluent mathematical thinkers
- Instruction focuses on mathematical knowledge and processes rather than specific strategies based on problem types
Effective Mathematics Teaching Practices

Implement tasks that promote reasoning and problem solving

- Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Support productive struggle in learning mathematics

- Effective teaching of mathematics consistently provide students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Strategies to enable and support problem SOLVERS

1. Introduce a simpler problem
2. Elicit prior knowledge
3. Delay the question
4. Give students the answer first
5. Challenge students to create the context


When working on the Fruit Scales problem (Lappan and Smith 2012) with problem performers, the authors present only the first two of the three pictures.
Notice and Wonder Routines
Problem-Posing

- Students observe an image, video, or graphic and they are asked “What do you notice? What do you wonder?”
- They are asked to share things they notice and the teacher records suggestions
- Repeat the process with wondering
- The wondering then leads to a mathematical question to answer
What do you notice? What do you wonder?
How much of an avocado is edible?

After seeing an array of avocados, they wonder: Does the edible fraction depend on the size of the avocado?

To answer these question, they bought almost 20 avocados of various sizes and weighed them with and without skin and pit.

Three-Act Tasks

Three parts:

1. An engaging and perplexing “Act One”
2. An information and solution seeking “Act Two”
3. A solution discussion and solution revealing “Act Three”

Resources:
- Graham Fletcher
- Robert Kaplinsky
- Dan Meyer
- John Orr
- Kyle Pearce
- Andrew Stradel
Numberless Word Problems

There were 36 kids eating pizza. 20 of them were eating cheese pizza. The rest were eating pepperoni. How many were eating pepperoni?

How could you change this to a numberless word problem?
Numberless Word Problems

What becomes the focus when you rewrite the task as a numberless work problem? What do you envision?

There were some kids were eating pizza. Some of them were eating cheese pizza. The rest were eating pepperoni. How many were eating pepperoni?
Open Middle Tasks

A specific type of task with

- a “closed beginning”
- a “closed end”
- an “open middle”

Characteristics include:

- Multiple ways of solving them
- Involve optimization
- Appear simple and procedural in nature but turn out to be more challenging and complex
- Generally not as complex as performance tasks

BENCHMARK FRACTIONS

Directions: Use the digits 1 to 9, no more than once, to create three fractions that are as close to zero, one half and one as possible. NOTE: Close as possible is measured by adding up all the differences and making it the least possible value.

https://www.openmiddle.com
Looking forward...

### Problem-Based Learning and Project-Based Learning

<table>
<thead>
<tr>
<th>Similarities</th>
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<tbody>
<tr>
<td>Focus on an open-ended question or task</td>
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<tr>
<td>Provide authentic applications of content and skills</td>
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<tr>
<td>Build inter- and intra-personal student success skills</td>
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<tr>
<td>Emphasize student independence and inquiry</td>
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<tr>
<td>Are longer, more complex and multifaceted than traditional lessons or assignments</td>
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<td>Typical Differences</td>
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<tr>
<td>----------------------------------------------------------</td>
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<tr>
<td><strong>Single content area</strong></td>
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<tr>
<td><strong>Follows specific, traditionally prescribed steps</strong></td>
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<tr>
<td><strong>Product may be tangible or proposed, a solution to the problem</strong></td>
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<tr>
<td><strong>Case studies or fictitious scenarios</strong></td>
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<tr>
<td><strong>Shorter time frame (minutes or days)</strong></td>
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<tr>
<td><strong>Primarily teacher generated and led</strong></td>
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A new way to think about the problem-solving process

Next Steps...

● Reflect on the history of math education and consider how it has influenced teaching and learning in your classroom
● Find a balanced approach to problem solving
● Explore math resources for Problem and Project Based Learning Opportunities
● Look for across and within content connections